

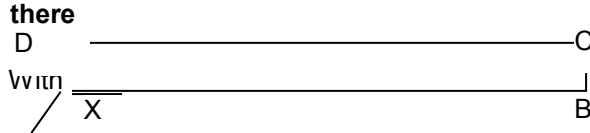
WTNP105 - Diffusion of air dissolved in water (plan HH2M)

Summary:

One considers here a problem at temperature and constant saturation. By boundary conditions suitable one imposes a water pressure and a steam pressure constants. A gas pressure is imposed on an edge of the field (worthless flows of the other with dimensions). Only the air pressures dryness and of dissolved air connected by the law of Henry evolve. This problem is brought back in an equation for the air pressure dryness of type "equation of heat". The reference solution will be then a thermal calculation ASTER. It is acted exactly of the same case test as the WTNP103 but as modeling HH2M.

1 Problem of reference

1.1 Geometry



Coordinates of the points (m) :

A	0	0	C	1	0.5
B	1	0	D	0	0.5

1.2 Properties of material

One gives here only the properties whose solution depends, knowing that the command file contains other data of material (moduli of elasticity, thermal conductivity...) who finally do not play any part in the solution of with the dealt problem.

Liquid water	Density ($kg.m^{-3}$)	10^3
	Specific heat with constant pressure ($J.K^{-1}$)	0.
	Dynamic viscosity of liquid water ($Pa.s$)	0,001
	thermal dilation coefficient of the liquid (K^{-1})	0.
	Permeability relating to water	$kr_w(S) = 0.5$
Vapor	Specific heat ($J.K^{-1}$)	0.
	Molar mass ($kg.mol^{-1}$)	0.01
Gas	Specific heat ($J.K^{-1}$)	0.
	Molar mass ($kg.mol^{-1}$)	0.01
	Permeability relating to gas	$kr_{gz}(S) = 0.5$
	Viscosity of gas ($kg.m^{-1}.s^{-1}$)	0,001
Dissolved air	Specific heat ($J.K^{-1}$)	0.
	Constant of Henry ($Pa.m^3.mol^{-1}$)	50000
Initial state	Porosity	1
	Temperature (K)	300
	Gas pressure (Pa)	1.01E5
	Steam pressure (Pa)	1000
	Capillary pressure (Pa)	1. ^{E6}
	Initial saturation in liquid	0.4
Constants	Constant of perfect gases	8.32
Homogenized coefficients	Homogenized density ($kg.m^{-3}$)	2200
	Isotherm of sorption	$S(P_c) = 0.4$

Coefficient of Biot	0
Fick Vapor ($m^2 \cdot s^{-1}$)	$FV = 0$
Fick dissolved air ($m^2 \cdot s^{-1}$)	$FA = 6 \cdot E - 10$
Intrinsic permeability (m^2)	$Kint = 1.E - 19$

1.3 Boundary conditions and loadings

On the whole of the field, one wants:

$$p_w = cte = p_w^0$$

$$\frac{1}{K_w} = 0 \Rightarrow \rho_w = cte = \rho_w^0$$

$$p_{vp} = cte = p_{vp}^0$$

$$F_{vp} = 0$$

$$S(p_c) = cte = S_0$$

$$T = cte = T^0$$

$$\phi = 1$$

$$M_{as}^{ol} = M_{vp}^{ol} = M_{ad}^{ol}$$

On all the edges: Hydraulic flows and worthless thermics.

One now will linearize p_{vp} according to p_w .

Writing of p_{vp} linear function of p_w :

Section 4.2.3 of the reference document Aster [R7.01.11] the relation gives us: $\frac{dp_{vp}}{p_{vp}} = \frac{M_{vp}^{ol}}{RT} \frac{dp_w}{\rho_w}$.

If this expression is linearized one obtains: $p_{vp} = \frac{p_{vp}^0}{RT} \frac{M_{vp}^{ol}}{\rho_w^0} p_w + \left[p_{vp}^0 - \frac{p_{vp}^0}{RT} \frac{M_{vp}^{ol}}{\rho_w^0} p_w^0 \right]$

that one can write in the form:

$$p_{vp} = Ap_w + B \quad \text{éq 1.3-1}$$

with $A = \frac{p_{vp}^0}{RT} \frac{M_{vp}^{ol}}{\rho_w^0}$ and $B = p_{vp}^0 - \frac{p_{vp}^0}{RT} \frac{M_{vp}^{ol}}{\rho_w^0} p_w^0$

Sur le bord gauche AD : $p_{gz} = 115000 Pa$
 $p_c = 10^6 Pa$

2 Reference solution

2.1 Method of calculating

Calculation of the conservation of the mass of air

The conservation of the gas mass is written:

$$\frac{dm_{air}}{dt} + \text{div}(\mathbf{M}_{as} + \mathbf{M}_{ad}) = 0 \quad \text{éq 2.1.1-1}$$

