# ADLV301 – Problem of coupling cavity-plate

**Summary:**

The objective of this CAS-test is to calculate the fields of pressure in a cavity of which one of the walls is flexible, the other walls being perfectly rigid. The excitation is mechanical and consists of a specific normal force applied to the flexible wall.

The vibroacoustic finite elements are used (modeling a: `3D_FLUIDE` and `FLUI_STRU`)

The boundary conditions are purely mechanical. They make it possible to create a deformation on the flexible wall, deformation which is with the source of the sound field in the cavity. The transmission between the plate and the fields of cavity is managed by elements of interface “fluid-structure”, suitable for modeling by vibroacoustic finite elements.

Dimensions of the cavity are sufficiently large and the thickness of the plate is low in order to observe an effect of coupling significant. Moreover, the excitation is eccentric in order to create a field of displacement on the plate and a sound field in the cavity which are three-dimensional.

An exact analytical solution exists. This CAS-test thus makes it possible to validate the taking into account of the coupling fluid-structure. In addition, this CAS-test validates the acoustic calculation of pressure in the vibroacoustic finite elements.

# 1 Problem of reference

## 1.1 Geometry

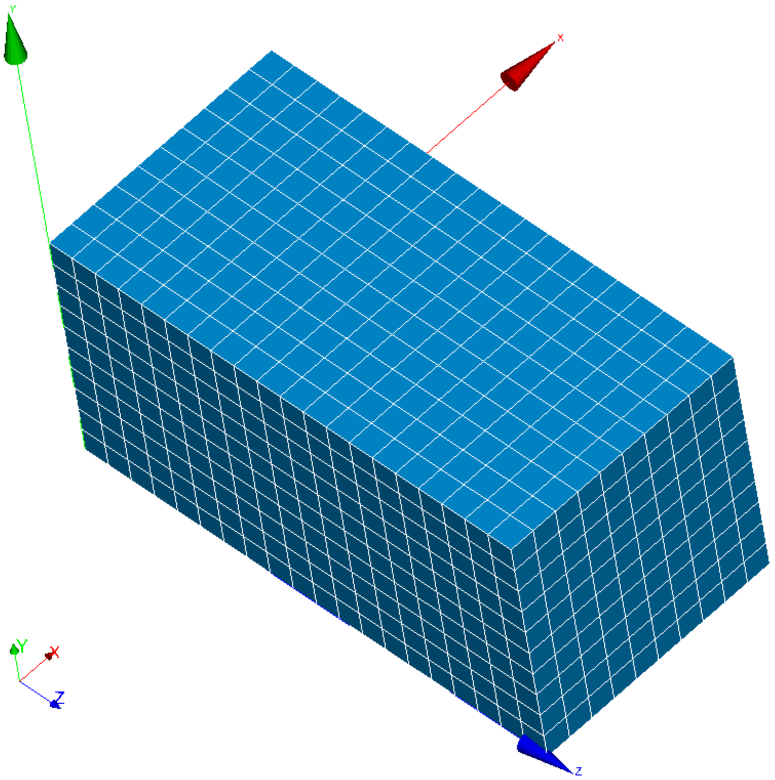


**Figure 1.1 Geometry of the problem and system of loading**

Parallelipipedic cavity:

- length: $l_x=1.0\,m$
- height: $l_y=1.0\,m$
- width: $l_z=2.0\,m$

Coordinates of the points:

- $N1$ : $x=0.6$ , $y=0.4$ , $z=0.0$
- $N2$ : $x=0.5$ , $y=0.5$ , $z=1.0$
- $N3$ : $x=0.5$ , $y=0.5$ , $z=2.0$

## 1.2 Properties of material

The material properties of the fluid are those of the air:

| | |
|---|---|
| $c_F=340\,\mathrm{m.s}^{-1}$ | Speed of sound |
| $\rho_F=1.2\,\mathrm{kg.m}^{-3}$ | Density |

The material properties of the plate are those of steel:

| | |
|---|---|
| $E=2.1\times10^{11}\,\mathrm{Pa}$ | Young modulus |
| $\nu=0.3$ | Poisson's ratio |
| $\rho_S=7800.0\,\mathrm{kg.m^{-3}}$ | Density |
| $t=0.005\,\mathrm{m}$ | Thickness |

## 1.3 Boundary conditions and loadings

Imposed displacement:

| | |
|---|---|
| Wall of the plate | $DX=0$ , $DY=0$ , $DZ=0$ |

Imposed loading :

| | |
|---|---|
| Not $N1$ | $F_z=1\,\mathrm{N}$ |

# 2 Reference solution

## 2.1 Method of calculating used for the reference solution

The reference solution for the problem of the modal representation of the plate is given in [1].

