Consistent Engineering Units
In Finite Element Analysis

New to FEA and don't want to read the full document? Here's the short summary if you need to work with a "units-free" FEA system. A units conversion table starts on page 4.

- What length units are you using? If you have imported geometry from elsewhere, then measure some distances to figure out what length units were used.
- Choose your preferred unit of force.
- Derive ALL other physical units from your chosen Force and Length units (assumes you will be using seconds as the standard unit of time!).
- For example, Length = mm, Force = N, then Pressure = Stress = Youngs Mod = N/mm^2 = MPa. F=ma, thus mass MUST be Tonnes because acceleration = mm/sec^2. Thus density MUST be $Tonnes/mm^3$, eg. $7.8x10^{-9}$ for a typical steel.
- Similarly, if Length = inch, Force = pounds, then Pressure = Stress = Youngs Mod = lbs/in^2 = psi. F=ma, thus mass MUST be slug-inch (this mass weighs ~386lbf on earth). Thus density MUST be $slug\text{-}inch/in^3$, eg. $7.3x10^{-4}$ for a typical steel.

Depending on the modelling software you use for your Finite Element Analysis (FEA), your modelling process often requires an understanding of consistent units ("coherent units").

Although some FEA modellers may provide a convenient method of declaring which unit you are applying at any instant, many FEA modellers and most FEA solvers are "units-free".

Therefore, it is critical to understand consistent units if you are:

- transferring a model or solver file to/from somewhere else;
- reviewing someone else's FEA work; or
- using any FEA modeller which leaves the units up to you.

When the units are left to the user, "consistent units" must be used.

SI Units

The easiest set of consistent units is the SI system of metres (meters), kilograms and seconds.

These derive other units that are commonly used by designers/engineers/analysts, such as Newtons, Joules, Watts, Pascals, and used to quantify measures such as density, acceleration, velocity, viscosity, specific heat and many more.

Using consistent units (coherent units) is the necessity to stick with units that work correctly together - not to mix units *or scale factors* which aren't properly related to each other.

For example, if using the basic SI units of metres, kilograms and seconds, then:

- **Density** *must* be in kg/m^3;

- **Acceleration** *must* be in metres/sec^2;
- **Force** *must* be in Newtons, because a Newton is the force it takes to accelerate 1kg at the rate of 1m/sec^2. (Force = mass x acceleration), and
- **Power** *must* be in Watts = (Newtons x metres) per second.

Some engineering organisations instruct their engineers to use basic SI units so there are no questions about unit conversion or consistent units. However, this has some practical downsides - and the use of millimetres by CAD designers is popular.
(*** Also, see foot note at the end of the doc for Nastran users using default 8 field format)

If you use millimetres, and prefer using Newtons for force, then this has specific consequences for all other units to preserve consistency:

- Applied **Pressures** are Newtons / mm^2, which is MegaPascals (MPa)
- **Stress results** are thus also MPa.
- **Young's Modulus** is also in units of pressure, and is thus also MPa.

What is less obvious is that in choosing millimetres and Newtons, you have also forced a change in your mass units. This is because F=ma, and so the question which needs to be answered is:

"What mass accelerates at 1mm/sec^2 when a force of 1N is applied?"
1mm/sec^2 is very slow (1000 times less than 1m/sec^2, of course) - *the answer is 1 Tonne*.

***Therefore,* density is in Tonnes per cubic mm** – eg. 1e-9 for water and 7.8e-9 for a typical steel.

So, if you are running any gravity analysis (having chosen millimetres and Newtons), then 1g is 9810 (mm/sec^2), point masses are in Tonnes and density is in Tonnes / mm^3. **Natural frequency analyses will be wrong if mass or density is inconsistent, despite no gravity loads being applied - so it is *unwise* to rely on inconsistent kg/mm^3 cancelling inconsistent acceleration in m/sec^2!**

Additional consequences of choosing millimetres and Newtons occur for heat transfer:

- **Conductivity** in standard SI: Watts per (metre x Kelvin)
  - Conductivity in your consistent units: milliWatts per (millimetre x Kelvin). Conveniently, 1W/m.K = 1mW/mm.K
- **Heat Flux** in Standard SI: Watts per square metre
  - Heat Flux in your consistent units: milliWatts per square millimetre. Therefore numeric input would be reduced by a factor of 1000 compared to Standard SI. Eg. 13W/m^2 = 0.013mW/mm^2.
- The same conversion is required for a **convection coefficient** - ie. W/m^2.K becomes mW/mm^2.K so numeric input also reduces by a factor of 1000.
- **Specific Heat** in Standard SI: Joules per (kilogram x Kelvin)
  - Specific Heat in your consistent units: milliJoules per (Tonne x Kelvin). Therefore numeric input would be increased by a factor of 1e6 compared to Standard SI. eg. 4180 [J/kg.K] becomes 4.18e9 [mJ/T.K]

Note that analysis results such as heat flux will obviously be presented in the same consistent units you have chosen to apply.

