WTNP121 – Modeling of a bar saturated with linear compressible liquid (monophasic flow) subjected to a shock with pressure

Summary:

This case test has a double objective:

- to validate the diagrams finished volumes developed for the modeling of the diphasic flows.
- to validate hydraulic modeling Finite elements saturated (modelings Puts NR)

The diphasic problem here will be degenerated in a monophasic problem which one knows the analytical solution. It is the monodimensional modeling of a bar saturated with water subjected to a shock with pressure.

1 Problem of reference

The objective of this case test is to compare the solution obtained with the various diagrams volumes finished with an analytical solution.

1.1 Analytical solution

The non stationary and monodimensional monophasic problem can be written in a general form of the type:

$$N\frac{\partial P}{\partial t}-K_{int.}\Delta P=0$$

$$P(t=0)=P_0$$

$$P(t,x=0)=0$$

$$\frac{\partial P}{\partial x}(t,x=L)=0$$

This problem admits an analytical solution obtained by development in Fourier series.

$$P=\sum_{k=0}^{\infty}\frac{4\mathrm{P}_0}{(2\mathrm{k}+1)\pi}\exp\left(\frac{-K_{int}}{N}\omega_k^2 t\right)\sin(\omega_k x)\ \text{ with }\ \omega_k=\left(k+\frac{1}{2}\right)\frac{\pi}{L}$$

One can reduce this series to a finished number K terms, according to the calculated moment.
This number of terms is in the following way given:

That is to say n_x the number of points x_i where the solution is evaluated at one moment t
One poses: $a_k^i=\frac{4}{(2\mathrm{k}+1)\pi}\exp\left(\frac{-K_{int}}{N}\omega_k^2 t\right)\sin(\omega_k x_i)$

So that the solution can be written: $P(x_i)=\sum_{k=0}^{K}P_0.a_k^i$

One chooses K such as: $\frac{1}{n_x}\sqrt{\sum_{i=1}^{nx}(a_k^i)^2}<\epsilon$

In practice, we took $\epsilon=10^{-10}$.
The paces of the analytical solution at times 1,10,100,1000 are shown on the figure 1 :

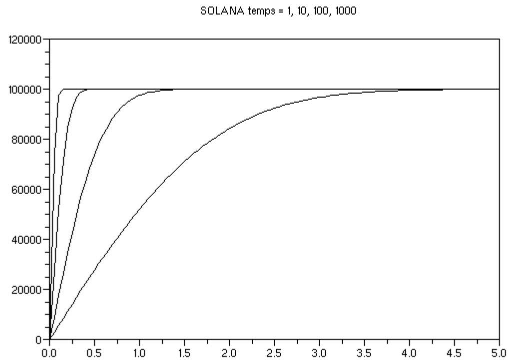


Figure 1: Analytical representation of the solutions

The following table gives the number of terms according to time:

Moments	Number series terms
1	194
10	64
100	22
1000	8

Table 1.1-1 : Representation amongst term according to time

1.2 Simplifying assumptions

It is considered that the medium is completely saturated with water and one imposes a worthless gas pressure on all the nodes. The biphasic system is then brought back to solve the following problem:

$$\frac{\partial(\varphi\,\rho_l)}{\partial t}-div\left(K_{int}\frac{\rho_l k_{rl}}{\mu_l}\nabla P_l\right)=0$$

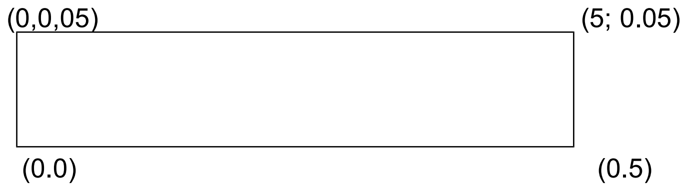
- The liquid is incompressible: $\rho_l=cst$
- The matrix is compressible and porosity evolves proportionally with the pressure of liquid: $\frac{\partial\varphi}{\partial P_l}=E_m$

The conservation equation of the mass for the liquid is thus written:

$$\rho_l E_m\frac{\partial P_l}{\partial t}-div\left(K_{int}\frac{\rho_l k_{rl}}{\mu_l}\nabla P_l\right)=0$$

1.3 Geometries

A bar is considered 1D of 5m of length. Concretely the field with a grid will make $[0\text{m},5m]\times[0\text{m};0{,}05m]$ (in the case of modeling in triangle, it is important not to have too “flattened” triangles, the choice height of the field is thus not pain-killer).