The reference solution for the problem of the modal representation of the cavity is given in [2].

The integration of these two models and the taking into account of their mutual interaction lead to a coupled modal solution whose equations are described hereafter (see also [3]).

The problem consists of the evaluation of the dynamic response of an elastic rectangular plate coupled to an acoustic cavity parallelepipedic, and subjected to a transverse specific force. The geometry of the system considered is represented in the preceding page.

The plate has a thickness $t$ and consists of a material characterized by constant usual mechanics: Young modulus $E$, structural factor of loss $\eta_S$, Poisson's ratio $\nu$ and density $\rho_S$. The fluid is characterized by a density $\rho_F$, a speed of sound $c_F$ and a factor of loss $\eta_F$.

The specific force (amplitude $F$) is applied to the position $(x_F, y_F)$.

The own pulsations and the clean modes of the structure are calculated by the following expressions ( $m, n=1,2,3,...$ ):

$$\omega_{mn}^S=\left(\frac{D}{\rho_S t}\right)^{1/2}\left(\left(\frac{m\pi}{L_x}\right)^2+\left(\frac{n\pi}{L_y}\right)^2\right) \text{ with } D=\frac{E t^3}{12(1-\nu^2)}$$

$$\Phi_{mn}^S(x,y)=A_{mn}^S \sin\left(\frac{m\pi x}{L_x}\right)\sin\left(\frac{n\pi x}{L_y}\right)$$

The coefficient $A_{mn}^S$ is selected so that the mode which is relative for him, is orthonormalisé compared to the structural mass:

$$\int_0^L\int_0^L \rho_s\left(\Phi_{mn}^S(x,y)\right)^2 t \,\mathrm{dx}\,\mathrm{dy}=1$$

What leads to the following value for $A_{mn}^S$ : $A_{mn}^S=\frac{2}{(\rho_S t L_x L_y)^{1/2}}$

The own pulsations and the acoustic clean modes are calculated by making the assumption of rigid walls for the cavity and lead to the following expressions ( $i, j, k=0,1,2,...$ ):

$$\omega_{ijk}^F=c\left(\left(\frac{i\pi}{L_x}\right)^2+\left(\frac{j\pi}{L_y}\right)^2+\left(\frac{k\pi}{L_z}\right)^2\right)^{1/2}$$

$$\Phi_{ijk}^F(x,y,z)=A_{ijk}^F\cos\left(\frac{i\pi x}{L_x}\right)\cos\left(\frac{j\pi y}{L_y}\right)\cos\left(\frac{k\pi z}{L_z}\right)$$

The coefficient $A_{ijk}^F$ is evaluated by the condition of standardisation compared to the acoustic mass:

$$\int_0^L \int_0^L \int_0^L \frac{1}{\rho_F c_F^2} \left(\Phi_{ijk}^F(x,y,z)\right)^2 \,\mathrm{dx}\,\mathrm{dy}\,\mathrm{dz}=1$$

and thus $A_{ijk}^F$ is worth: $A_{ijk}^F=\left(\frac{\rho_F c_F^2}{L_x^{'} L_y^{'} L_z^{'}}\right)^{1/2}$

with:

$$\begin{aligned} L_x^{'} &= L_x \text{ si } i=0 & L_y^{'} &= L_y \text{ si } j=0 & L_z^{'} &= L_z \text{ si } k=0 \\ &=0 \text{ si } i>0 & &=0 \text{ si } j>0 & &=0 \text{ si } k>0 \end{aligned}$$

The coupled system is described by the following equations:

$$\begin{bmatrix} \Lambda_S & \mathrm{E} \\ \omega^2 \mathrm{E}^T & \Lambda_F \end{bmatrix} \begin{pmatrix} X_S \\ X_F \end{pmatrix} = \begin{pmatrix} F^S \\ 0 \end{pmatrix}$$

with:

- $X_S$ and $X_F$ are the structural and acoustic modal factors of participation respectively;
- $\Lambda_S=\mathrm{diag}\left\{\left(\omega_{mn}^S\right)^2-\omega^2\right\}$
- $\Lambda_F=\mathrm{diag}\left\{\left(\omega_{ijk}^F\right)^2-\omega^2\right\}$
- $F^S=\Phi_{mn}^S(x_F, y_F)\cdot F$
- $\mathrm{E}_{\{mn\}\{ijk\}}=\int_0^L \int_0^L \Phi_{mn}^S(x,y)\,\Phi_{ijk}^F(x,y,0)\,\mathrm{dx}\,\mathrm{dy}=A_{mn}^S A_{ijk}^F c_x c_y$
- $c_x=-\frac{L_x}{\pi}\left(\frac{\cos(m-i)\pi}{2(m-i)}+\frac{\cos(m+i)\pi}{2(m+i)}\right)$
- $c_y=-\frac{L_y}{\pi}\left(\frac{\cos(n-j)\pi}{2(n-j)}+\frac{\cos(n+j)\pi}{2(n+j)}\right)$

