Computation of T-stress by extrapolation of displacement field

Abstract:

The computation of T-stress is implemented for linear, homogeneous and elastic-isotropic materials with plane crack mesh (FEM only). The accuracy of T-stress results obtained by extrapolation of the displacement field method is clearly improved if the meshing elements are quadratic. It is also highly recommended to use “Barsoum” type elements in the crack tip (elements where the mid-side nodes are located at the quarter of edges).

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1 Theory

Based on isotropic-linear-elasticity theory, when an elastic cracked body is subjected to external forces (see Fig.1), the stress field in the vicinity of a crack tip can be expressed by the Williams' expansion. The stresses near the crack-tip can be written according to equations 1-2, where σ_{ij} is the stress tensor, r represents the distance of element from crack tip, and θ is the angle of element with respect to a polar axes located at the crack tip [1].

$$\sigma_{rr}=\frac{K_1}{\sqrt{(2\pi r)}}\left(\frac{5}{4}\cos\left(\frac{\theta}{2}\right)-\frac{1}{4}\cos\left(\frac{3\theta}{2}\right)\right)+\frac{K_2}{\sqrt{(2\pi r)}}\left(\frac{-5}{4}\sin\left(\frac{\theta}{2}\right)+\frac{3}{4}\sin\left(\frac{3\theta}{2}\right)\right)+T\cos^2(\theta)$$

Equation 1

$$\sigma_{\theta\theta}=\frac{K_1}{\sqrt{(2\pi r)}}\left(\frac{3}{4}\cos\left(\frac{\theta}{2}\right)+\frac{1}{4}\cos\left(\frac{3\theta}{2}\right)\right)+\frac{K_2}{\sqrt{(2\pi r)}}\left(\frac{-3}{4}\sin\left(\frac{\theta}{2}\right)+\frac{3}{4}\sin\left(\frac{3\theta}{2}\right)\right)+T\sin^2(\theta)$$

Equation 2

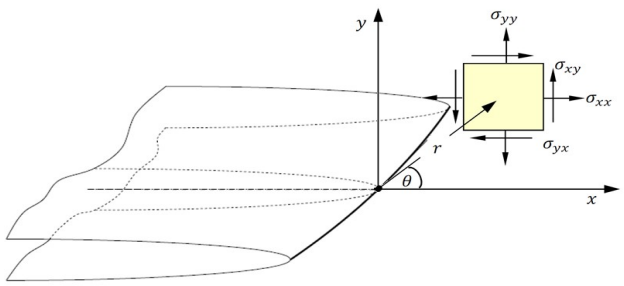


Fig.1 Stress in a polar coordinate system ahead of a crack in an infinite plate

The non-singular term is known as T-stress, and characterises the crack tip constraint; this is particularly useful in the assessment of the integrity of structures containing defects. A robust method for obtaining T-stress is via the displacement field along the crack faces. This method is based on the asymptotic development of the displacement field in the crack tip. For $\theta=-\pi$ or $\theta=+\pi$ the singular term of σ_{rr} vanished and hence near the crack tip, T-stress can be represented by :

$$T=E'\frac{[U_{\theta=\pi}(r)]-[U_{\theta=\pi}(0)]}{r}$$

where $[U]$ is the displacement step between the crack lips and hence $[U_{\theta=\pi}(0)]$ denotes displacement of crack tip; details of the method are given elsewhere [2]. E' is defined for plane stress and plane strain conditions as:

$$E'=\begin{cases}E & Plane\ stress\\ \dfrac{E}{1-\upsilon^2} & Plane\ strain\end{cases}$$

Note:

- *It is also possible to compute T-stress according to the stress fields, but the values of the vector forced on the lips of crack are less accurate than displacements (resulting from a stress singularity in crack tip).*
- *The user must take care to refine the mesh in the vicinity of crack tip so that enough points are available for extrapolation.*

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- *The method used here is theoretically less accurate than the integration integral method for computation of T-stress since the integral is taken over a domain of elements surrounding the crack and errors in local solution parameters have less effect on the evaluated quantities. However, i n this method no spec ia l crack tip elements are required and the determination of node sets for the calculation of T-stress is simpler than in the J-integral approach. Thus it makes possible to obtain easily relatively reliable values of T-stress for any type of mesh strategy. The comparison of the different methods of calculating is always useful to estimate the accuracy of the results obtained.*