1.4 Grids

One tested this case on two grids, one composed of 100 quadrangles (modelings A with D and M) and the other of 200 triangles (modelings E with H).

Modelings I, J, K, L and NR consituent an extension $3D$ (bar of section 1×1), the grid consists of 100 hexahedrons.

1.5 Properties of materials

One gives here only the properties whose solution depends, knowing that the command file contains other data of material which do not play any part in the solution of with the dealt problem.

Liquid	Relative permeability	1
	Viscosity $(kg.m^{-1}.s^{-1})$	1
	Module of compressibility	0
	Density of the liquid (kg/m^3)	1
Homogenized parameters	Permeability (m^2)	10^{-13}
	Porosity	$0{,}5$
	Storage	10^{-10}
	Saturation in liquid	1

Table 1.5-1 : Properties of materials

1.6 Boundary conditions and initial

The limiting conditions are the following ones:

- conditions of Neumann on the right of field: $\dfrac{\partial P_l}{\partial x}(t, x=5, y)=0$
- conditions of Dirichlet on the left part of the field: $P_l(t, x=0, y)=0$

The initial pressure of liquid is of $P_l(t=0, x, y)=10^4 Pa$.

1.7 Duration of simulation and not of time

The duration of simulation is of $100\,s$ and the number of steps of time is of 100.

Note: By preoccupations with an information, the following tests are presented until $1000\,s$ *whereas one limits without Code_Aster simulations to* $100\,s$ *.*

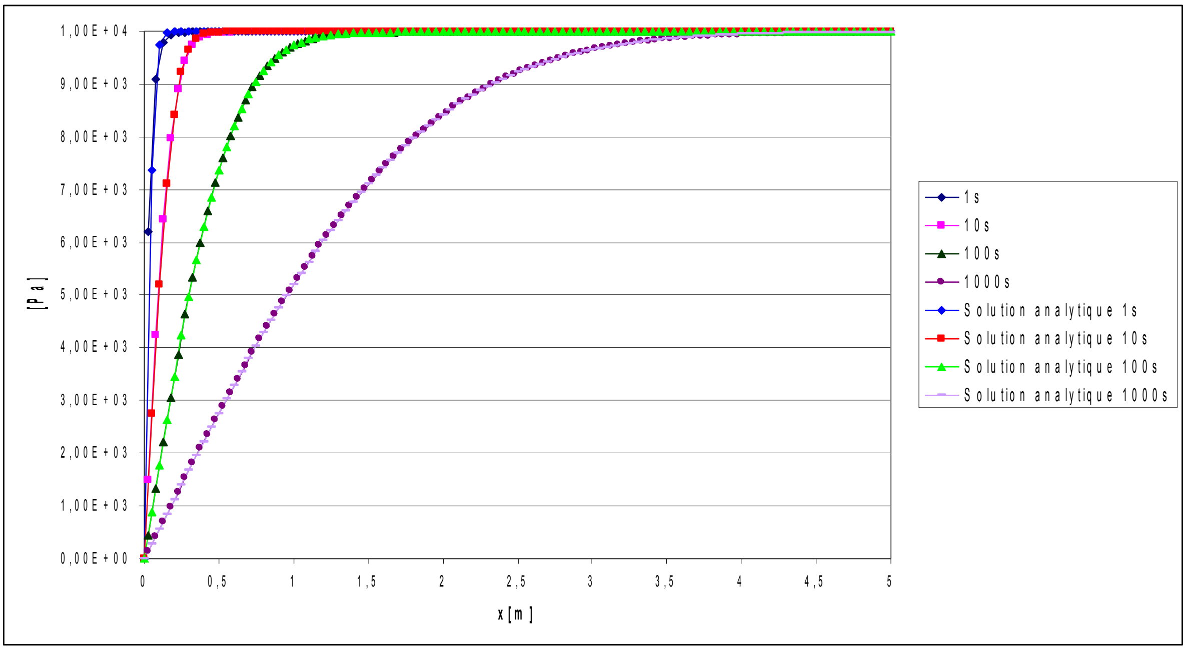
2 Modeling A

2.1 Characteristics of modeling A

Modeling `D_PLAN_HH2SUDM`. This modeling corresponds with modeling Volume Finished with decentring on the mesh neighbor for terms of mobilities (the fickiens terms are realised). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. One uses a grid made up of 100 elements `QUAD9`.

