FDLV110 - Calculation of mass added on modes obtained by under-structuring

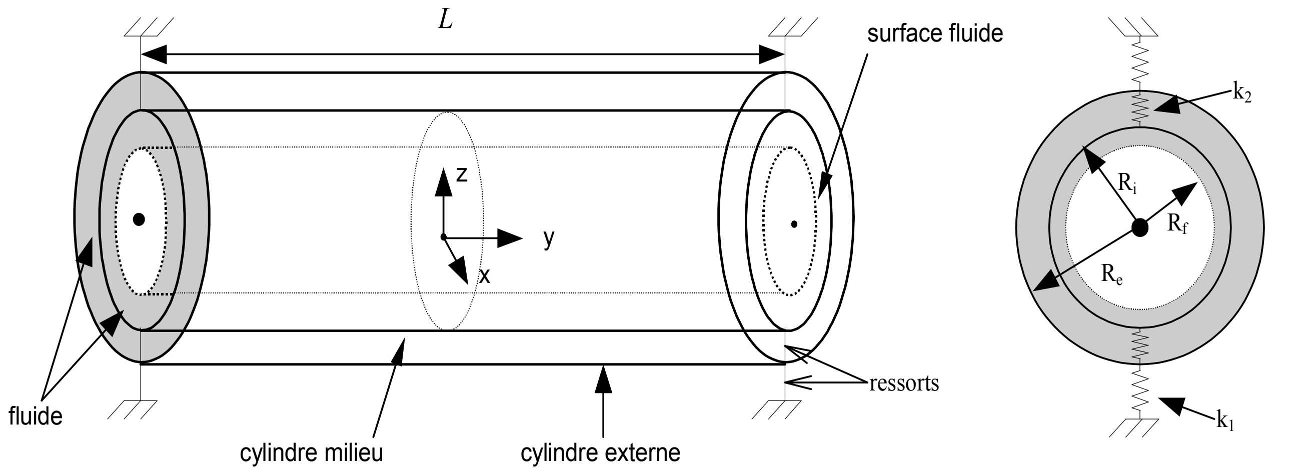
Summary:

This test of the field of the modal analysis and the interaction fluid-structure implements the calculation of mass added on a structure made up of three concentric cylinders separated by two rings from fluid (water) which one supposes the behavior governed by the potential theory (fluid true, incompressible at rest). The model is three-dimensional for water. The structure is represented by elements of type thin hull in modeling A (the structure is rigid in the reference solution). This one is characterized by two clean modes evaluated by dynamic under-structuring, with interface of the type CRAIG-BAMPTON.

The interest of the test lies in the use of the functionality 'NOEUD_DOUBLE'L' operator 'CALC_MATR_AJOU'. This functionality makes it possible to calculate the effects of added mass D' a structure represented by a surface grid (without thickness) which is bathed in a fluid. The fluids chosen in this CAS-test are different densities on both sides of the intermediate cylinder (water at different temperatures).

1 Problem of reference

1.1 Geometry



$L=50\text{m}$; $R_f=1\text{m}$; $R_i=\frac{5}{3}m$; $R_e=3\text{m}$; $k_1=10^9 N.m^{-1}$; $k_2=0.5\,10^7 N.m^{-1}$; $\rho_{fluide}=1000\,kg.m^{-3}$; $\rho_s=7800\ kg.m^{-3}$; thickness of the hull: $50\,cm$.

1.2 Properties of materials

Fluid: density $\rho_1=1000\,kg.m^{-3}$; $\rho_2=750\,kg.m^{-3}$.

Structure: $\rho_s=7800\,kg/m^3$; $E=2.1\,10^{11}\,Pa$; $\nu=0.3$ (steel).

1.3 Boundary conditions and loadings

The external cylinder on the one hand is connected to a fixed frame via the four springs of unit stiffness k_1 , connected in addition to the cylinder medium by four springs of unit stiffness k_2 . The two structures are rigid in this reference solution.

