# SZLZ105 - Counting of cycles by RAINFLOW and calculation of the damage

**Summary:**

Transitory linear elastic problem quasi-static in mechanics of the structures.

Calculation of the final damage in an element subjected to a cyclic loading, with a linear elastic behavior.

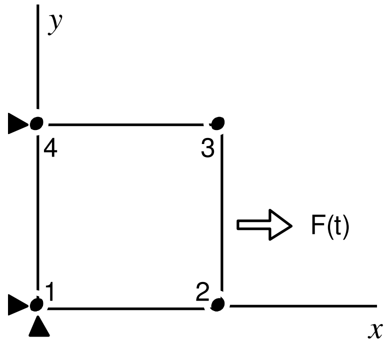
A modeling in plane constraints and a modeling in 3D.

This test validates the method of counting of cycles (RAINFLOW) established in the operator CALC_FATIGUE as well as the method of calculating of the damage in imposed constraint (curve of Wöhler) or imposed deformation (curve of Manson-Whetstone sheath). The reference solution is an analytical solution.

It also validates the calculation of the constraints and deformations equivalent using the options SIEQ_ELGA, SIEQ_ELNO, EPEQ_ELGA, EPEQ_ELNO, EPMQ_ELGA and EPMQ_ELNO.

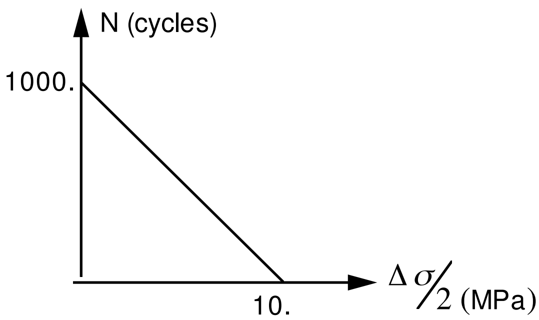
# 1 Problem of reference

## 1.1 Geometry

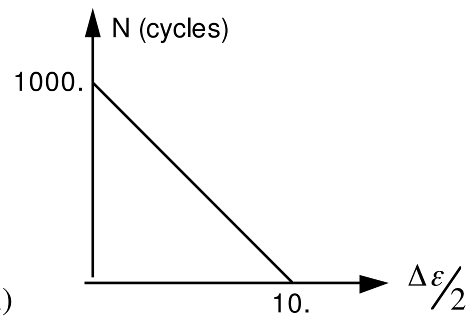



## 1.2 Material properties

Linear elasticity: $E=1.MPa$ $\nu=0.3$



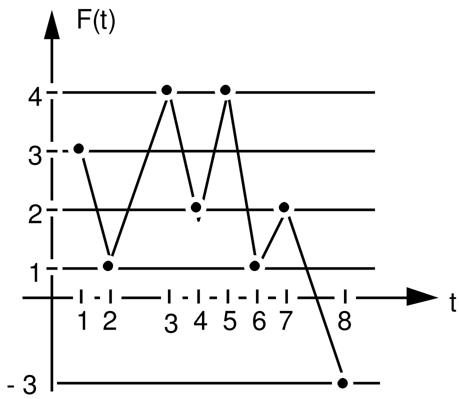

Courbe de Wöhler




Courbe de Manson-Coffin

## 1.3 Boundary conditions and loadings

- Blocked on face $1-4$ according to $X$ - node $1$ blocked according to $Y$.
- In unit simple traction on the face $2-3$.
- Loading $F(t)$ in teeth of saw (according to the Article of Downing and Socie 1982) [bib1].




## 1.4 Initial conditions

Worthless constraints and deformations.

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

Analytical solution

- calculation of the constraints and deformations. For a loading in simple traction, one obtains a homogeneous state of stress uniaxial in any point:

$$\sigma=\begin{bmatrix}\sigma & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \text{ and } \varepsilon=\begin{bmatrix}\varepsilon & 0 & 0\\ 0 & \gamma & 0\\ 0 & 0 & \gamma\end{bmatrix}$$

the equivalent sizes are thus $\begin{cases}\sigma_{VMIS}=|\sigma|=\sigma_{TRESCA}\\ \sigma_{VMIS-SG}=\sigma\end{cases}$

and $\begin{cases}\varepsilon_{INVA-2}=\dfrac{2}{3}|\varepsilon-\gamma|\\ \varepsilon_{INVA-2SG}=\dfrac{2}{3}|\varepsilon-\gamma|*sign\left[\dfrac{\varepsilon+2\gamma}{3}\right]\end{cases}$

- then manual calculation of the cycles by the method of RAINFLOW, as well as amplitudes of loading ( $\frac{\Delta\sigma}{2}$ or $\frac{\Delta\varepsilon}{2}$ ).

| cycles | $\Delta\sigma/2$ | $\Delta\varepsilon_{INVA-2}/2$ |
|---|---|---|
| 1 | 1. | 0.8667 |
| 2 | 0.5 | 0.433315 |
| 3 | 1. | 0.8667 |
| 4 | 3.5 | 3.03335 |

- finally carryforward of these values on the curves of Wöhler or Manson-Whetstone sheath to consider the damage unit at each cycle $i$, that is to say $Du_i=\frac{1}{N_i}$ ( $N_i$ being the number of cycles with rupture for a given amplitude), as well as the cumulated damage $D=\sum_i Du_i$ (linear rule of office plurality TO MINE).

**Note:**

> *One will use as equivalent constraint* $\sigma_{VMIS-SG}$ *and like equivalent deformation* $\varepsilon_{INVA-2SG}=\frac{2}{3}|\varepsilon-\gamma|\times sign\left[\frac{\varepsilon+2\gamma}{3}\right]$.