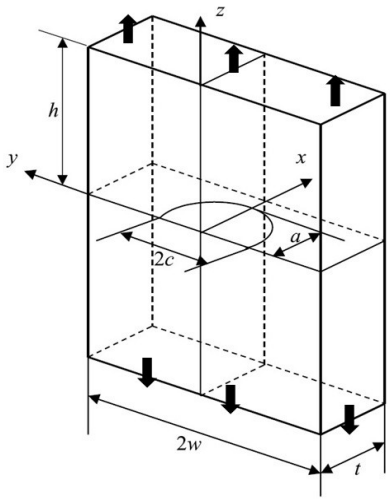
2 Implementation of extrapolation method

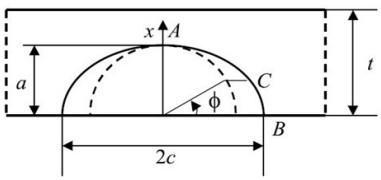
The extrapolation displacement method is implemented in the POST_T_Q operator which starts by computing the displacement field on the global structure. Since the definition of T-stress are only asymptotically true; extrapolation is restricted to the vicinity of the crack tip limited by a maximum distance $dmax$ around the crack. $dmax$ is a parameter of the ABSC_CURV_MAXI operator. In the case of a meshed crack ABSC_CURV_MAXI is optional. If $dmax$ is not defined in the .comm file, by default it is automatically assigned in POST_T_Q operator as equal to three times the maximum size of the mesh elements connected to the nodes on the crack front. The general principle of T-stress computation is defined as follows:

1. Loop on the crack tip nodes
2. Extract the crack tip displacement,
3. Define the plane normal to crack and the crack tip,
4. Project the displacement field in $\theta=+\pi$ and $\theta=-\pi$,
5. Calculate the difference of the displacement value,
6. The displacement difference is divided by r with material multiplicative factor will obtaine and then extrapolate into $r=0$. If the solution is perfect, one should obtain a line. Actually, we obtain almost a line with a mesh of the type “Barsoum”.

3 Accuracy of suggested method

The extrapolation displacement method was validated on the test case (SSS ?) , for which analytical solutions are known [3]. This test case, as detailed in (T1.odt), considers an elliptic crack in a three-dimensional body subject to a tensile load. A cube by size of $(h=w=t=16)$, containing a horizontal circular crack of radius $c=a=1$ is considered (see Fig.2). The value of far-field stress, σ , was prescribes as 1 MPa. Due to symmetry, only one quarter of the elliptical crack is considered. The crack is meshed with fine quadratic mesh type in the vicinity of crack tip. The results are compared with reference solution (see Fig.3).





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Fig.2 Geometric configuration of the testcase shown in half symmetry [3]

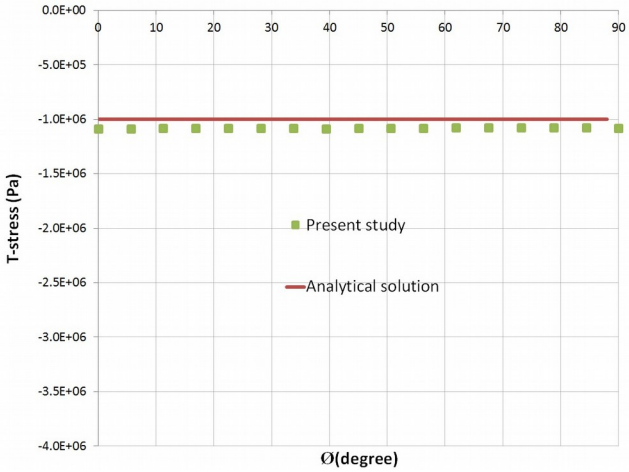


Fig.3 T-stress compression

4 Conclusion

The results obtained with extrapolation displacement method are in a good agreement, with less than 5% error compared to the analytical solution of T-stress. It should be noticed that the asymptotic relation of displacements is valid only for r tending towards 0. Therefore, it is necessary to take care not to choose too large a domain of extrapolation (distance $dmax$ from operator POST_T_Q of about 4 to 5 elements).

5 Bibliography

1 L. Novotný, Calculation of T– stress on 3D specimens with crack. Procedia Engineering. 48, 489 – 494. 2012,

2 S.A. Zahedi, A. Jivkov, Two-parameter fracture characterization of a welded pipe in the presence of residual stresses – Procedia Structural Integrity. 2, 777-784, 2016.

3 X. Wang, Elastic T-stress solutions for semi-elliptical surface cracks in finite thickness plates. Engineering Fracture Mechanics 70, 731–756, 2003.

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6 Description of the document versions

Index	Version Aster	Auteur(s) or contributeur(s), organisme	Description of the modifications
A	13.4	A. Zahedi EDF Energy UKC	Initial document

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