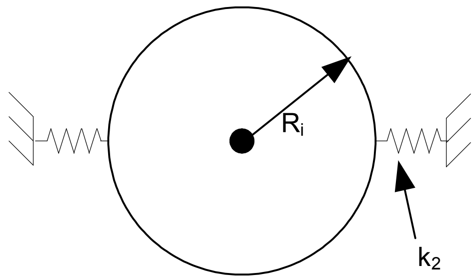
2 Reference solution

One calculates the clean modes of the system after having checked those of each substructure. One evaluates then the mass added on the modes in air.

2.1 Decomposition in substructures

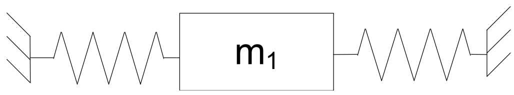
First substructure: intermediate cylinder

The first substructure is consisted by the intermediate cylinder and of four springs of stiffness $k_2 = 10^7 N.m^{-1}$. These springs are embedded with the interface with the external cylinder which constitutes the second substructure (interface of the type CRAIG-BAMPTON).



Mass of cylinder 1: $m_1 = 2.041\, 10^6 kg$

The cylinder being rigid, its movement can be modelled by a system mass-arises with a degree of freedom:

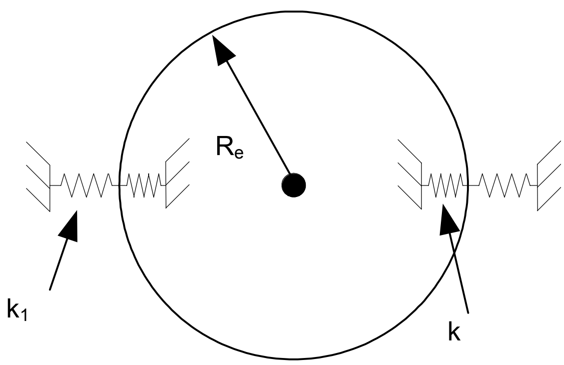


Achacune of its ends, the cylinder is connected to two springs in parallel: the equivalent stiffness of each one is $k' = 2k_2$

The Eigen frequency is worth then: $f\square = \frac{1}{2\pi}\sqrt{\frac{2k'}{m_1}} = \frac{1}{2\pi}\sqrt{\frac{4k_2}{m_1}}$, that is to say: $f\square = 0.705\, Hz$

Second substructure: external cylinder

The second substructure is the external cylinder connected on the one hand to the interface by the same springs, on the other hand with a fixed frame:



Mass of cylinder 2: $m_2 = 3{,}674\,10^6\,kg$

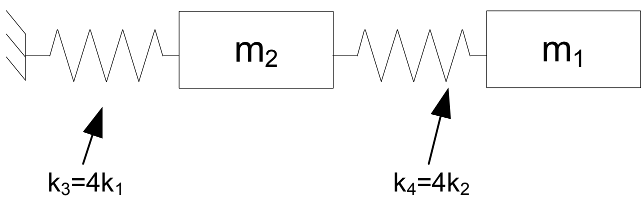
Equivalent stiffness of a fastener of this cylinder by the system of springs in series k_1 and k_2 being worth $9{,}9\,10^6\,N.m^{-1}$ (four fasteners of the same type connect in parallel the cylinder to an embedding), the Eigen frequency is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{4k_2k_1/(k_2+k_1)}{m_1}}$$, that is to say: $f = 0.522\,Hz$

N.B. : the third cylinder (interior cylinder) was not modelled in our case because it agint of a fixed cylinder. It thus constitutes a fixed wall of the fluid field.