## 2.2 Results of reference

- Being given the values of the parameters of loading used, one obtains simply at the end of the loading (increment 8) $\sigma=-3.$ $\varepsilon=-3.$ $\gamma=0.9$ $\varepsilon_{INVA-2}=2.6$ .
- For the calculation of the damage, one obtains:

$$D_{Wöhler}=4{,}8133\,.\,10^{-3}=\sum_{i=1}^{4} Du_i$$

$$D_{Manson}=4{,}67\,.\,10^{-3}=\sum_{i=1}^{4} Du_i$$

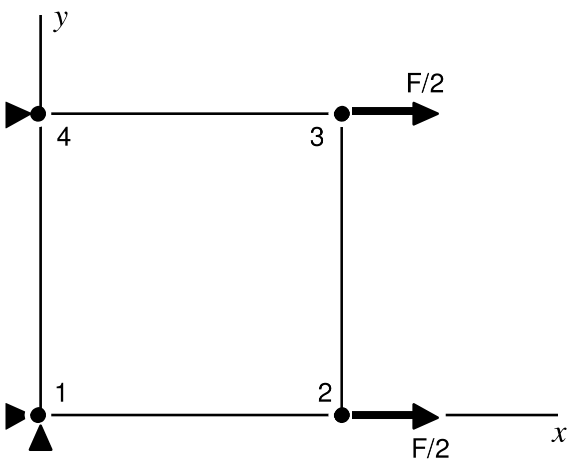
## 2.3 Bibliographical references

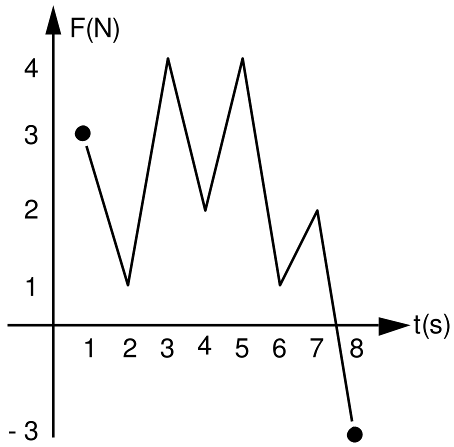
1. DOWNING and SOCIE, 1982. “Simple Rainflow counting algorithms”. Int. J. Tiredness, January 1982 (p. 31).

# 3 Modeling A

## 3.1 Characteristics of modeling

**Modeling in plane constraints:**




## 3.2 Characteristics of the grid

1 mesh QUAD4.

Square width = 1
thickness = 1

## 3.3 Sizes tested and results

| Identification | Reference |
|---|---|
| in all nodes at the end of the loading in constraint or deformation | |
| Wöhler damage | $4.8133\ 10^{-3}$ |
| Damage Manson-Whetstone sheath | $4.6705\ 10^{-3}$ |
| $\sigma$ | -3. |
| $\sigma_{VMIS}$ | 3. |
| $\sigma_{TRESCA}$ | 3. |
| $\sigma_{VMIS-SG}$ | -3. |
| $\varepsilon$ | -3. |
| $\gamma$ | 0.9 |
| $\varepsilon_{INVA-2}$ | 2.6 |
| $\varepsilon_{INVA-2}SG$ | -2.6 |

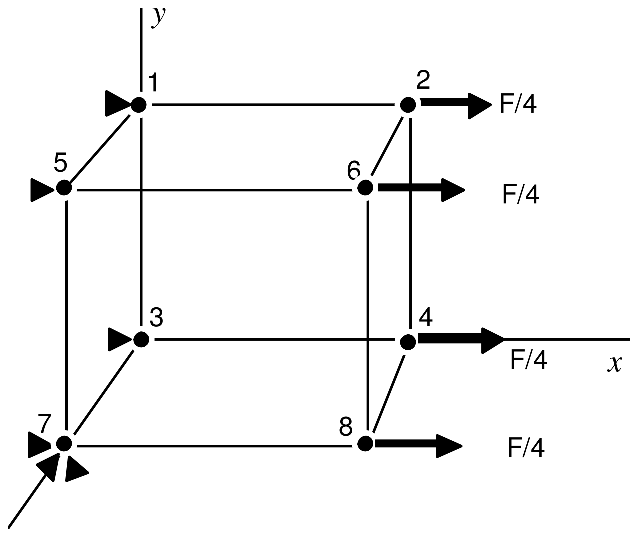
## 3.4 Remarks

Fast test in time calculation.

# 4 Modeling B

## 4.1 Characteristics of modeling

Modeling in `3D` :




## 4.2 Characteristics of the grid

1 mesh `HEXA8`.
cubic of width = 1

## 4.3 Sizes tested and results

| Identification | Reference |
|---|---|
| in all nodes at the end of the loading | |
| Wöhler damage | $4.8133\ 10^{-3}$ |
| Damage Manson-Whetstone sheath | $4.6705\ 10^{-3}$ |
| $\sigma$ | -3. |
| $\sigma_{VMIS}$ | 3. |
| $\sigma_{TRESCA}$ | 3. |
| $\sigma_{VMIS-SG}$ | -3. |
| $\varepsilon$ | -3. |
| $\gamma$ | 0.9 |
| $\varepsilon_{INVA-2}$ | 2.6 |
| $\varepsilon_{INVA-2} SG$ | -2.6 |
| $(\varepsilon-\varepsilon^{th})_{INVA-2}$ | 2.6 |
| $(\varepsilon-\varepsilon^{th})_{INVA-2} SG$ | -2.6 |

## 4.4 Remarks

Same results and reference that in plane constraints.

# 5 Summary of the results

This test validates the method and the calculation of the damage of Wöhler and Manson-Whetstone sheath.

Results of *Code_Aster* are identical to those obtained analytically.