2.2 Results

One traces the profiles of pressure of liquid at various moments as well as the analytical solution at these same moments. The results are identical.



Drawing 2: Pressure of liquid

2.3 Values tested

One carries out tests on 4 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time (S)**	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}075\,;0)$ *N304*	100 S	-1,33E+003	7%
$(0{,}075\,;0{,}5)$ *NQ95*	100 S	-1,33E+003	9%
$(0{,}075\,;1)$ *N293*	100 S	-1,33E+003	7%
$(0{,}05\,;0{,}05)$ *N469*	100 S	-8,93E+002	7,5%

Table 2.3-1 : Values tested

3 Modeling B

3.1 Characteristics of modeling B

Modeling `D_PLAN_HH2SUDA`. This modeling corresponds with modeling Volume Finished diecentered on the edges for mobilities (the fickiens terms are centered). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. One uses a grid made up of 100 elements `QUAD9`.

3.2 Results

The results are identical to those obtained with modeling volumes finished eccentric on the mesh.

3.3 Values tested

One carries out tests on 4 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\,\%$.

Points (x,y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}075;0)$ *N304*	100 S	-1,33E+003	7%
$(0{,}075;0{,}5)$ *NQ95*	100 S	-1,33E+003	9%
$(0{,}075;1)$ *N293*	100 S	-1,33E+003	7%
$(0{,}05;0{,}05)$ *N469*	100 S	-8,93E+002	7,5%

Table 3.3-1 : values tested

4 Modeling C

4.1 Characteristics of modeling C

Modeling `D_PLAN_HH2SUC`. This modeling corresponds with modeling Finished Volumes centered (for the darcéens terms and fickiens). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 100 elements `QUAD9`.

4.2 Results

The results are identical that those obtained with modeling Volumes finished eccentric on the mesh.

4.3 Values tested

One carries out tests on 4 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\,\%$.

Points (x,y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}075\,;0)$ $N304$	100 S	-1,33E+003	7%
$(0{,}075\,;0{,}5)$ $NQ95$	100 S	-1,33E+003	9%
$(0{,}075\,;1)$ $N293$	100 S	-1,33E+003	7%
$(0{,}05\,;0{,}05)$ $N469$	100 S	-8,93E+002	7,5%

Table 4.3-1 : Values tested

5 Modeling D

5.1 Characteristics of modeling D

Modeling `D_PLAN_HH2S`. This modeling corresponds with modeling classical Finite elements. The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 100 elements `QUAD8`.

5.2 Results

The results are identical to those obtained with modeling volumes finished eccentric on the mesh.

5.3 Values tested

One carries out tests on 2 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}05;0)$	100 S	-8,89E+002	1%
$(0{,}05;1)$	100 S	-8,89E+002	1%