The coupled dynamic response (displacement $\mathrm{u}$ for the structure and acoustic pressure $\mathrm{p}$ for the fluid) can then be calculated by:

$$u(x,y)=\sum_{m,n} \Phi_{mn}^S(x,y)\,X_{mn}^S$$
$$p(x,y,z)=\sum_{i,j,k} \Phi_{ijk}^F(x,y,z)\,X_{ijk}^F$$

## 2.2 Results of reference

One calculates the pressure at the points $N1$, $N2$, $N3$.

## 2.3 Uncertainty on the solution

Analytical solution.

## 2.4 Bibliographical references

[1] acoustic Radiation of the structures, Vibroacoustique, Interaction fluid-structure, p. 27 of Claude Lesueur – Collection of the direction of the Studies and Searchs for EDF.
[2] acoustic Radiation of the structures, Vibroacoustique, Interaction fluid-structure, p. 80 of Claude Lesueur – Collection of the direction of the Studies and Searchs for EDF.
[3] Derivation of has reference solution for has Cavity-Punt problem by J. - P. Coyette de Numerical Integration Technologies – internal Note.

# 3 Modeling A

## 3.1 Characteristics of modeling A

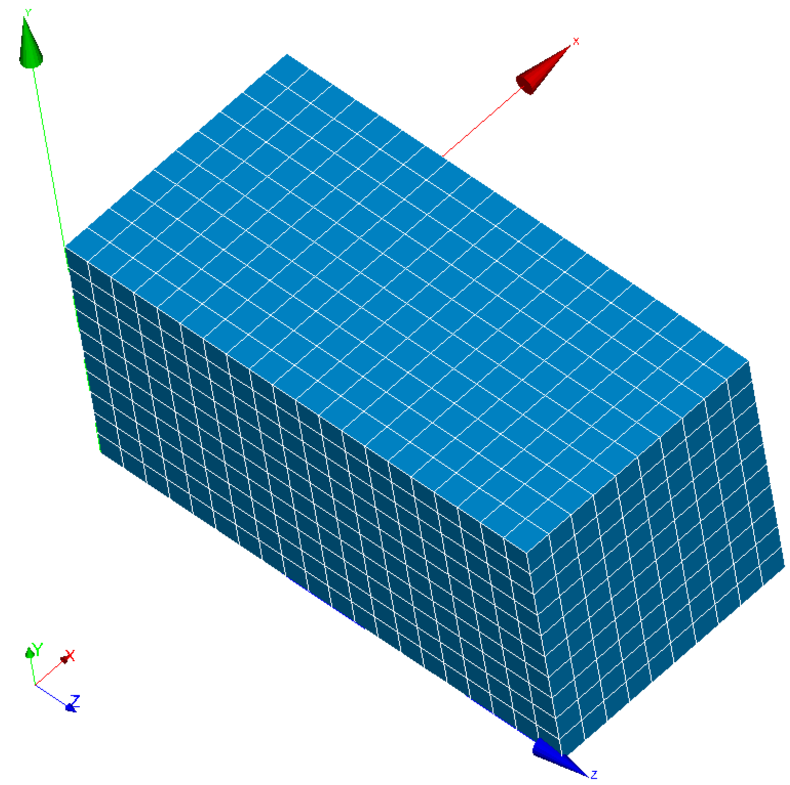


**Figure 3.1. Grid of modeling A**

Modeling 3D_FLUIDE and FLUI_STRU.

Cutting: 10 elements along axis X;
10 elements along the axis there;
20 elements along axis Z.

## 3.2 Characteristics of the grid

Many nodes: 2541
Many meshs and types: 2000 HEXA8 and 1100 QUAD4

## 3.3 Sizes tested and results

Frequency 100 $Hz$ :

| Position | Value reference | % difference |
|---|---|---|
| $N1$ | $-0.0584+0.\ i$ | $18\ \%$ |
| $N2$ | $-0.02536+0.\ i$ | $2\ \%$ |

| | | |
|---|---|---|
| $N3$ | $0.09224+0.\ i$ | $0.2$ % |

# 4 Summary of the results

The results are satisfactory but the pressure calculated with the right of the point of application of the load remains nevertheless vague.