Modes in air of the structure supplements (intermediate cylinder and external cylinder)

It is a system with two degrees of freedom:



The Eigen frequencies of this system are given by the exact formula [bib2]:

$$f_i = \frac{1}{2^{3/2}\pi}\sqrt{\frac{k_3}{m_2}+\frac{k_4}{m_2}+\frac{k_4}{m_1}\pm\sqrt{\left(\frac{k_3}{m_2}+\frac{k_4}{m_2}+\frac{k_4}{m_1}\right)^2-4\frac{k_3k_4}{m_1m_2}}} ,$$

that is to say

$$f_1 = 0.497\,Hz \text{ and } f_2 = 5.263\,Hz .$$

The two clean modes admit, for digital value:

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}_1 = \begin{pmatrix} 1 \\ 5.\,10^{-3} \end{pmatrix} \text{ et } \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}_2 = \begin{pmatrix} -\,9.\,10^{-3} \\ 1 \end{pmatrix} .$$

2.2 Calculation of the matrix of added mass

Fluid potentials

Beginning again [bib1], it is established that:

$$\phi_1^{(1)} = \left(R_i^2 \left(\frac{R_i^2 + R_f^2}{R_i^2 - R_f^2} + \frac{R_e^2 + R_i^2}{R_e^2 - R_i^2} \right); \left(5.e^{-3}\right) \times R_e^2 \frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right)$$

et

$$\phi_1^{(2)} = \left(\left(-9.e^{-3}\right) \times R_i^2 \left(\frac{R_i^2 + R_f^2}{R_i^2 - R_f^2} + \frac{R_e^2 + R_i^2}{R_e^2 - R_i^2} \right); R_e^2 \frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right)$$

The shape of the matrix of added mass, in this configuration, is:

$$M_a = \begin{bmatrix} M_a^{11} & M_a^{12} \\ M_a^{21} & M_a^{22} \end{bmatrix}$$

With:

$$M_a^{11} = \rho\pi L \left[R_i^2 \left(\frac{R_i^2 + R_f^2}{R_i^2 - R_f^2} + \frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right) \right] = 1{,}753.10^6 \text{ kg},$$

$$M_a^{22} = \rho\pi L \left[R_e^2 \left(\frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right) \right] = 2{,}676.10^6 \text{ kg},$$

$$M_a^{12} = \rho\pi L \left[(5.10^{-3}) \times R_i^2 \left(\frac{R_i^2 + R_f^2}{R_i^2 - R_f^2} + \frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right) - (9.10^{-3}) \times R_e^2 \left(\frac{R_i^2 + R_e^2}{R_e^2 - R_i^2} \right) \right] = -15318 \text{ kg}.$$

The coefficient of inertial coupling M_a^{12} is regarded as negligible in front of the coefficients of added auto--mass M_a^{11} and M_a^{22}. The Eigen frequencies of the system depend, at first approximation, only of these two last coefficients.

2.3 Results of reference

Analytical result.

2.4 References bibliographical

- ROUSSEAU G., LUU H.T. : Mass, damping and stiffness added for a vibrating structure placed in a potential flow - Bibliography and establishment in *Code_Aster* - HP-61/95/064
- BLEVINS R.D.: Formulated for Natural frequency and shape mode, ED. Krieger

3 Modeling A

3.1 Characteristics of modeling

For the system 3D on which one calculates the added coefficients:

Cylinder:	2400 meshs `QUAD4` elements of hulls `MEDKQU4` 12 meshs `SEG2` elements springs `MECA_DIS_T_L`
Fluid:	3600 meshs `QUAD4` thermal elements `THER_FACE4` on cylindrical surfaces
	7200 meshs `HEXA8` thermal elements `THER_HEXA8` in fluid annular volume

3.2 Values tested

	Frequencies analytical in air (Hz)
First mode in air	0,497
Second mode in air	5,263

	Theoretical added mass (kg)
M^{11}	$1,753\ 10^{6}$
M^{22}	$2,675\ 10^{6}$

	Frequencies analytical of the modes out of water (Hz)
First mode out of water	0,365
Second mode out of water	4,004

4 Summary of the results

The calculation of mass added on modes estimated by under-structuring is satisfactory. This made it possible to validate the option 'NOEUD_DOUBLE' order 'CALC_MATR_AJOU'. L' variation observed on the second coefficient of added mass is explained by the discretization of the second cylinder. The number of elements is a little insufficient to calculate in an exact way the integral of the field of pressure on the structure.