Theorists correctly note that thermal units for heat transfer (or any "separate" physics) can be independently self-consistent from the structural units - but if the analysis later needs to become fully-coupled mech / fluids / thermal / etc, then 100% consistency across the disciplines is compulsory, and may as well have started that way.

Imperial Units

For those who use Imperial Units, the same necessity to use a consistent set of units still applies. The relationships are less tidy, because these measures were invented before real science. If doing any *coupled mechanical / thermal / fluids analysis*, then it is best to avoid using Imperial Units, as material properties are unlikely to be listed in any consistent units measure.

For engineering, a consistent set of Imperial Units is slug (mass), foot (length) and seconds (time).

Note that a pound is not a unit of mass *for engineers or scientists* and should never be used as a unit of mass. Science required the "invention" of a slug as a real mass unit in the Imperial system.

A pound is a unit of force in engineering and FEA.

1 pound is the unit of force which accelerates 1 slug at 1 foot/sec^2. A mass of one slug *weighs* about 32 *pounds force* on earth (1g = 32.19 ft/sec^2 and F = ma).

Again, the use of feet for length is often inconvenient, so inches are common for modelling - which thus means some conversions are required to preserve a consistent set of units.

The obvious consequence of using inches is that applied pressures, stress results and Youngs Modulus are all in psi (pounds per square inch). Many sources also supply material constants in these units.

Not so obvious, if using pounds force and inches, is that mass is now in units of "12 x slugs" = "slug-inch". Although some have named this mass unit the "slinch" or "blob", *we prefer the colloquial name*, *the "snail".* (Thanks to Dr Tim Coates for supplying this useful name for the unit - we are unsure as to whether the term has earlier origins).
Thus density must be in units of snails per cubic inch, which for a typical steel would be 7.3e^{-4}. A snail weighs about 386 pounds force on earth (it is a large snail).

If doing thermal analysis using Imperial Units (in inches), then consistent units would be:

- inch pounds for **Work**
- inch pounds per second for **Power**
- (inch pounds per second) per square inch for **Heat Flux**
- inch pounds per (snail x degree) for **Specific Heat**.

Most sources do not supply material constants / coeffs / equations in these units (even densities are often misquoted as weight densities), so substantial conversion and care is required.

Below is a table of some typical mechanical and thermal units with their equivalent values in four columns of commonly used consistent unit sets for FEA. Note that it is possible to use "hybrid" combinations of consistent units, where certain properties (eg. energy, power, temperature) may use different self-consistent units values compared to the remainder of the units set. This requires extreme caution if the physics is coupled and our conservative recommendation is never to use such an approach.

Note that due to some minor variations in the definitions of some values (such as g, BTU and derived properties), other sources may show a difference of a few units variation in the 4th significant figure for some of the values in the table. For the approximations associated with FEA, such variations are insignificant.