Table 5.3-1 : Values tested

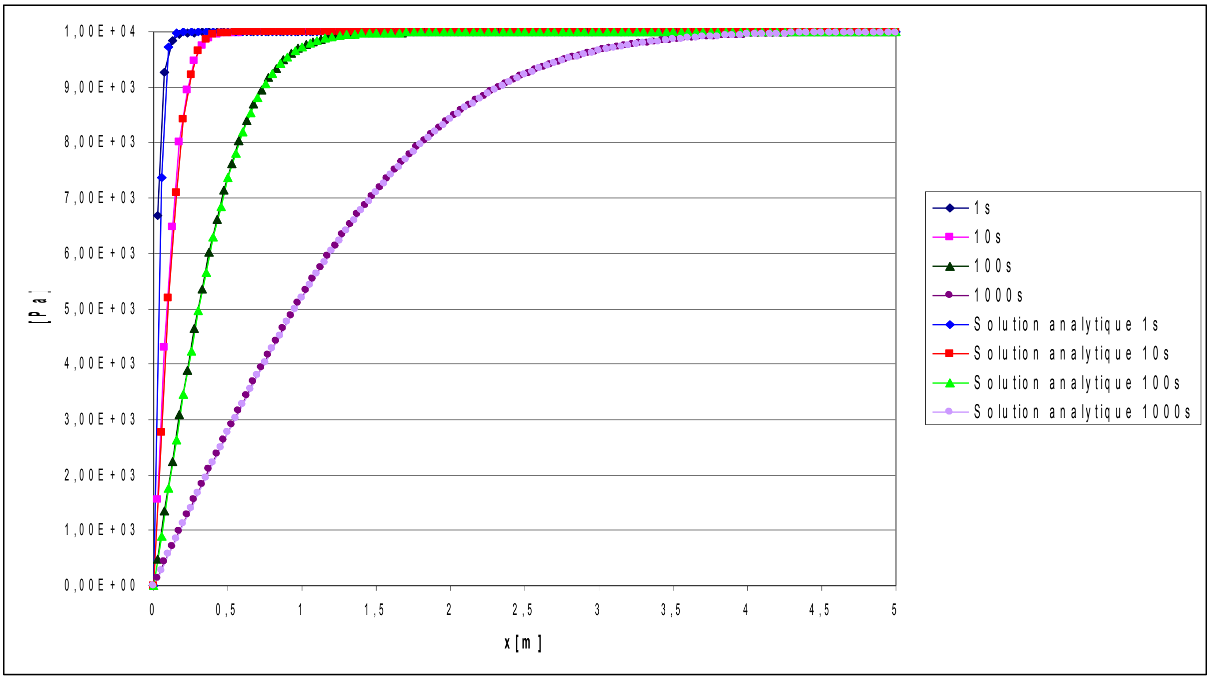
6 Modeling E

6.1 Characteristics of modeling E

Modeling `D_PLAN_HH2SUDM`. This modeling corresponds with modeling Volume Finished with decentring on the mesh neighbor for terms of mobilities (the fickiens terms are realised). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 200 elements `TRIA7`. The goal is here to validate this diagram on meshs triangles.

6.2 Results

One traces the profiles of pressure of liquid at various times. The results correspond well to the analytical solution.



Drawing 3: Pressure of liquid

6.3 Values tested

On two nodes we compared the results with the analytical solution and on the third node we carried out a test of nonregression.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}075;0)$ $N360$	100 S	-1,33E+003	3%
$(0{,}075;0{,}025)$ $N505$	100 S	-1,33E+003	3%
$(1675;0{,}0158)$ $NT70$	100 S	-3,13E+002	0,01%

Table 6.3-1 : Values tested

7 Modeling F

7.1 Characteristics of modeling F

Modeling `D_PLAN_HH2SUDA`. This modeling corresponds with modeling Finished Volumes diecentered on the edges for mobilities (the fickiens terms are centered). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 200 elements `TRIA7`.

7.2 Results

The results are identical to those obtained with modeling volumes finished eccentric on the mesh.

7.3 Values tested

One carries out tests on 3 nodes with $t=100\,s$.
On two nodes we compared the results with the analytical solution and on the third node we carried out a test of nonregression.

Points (x,y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}075\,;0)$ *N360*	100 S	-1,33E+003	3%
$(0{,}075\,;0{,}025)$ *N505*	100 S	-1,33E+003	3%
$(1675\,;0{,}0158)$ *NT70*	100 S	-3,13E+002	0,01%

Table 7.3-1 : Values tested

8 Modeling G

8.1 Characteristics of modeling G

Modeling `D_PLAN_HH2SUC`. This modeling corresponds with modeling Volume Finished centered (for the darcéens terms and fickiens). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 200 elements `TRIA7`.

8.2 Results

The results are identical that those obtained with modeling Volumes finished eccentric on the mesh.

8.3 Values tested

On two nodes we compared the results with the analytical solution and on the third node we carried out a test of nonregression.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error ($\%$)**
$(0{,}075;0)$ *N360*	100 S	-1,33E+003	3%
$(0{,}075;0{,}025)$ *N505*	100 S	-1,33E+003	3%
$(1675;0{,}0158)$ *NT70*	100 S	-3,13E+002	0,01%

Table 8.3-1 : Values tested

9 Modeling H

9.1 Characteristics of modeling H

Modeling `D_PLAN_HH2S`. This modeling corresponds with modeling Finite elements. The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 200 elements `TRIA6`.