It is written that the total water mass and the total mass of air are preserved (because there is no gas water flow nor at the edge) and one obtains:

$$m_{air} = m_{as} + m_{ad} = S_0(\rho_{ad} - \rho_{ad}^0) + (1 - S_0)(\rho_{as} - \rho_{as}^0)$$

thus

$$d(m_{as} + m_{ad}) = S_0 d\rho_{ad} + (1 - S_0) d\rho_{as} \quad \text{éq 2.1.1-2}$$

$$d\rho_{as} = \frac{M_{as}^{ol}}{RT} dP_{as} \quad \text{and} \quad d\rho_{ad} = \frac{M_{ad}^{ol}}{K_H} dP_{as}$$

$$\frac{dm_{air}}{dt} = \left[S_0 \frac{M_{as}^{ol}}{K_H} + (1 - S_0) \frac{M_{as}^{ol}}{RT} \right] \frac{dP_{as}}{dt}$$

Calculation speeds:

$$\frac{\mathbf{M}_{as}}{\rho_{as}} = \lambda_{gz} (-\nabla P_{as}) \quad \text{éq 2.1.1-3}$$

since $F_{vp} = 0$ and $\nabla P_{vp} = 0$

and

$$\mathbf{M}_{ad} = \rho_{ad} \lambda_{lq} (-\nabla P_{lq}) - F_{ad} \nabla C_{ad} \quad \text{with} \quad C_{ad} = \rho_{ad}$$

Like $\nabla P_{lq} = \nabla P_w + \nabla P_{ad} = \nabla P_{ad} = \frac{RT}{K_H} \nabla P_{as}$

$$\mathbf{M}_{ad} = \rho_{ad} \lambda_{lq} \frac{RT}{K_H} (-\nabla P_{as}) - \frac{M_{ad}^{ol}}{K_H} F_{ad} \nabla P_{as}$$

[éq 2.1.1-1] can then be simplified in the following form:

$$C \frac{dP_{as}}{dt} = L \text{div}(\nabla P_{as})$$

$$\text{with} \quad C = S_0 \frac{M_{as}^{ol}}{K_H} + (1 - S_0) \frac{M_{as}^{ol}}{RT}$$

$$L = \rho_{as}^0 \lambda_{gz} + \frac{RT}{K_H} \rho_{ad}^0 \lambda_{lq} + \frac{M_{as}^{ol}}{K_H} F_{ad}$$

Equation of the heat which one treats by a thermal calculation Aster.

2.2 Results of reference

With the preceding digital values, one finds:

$$P_{as} = 10^5 \Rightarrow P_{ad}^0 = \frac{RT}{K_H} P_{as}^0 = 4992$$
$$\rho_{as}^0 = \frac{M_{as}^{ol}}{RT} P_{as}^0 = 0.4 \text{ and } \rho_{ad}^0 = \frac{M_{ad}^{ol}}{RT} P_{ad}^0 = 0.02$$

$$\rho_{vp}^0 = \rho_{vp} = 4.10^{-3}$$

The constants of the equation of heat are then:

$$C = 2,4810^{-6}$$

$$L = 1,4.10^{-16}$$

2.3 Uncertainties on the solution

Uncertainties are rather large being given that the quasi-analytical solution (fruit of a thermal calculation) is a solution approached because of linearization of the equations.

3 Modeling A

3.1 Characteristics of modeling

Modeling in plane deformations D_PLAN_HH2MD.

3.2 Characteristics of the grid

20 elements QUAD8.

3.3 Sizes tested and results

It is pointed out that the temperature resulting from thermal calculation corresponds to the air pressure dryness of our calculation thermo-hydro-mechanics. The steam pressure being constant one a:

$$P_{gz} = PRE2^{DDL} + PRE2^{init} = P_{vp}^0 + P_{as}$$

$X (m)$	Time (s)	PRE2	Tolerance (%)
0.2	3E9	1.120e4	5.0
0.2	5E9	1.224e4	5.0

4 Modeling B

4.1 Characteristics of modeling

Even modeling that of A into selective, model D_PLAN_HH2MS.

4.2 Characteristics of the grid

20 elements QUAD8.

4.3 Sizes tested and results

It is pointed out that the temperature resulting from thermal calculation corresponds to the air pressure dryness of our calculation thermo-hydro-mechanics. The steam pressure being constant one a:

$$P_{gz} = PRE2^{DDL} + PRE2^{init} = P_{vp}^0 + P_{as}$$

$X (m)$	Time (s)	PRE2	Tolerance (%)
0.2	3E9	1.120e4	5.0
0.2	5E9	1.224e4	5.0

5 Modeling C

5.1 Characteristics of modeling

Even modeling that of A under-integrated, model D_PLAN_HH2M_SI.

5.2 Characteristics of the grid

20 elements QUAD8.

5.3 Sizes tested and results

It is pointed out that the temperature resulting from thermal calculation corresponds to the air pressure dryness of our calculation thermo-hydro-mechanics. The steam pressure being constant one a:

$$P_{gz} = PRE2^{DDL} + PRE2^{init} = P_{vp}^0 + P_{as}$$

$X (m)$	Time (s)	PRE2	Tolerance (%)
0.2	3E9	1.120e4	5.0
0.2	5E9	1.224e4	5.0

6 Summary of the results

The results are in very good agreement with the semi-analytical solution.