	Each column represents a typical consistent unit set for FEA. The length units of your geometry dictate which single consistent column set to use.			
Property/value	**SI, Standard: N,m,kg,s,^{o}K**	**SI, mm: N,mm,T,s,^{o}K**	**Imperial, inches: Lbf, in, snail,s,^{o}F Snail = slug-inch =386.2 "lb mass"**	**Imperial, feet: Lbf,ft,slug,s,^{o}F Slug = 32.19 "lb mass"**
1 pound (Force)	4.449 Newton	4.449 N	1.000 lbf	1.000 lbf
1 Newton (Force)	1.000 N	1.000 N	0.2248 lbf	0.2248 lbf
1 kg (Mass)	1.000 kilogram	1.000e^{-3} Tonne	5.709e^{-3} snails	6.851e^{-2} slugs
1 slug (Mass)	14.60 kg	1.460e^{-2} T	8.333e^{-2} snails	1.000 slug
1 snail (Mass)	175.2 kg	0.1752 T	1.000 snail	12.00 slugs
1 "pound mass" lbm	0.4535 kg	4.535e-^{4} T	2.589e^{-3} snails	3.107e^{-2} slugs
1 metre (Length)	1.000 m	1000 mm	39.37 in	3.281 ft
1 inch (Length)	0.0254 m	25.4 mm	1.000 in	8.333e^{-2} ft
1 foot (Length)	0.3048 m	304.8 mm	12.00 in	1.000 ft
1 Pascal (Pressure)	1 N/m^2	1.000e^{-6} MPa	1.450e^{-4} psi	2.088e^{-2} lbf/ft^2
1 psi (Pressure)	6896 Pa	6.896e^{-3} MPa	1.000 psi	144.0 lbf/ft^2
1 MPa (Pressure)	1.000e^{6} Pa	1.000 MPa	145.0 psi	20885 lbf/ft^2
1 g (Acceleration)	9.810 m/sec^2	9810 mm/sec^2	386.2 in/sec^2	32.19 ft/sec^2
1 m/sec^2 (Accel.)	1.000 m/sec^2	1000 mm/sec^2	39.37 in/sec^2	3.281 ft/sec^2
1 ft/sec^2 (Acceleration)	0.3048 m/sec^2	304.8 mm/sec^2	12.00 in/sec^2	1.000 ft/sec^2
1kg/m^3 (Density)	1.000 kg/m^3	1.000e^{-12} T/mm^3	9.356e^{-8} snails/in^3	1.940e^{-3} slugs/ft^3
1 lbm/ft^3 (Density)	16.02 kg/m^3	1.602e^{-11} T/mm^3	1.498e^{-6} snails/in^3	3.107e^{-2} slugs/ft^3
1 lbm/in^3 (Density)	2.768e^{4} kg/m^3	2.768e^{-8} T/mm^3	2.589e^{-3} snails/in^3	53.69 slugs/ft^3
1 Joule (Energy)	1.000 N.m ie. Joule	1000 N.mm ie. mJ	8.849 in.lbf	0.7374 ft.lbf
1 BTU (Energy)	1,055 Joules	1.055e^{6} mJ	9334 in.lbf	777.8 ft.lbf
1 Calorie (Energy)	4.186 Joules	4186 mJ	37.04 in.lbf	3.086 ft.lbf
1 Watt (Power)	1.000 N.m/sec ie. Watt	1000 N.mm/sec ie. mW	8.849 in.lbf/sec	0.7374 ft.lbf/sec
1 BTU/hr (Power)	0.2930 Watts	293.0 mW	2.593 in.lbf/sec	0.2161 ft.lbf/sec
1 BTU/sec (Power)	1055 Watts	1.055e^{6} mW	9334 in.lbf/sec	777.8 ft.lbf/sec
Continued next page				

1 HP (Power)	745.8 Watts	$7.458e^{5}$ mW	6600 in.lbf/sec	550.0 ft.lbf/sec
1 Calorie/sec (Power)	4.186 Watts	4186 mW	37.04 in.lbf/sec	3.086 ft.lbf/sec
1 Watt/m^2 (Heat Flux)	1.000 W/m^2	$1.000e^{-3}$ mW/mm^2	$5.709e^{-3}$ in.lbf/sec.in^2	$6.851e^{-2}$ ft.lbf/sec.ft^2
1 BTU/hr/ft^2 (Heat Flux	3.155 W/m^2	$3.155e^{-3}$ mW/mm^2	$1.801e^{-2}$ in.lbf/sec.in^2	0.2161 ft.lbf/sec.ft^2
1 HP/ft^2 (Heat Flux)	8028 W/m^2	8.028 mW/mm^2	45.83 in.lbf/sec.in^2	550.0 ft.lbf/sec.ft^2
1 calorie/sec/in^2 (Heat Flux)	6488 W/m^2	6.488 mW/mm^2	37.04 in.lbf/sec.in^2	444.5 ft.lbf/sec.ft^2

Temperature given in
Kelvin (#K)
Celsius (#C)
Fahrenheit (#F)
	K = #F x 5/9 + 255.37
	K = #C x 1 + 273.15
	C = #F x 5/9 - 17.778
	C = #K x 1 - 273.15
			F = #K x 1.8 - 459.67
			F = #C x 1.8 + 32	