9.2 Results

The results are identical to those obtained with modeling volumes finished eccentric on the mesh.

9.3 Values tested

One carries out tests on 2 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\,\%$.

Points (x,y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}05\,;0,)$ $N203$	100 S	-8,93E+002	3%
$(0{,}05\,;0{,}005)$ $N103$	100 S	-8,93E+002	3%

Table 9.3-1 : Values tested

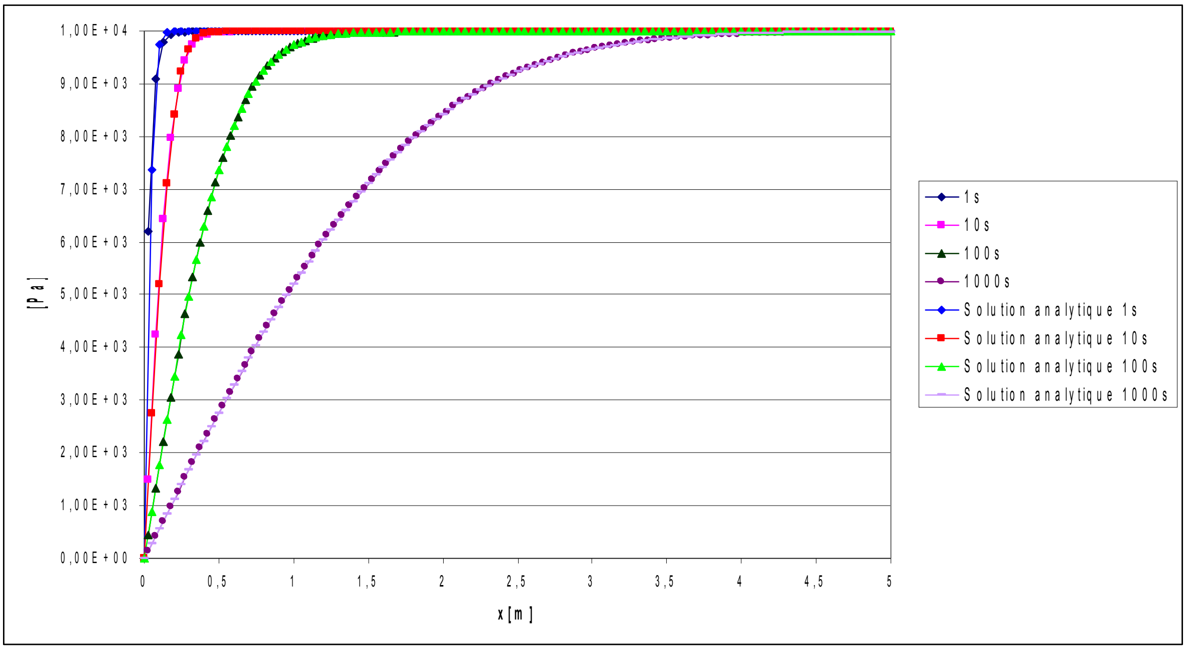
10 Modeling I

10.1 Characteristics of modeling I

Modeling `3D_HH2SUDM`. This modeling corresponds with modeling Volume Finished with decentring on the mesh neighbor for terms of mobilities (the fickiens terms are realised). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 100 elements `HEXA27`. It is a question here of validating the diagrams on a test $3D$.

10.2 Results

One traces the profiles of pressure of liquid at various times. The results are identical to the analytical solution.



Drawing 4: Pressure of liquid

10.3 Values tested

One carries out tests on 3 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error**
$(-49{,}5\,;0{,}5\,;0)$ *NH4*	100 S	-8.263E+03	1%
$(-1{,}5\,;0\,;-0{,}5)$ *NH195*	100 S	-9.990E+03	1%
$(-49{,}5\,;0\,;0{,}5)$ *NH1*	100 S	-8.263E+03	1%

Table 10.3-1 : Values tested

11 Modeling J

11.1 Characteristics of modeling J

Modeling `3D_HH2SUDA`. This modeling corresponds with modeling Volume Finished diecentered on the edges for mobilities (the fickiens terms are centered). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 100 elements `HEXA27`.