1 J/kg.K (Specific Heat)	1.000 N.m/kg.K	$1.000e^{6}$ N.mm/T.K	861.1 in.lbf/snail.^{o}F	5.980 ft.lbf/slug.^{o}F
1 BTU/lbm.F (Specific Heat)	4186 J/kg.K	$4.186e^{9}$ mJ/T.K	$3.605e^{6}$ in.lbf/snail.^{o}F	$2.503e^{4}$ ft.lbf/slug.^{o}F
1 kcal/lbm.F (Specific Heat)	$1.662e^{4}$ J/kg.K	$1.662e^{10}$ mJ/T.K	$14.31e^{6}$ in.lbf/snail.^{o}F	$9.936e^{4}$ ft.lbf/slug.^{o}F
1 W/m.K (Conductivity)	1.000 W/m.K	1.000 mW/mm.K	0.1249 in.lbf/sec.in.^{o}F	0.1249 ft.lbf/sec.ft.^{o}F
1 BTU/hr.ft.^{o}F (Conductivity)	1.730 W/m.K	1.730 mW/mm.K	0.2161 in.lbf/sec.in.^{o}F	0.2161 ft.lbf/sec.ft.^{o}F
1 W/m^2.K (Convection)	1.000 W/m^2.K	$1.000e^{-3}$ mW/mm^2.K	$3.172e^{-3}$ in.lbf/sec.in^2.^{o}F	$3.806e^{-2}$ ft.lbf/sec.ft^2.^{o}F
1 BTU/hr.ft^2.^{o}F (Convection)	5.678 W/m^2.K	$5.678e^{-3}$ mW/mm^2.K	$1.801e^{-2}$ in.lbf/sec.in^2.^{o}F	0.2161 ft.lbf/sec.ft^2.^{o}F
1 kcal/hr.ft^2.^{o}F (Convection)	22.53 W/m^2.K	$2.253e^{-2}$ mW/mm^2.K	$7.146e^{-2}$ in.lbf/sec.in^2.^{o}F	0.8576 ft.lbf/sec.ft^2.^{o}F

A further set of metric consistent units could be milliNewton, mm, kg, kPa, kg/mm^3, mm/sec^2, microJoule, microWatt, microWatt/mm^2 (heat flux), microJoule/kg.K (specific heat). This choice gives the "advantage" of using kg instead of Tonnes for mass and density.

Preferences such as kN and mm (common in civil/structural environments) can be chosen, but their derived consistent unit set includes GPa (kN/mm^2), mm/sec^2, kiloTonne (1kN=1kT x 1mm/sec^2), kiloTonne/mm^3, Joule (kN.mm), Watt (kN.mm/sec), Watt/mm^2 and Joule/kiloTonne.K

Our recommendations:

1. ALWAYS use consistent units. That means the base units of Mass, Length, Time, Temp must comply with / derive all associated units you are using in your analysis (such as pressure, force, velocity, density, acceleration, power, convection, viscosity etc.). Then you won't get caught out when a static gravity analysis works perfectly using mm, N, MPa, ~~kg/mm~~^{3} and ~~m/sec~~^{2}, but is instantly wrong as soon as you run a simple natural frequency analysis, or various other more advanced analyses. N, mm, sec, MPa, T/mm^3 and mm/sec^2, mJ, mW are one example set which could always be used for analysis because these units are consistent.

2. Use metric if you can - the availability of material data and constants in workable imperial units (particularly in thermal and fluids analysis) typically requires arduous conversion to maintain a consistent set of units for analysis. However, units conversion of any sort is best minimised, to avoid becoming responsible for losing the next Mars Climate Orbiter!

3. PARAM, WTMASS is available in NASTRAN so analysts can use force units as masses (eg. non-structural mass in pounds per square foot). We believe this practice should be rigorously avoided (except by analysts of the highest expertise), and that proper consistent units are always used. It is much easier to convert imperial inputs to real mass units (such as slugs and slug-inches, rather than non-mass units such as pounds) than to test the full scope of all NASTRAN analysis types for which WTMASS correctly operates.

These guidelines have been assembled by and for Nastran users; however, the information is equally applicable to users of Femap, Ansys, Abaqus, MSC.Software, MSC Marc, Patran, Ideas, Strand, Algor, Cosmos, CosmosWorks and many others.

EnDuraSim has decades of hands on experience in FEA, particularly using Femap and Nastran.
We provide general training in FEA or specific training for Femap and NX Nastran.
We provide quality FEA consulting, typically fixed price contract for defined scope of work.
We provide yearly unlimited (within reason) technical support for *Femap with NX Nastran*.
We provide Femap API programming products and services to enhance analyst productivity.
Contact us for further information.

Footnote regarding precision and the Nastran 8 character field format.
***There are some particular problems to be avoided when using metres with the very common Nastran "small field" format. With only 8 characters wide per field, a location at -1.0154mm will be written as -1.015-3 (metres) which is effectively only 4 significant figures. For models with fine details or where precise contact is involved, precision to 0.001 mm may not be enough. In extreme cases, closely spaced nodes can be written out at the same coordinate location, which will typically produce NOGO fatal error messages due to element quality. The issue also remains relevant for FEA models in mm units, but is less likely to be exposed for common structure sized models.

So, if you are using metres and Nastran (or any other solver which uses text analysis files), we recommend you use large field (double precision) format if it is not the default in your modelling system. We also recommend using large field format if you are modelling small details which are "distant from the model origin", or involve precision contact (both regardless of units) to achieve sufficient significant figures.