11.2 Results

The results are identical to those obtained with modeling volumes finished eccentric on the mesh.

11.3 Values tested

One carries out tests on 3 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(-49{,}5\,;0{,}5\,;0)$ *NH4*	100 S	-8.263E+03	1%
$(-1{,}5\,;0\,;-0{,}5)$ *NH195*	100 S	-9.990E+03	1%
$(-49{,}5\,;0\,;0{,}5)$ *NH1*	100 S	-8.263E+03	1%

Table 11.3-1 : Values tested

12 Modeling K

12.1 Characteristics of modeling K

Modeling `3D_HH2SUC`. This modeling corresponds with modeling Volume Finished centered (for the darcéens terms and fickiens). The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 100 elements `HEXA27`.

12.2 Results

The results are identical that those obtained with modeling Volumes finished eccentric on the mesh.

12.3 Values tested

One carries out tests on 3 nodes with $t=100\,s$ by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\,\%$.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(-49{,}5\,;0{,}5\,;0)$ *NH4*	100 S	-8.263E+03	1%
$(-1{,}5\,;0\,;-0{,}5)$ *NH195*	100 S	-9.990E+03	1%
$(-49{,}5\,;0\,;0{,}5)$ *NH1*	100 S	-8.263E+03	1%

Table 12.3-1 : Values tested

13 Modeling L

13.1 Characteristics of modeling L

Modeling `3D_HH2S`. This modeling corresponds with modeling Finite elements. The hydraulic mixing rate is `LIQU_AD_GAZ_VAPE`. The grid consists of 100 elements `HEXA20`.

13.2 Results

The results are very close to those obtained with modeling volumes finished eccentric on the mesh.

13.3 Values tested

One carries out tests on 3 nodes with 1 moment by comparing the results with the analytical solution. One also tests the first node in nonregression with a relative error authorized of $0{,}01\%$.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(-49; -0{,}5; 0{,}5)$ $N6$	100	-9.090E+03	1%
$(-47: -0{,}5; 0{,}5)$ $N16$	100	-9.989E+03	1%
$(-48; -0{,}5; -0{,}5)$ $N716$	100	-9.947E+03	1%

Table 13.3-1 : Values tested

14 Modeling M

14.1 Characteristics of modeling M

Modeling `D_PLAN_HS`. This modeling corresponds with modeling Finite elements. The hydraulic mixing rate is `LIQU_SATU`. The grid consists of 100 elements `QUAD8`.

14.2 Results

The results are very close to those obtained with modeling finite elements `D_PLAN_HH2S` (modeling D).

14.3 Values tested

One carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}5 ; 0)$ $N104$	100	-8.89E+02	1%
$(0{,}5 ; 1)$ $N103$	100	-8.89E+02	1%

Table 14.3-1 : Values tested

15 Modeling NR

15.1 Characteristics of modeling NR

Modeling `3D_HS`. This modeling corresponds with modeling Finite elements. The hydraulic mixing rate is `LIQU_SATU`. The grid consists of 100 elements `HEXA20`.

15.2 Results

The results are very close to those obtained with modeling finite elements `D_PLAN_HH2S` (modeling D).

15.3 Values tested

One carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution.

Points (x, y)	**Time** (s)	**PRE1** ***Aster***	**Authorized relative error (** $\%$ **)**
$(0{,}05\,;-0{,}5\,;-0{,}5)$ *N108*	100	-8.89E+02	1%
$(0{,}05\,;0{,}5\,;0{,}5)$ *N306*	100	-8.89E+02	1%

Table 15.3-1 : Values tested

16 Summary of the results

This case test makes it possible to test the diagrams volumes finished in various configurations:

- the 3 diagrams finished volumes: centered, decentred edge, decentred mesh
- in $2D$ and in $3D$
- on various types of meshs (triangles and rectangles for $2D$, hexahedrons for $3D$)

These same cases are also carried out with the classical diagrams finite elements. All the results are identical to the analytical solution.