WTNV109 - Hydrous and mechanical loading of a saturated porous environment

Summary:

One considers a three-dimensional problem of coupling thermo-hydro-mechanics of a saturated porous environment.

This test consists in studying the effect of mechanics and hydraulics on thermics. One stretches the element by imposing a displacement in the direction to him z , one imposes a water pressure on the whole of the field and one studies the effect of these loadings on the temperature of the model. It is a question of looking at the very weak coupling of poromecanic towards thermics. One limits oneself to the first step of time.

The studied models are $2D$ plans (DPQ8 and DPTR6) and $3D$ voluminal (HEXA20) with a linear behavior for thermal hydraulics and it.

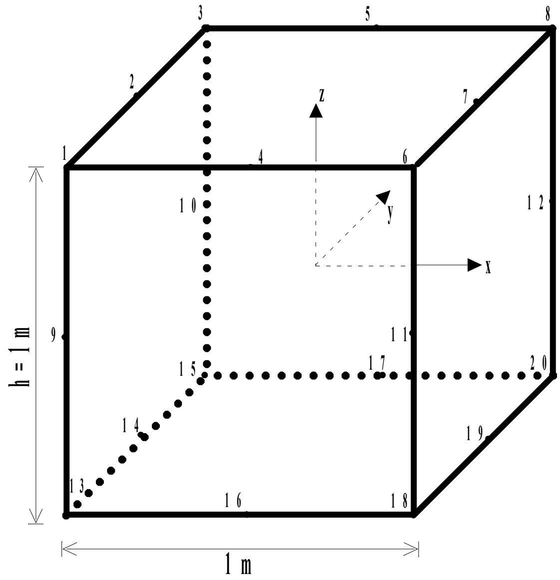
1 Problem of reference

1.1 Presentation

One studies in this case test the behavior thermo-hydro-mechanics of a saturated porous environment consisted only one fluid: water in its liquid phase. It acts in *Code_Aster* of a modeling `THM`. The associated law of behavior of the fluid is of type `LIQU_SATU`.

1.2 Geometry

One considers a cube of 1m of with dimensions centered on the center of the axis $(-0,5 \leqslant x \leqslant 0,5 ; -0,5 \leqslant y \leqslant 0,5 ; -0,5 \leqslant z \leqslant 0,5 ;)$.



1.3 Properties of material

solid	Density $(kg.m^{-3})$	$2.\times 10^3$
	Drained Young modulus $E(Pa)$	$225.\times 10^6$
	Poisson's ratio	$0.$
	Thermal dilation coefficient of the solid (K^{-1})	$8.\times 10^{-6}$
Fluid	Density $(kg.m^{-3})$	10^3
	Heat with constant pressure $(J.K^{-1})$	2.85×10^6
	Thermal dilation coefficient of the liquid (K^{-1})	10^{-4}
	Derived from the conductivity of the fluid compared to the temperature	$0.$
Thermics	Homogenized conductivity $(W.K^{-1}m^{-1})$	1.7
	Derived from the conductivity homogenized compared to the temperature	$0.$

Coefficients of homogenisation	Coefficient of $Biot$ Porosity	10^{-12} 0.4
Homogenized coefficients	Density $(kg.m^{-3})$ Heat with constant constraint $(J.K^{-1})$	1.6×10^{3} 2.85×10^{6}

1.4 Boundary conditions and loadings

- Complete element:
  pressure of the fluid $PRE1=500.0\,Pa$ (not of flow nor of variation of the water mass)

- Lower face:

  displacements $u_x=0.0\,m\,,u_y=0.0\,m\,,u_z=0.0\,m.$

- Higher face:
  displacement $u_z=10^{-3}\,m$

1.5 Initial conditions

The fields of displacement, pressure, temperature are initially all worthless, the temperature of reference is worth $T_0=273\,°K$.

2 Reference solution

2.1 Method of calculating

The reference solution is unidimensional because it depends only on the vertical coordinate. One will thus reason on a linear axis according to Z.

If gravity is neglected and that it is considered that there is no external source of heat, the equation of thermics is given by the following expression:

$$h_l^m \frac{d\,m_l}{dt} + \frac{\delta Q'}{dt} Div\, M_l + divq = 0 \qquad (1)$$

Where h_l^m is the mass enthalpy of water, m_l its mass, M_l its flow, Q heat flow and Q' not convectée heat.
The conservation equation of the mass is the following one:

$$\frac{d\,m_l}{dt} + Div\, M_l = 0 \qquad (2)$$

In the absence of hydrous flow, the equation (1) is simplified to become:

$$\frac{\delta Q'}{dt} + divq = 0 \qquad (3)$$

In this equation, quantity Q' represent the heat received by the system in a transformation for which there are no contributions of heat per entry of fluid. One chooses a weak variation of pressure and parameters so that the temperature varies little and thus that:

$$\frac{d\,Q'}{T} = \frac{d\,Q'}{T_0}$$

dQ' consequently this expression has:

$$dQ' = 3\alpha_0 K_0 T_0\, d\,\varepsilon_v + C_0^\varepsilon dT - 3\alpha_l^m T_0\, dp_l$$

with:

- α_0 the thermal dilation coefficient homogenized equivalent with that of the solid α_s .
- K_0 the drained coefficient of elasticity.
- C_0^ε heat has constant deformation which has as an expression $C_0^\varepsilon = C_0^\sigma - 9T_0 K_0 \alpha_0^2$ and C_0^σ heat with constant constraint.
- α_l^m The relative thermal dilation coefficient of the liquid compared to the skeleton, it has as an expression: $\alpha_l^m = (b-\phi)\alpha_s + \phi\alpha_l$ with B the coefficient of Biot, ϕ porosity and α_l the dilation coefficient of the liquid.
- $d\,\varepsilon_v$ Voluminal variation of deformation.
- dp_l Variation of pressure of liquid.

In addition, the heat flow has the following expression: $q = -\lambda_T \partial \frac{T}{\partial z}$ where λ_T is the thermal coefficient of conductivity.
While replacing dQ' and Q by their value in the equation (3) and limiting itself to only step of time Δt , one obtains:

$$3\alpha_0 K_0 T_0 \Delta\varepsilon_v - 3\alpha_l^m T_0 \Delta p_l + C_0^\varepsilon \Delta T = -\Delta t\, div\, q = \lambda_l \Delta t \frac{\partial^2 T}{\partial z^2}$$

By considering initially worthless temperatures and displacements, one can write:
$\Delta T = T(t) - T(0) = T$ and $\Delta\varepsilon = \varepsilon(t) - \varepsilon(0) = \varepsilon$ and $\Delta p_l = p_l(t) - p_l(0) = p_l$
Thus with the first step of time, one will have:

$$T - a\frac{\partial^2 T}{\partial z^2} = b$$

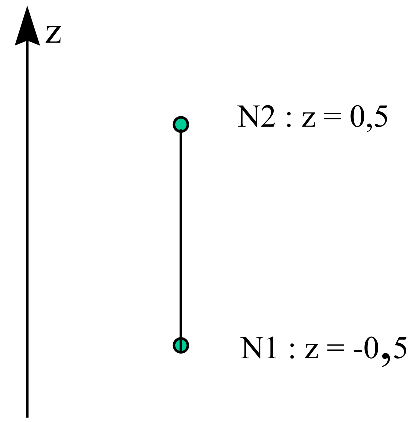
with $a = \dfrac{\lambda_l \Delta T}{C_0^\varepsilon}$ and $b = -\dfrac{3\alpha_0 K_0 T_0 \varepsilon - 3\alpha_l^m T_0 p_l}{C_0^\varepsilon}$

The variational formulation of this expression (in the unidimensional case) is then the following one:

$$\int_\Omega T.\hat{T}\,dz + a\int_\Omega \frac{\partial T}{\partial z}\frac{\partial \hat{T}}{\partial z}dz - a\int_{\partial\Omega}\frac{\partial T}{\partial z}\hat{T}\,dz = \int_\Omega b.\hat{T}\,dz \quad (4)$$

To establish the analytical solution a single element of degree 1 is considered since in modeling THM the thermohydraulic part is treated by linear elements.

That is to say a linear element:



One points out the boundary conditions: $\dfrac{\partial T}{\partial z} = 0$ at 2 ends (z=0,5 and Z = -0.5)

The temperature on the basis as of functions of forms is written in the following way:

$$T(z,t) = \sum_{i=1}^{i=2} T^i(t)\lambda_i(z)$$

with

$$\lambda_1(z) = 0{,}5(1+2z)$$

and

$$\lambda_2(z) = 0{,}5(1-2z)$$

The following matrices then are introduced:

$$[A] = [A_{ij}]\,;\, A_{ij} = \int_{-0,5}^{0,5} \lambda_i \lambda_j\, dz$$

$$[B]=[B_{ij}]\,;\,B_{ij}=\int_{-0,5}^{0,5}\frac{\partial\lambda_i}{\partial z}\frac{\partial\lambda_j}{\partial z}dz$$

what leads to:

and

$$[A]=\frac{1}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \qquad [B]=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

One notes then classically:

$$\{T\}=\begin{Bmatrix}T^1\\ T^2\end{Bmatrix}$$

and

$$\{\frac{\partial T}{\partial t}\}=\begin{Bmatrix}\frac{\partial T^1}{\partial t}\\ \frac{\partial T^2}{\partial t}\end{Bmatrix}$$

The equation (4) becomes then

$$[A]\{T\}+a[B]\{T\}-a[A]\{\frac{\partial T}{\partial t}\}=[A]\{b\}$$

with the boundary conditions imposed (null displacement in bottom and worthless flows of temperatures at the two ends), one a:

$$\{b\}=\begin{Bmatrix}0\\ b\end{Bmatrix}\,et\,\{\frac{\partial T}{\partial t}\}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$$

What with final gives us

$$\begin{Bmatrix}T^1\\ T^2\end{Bmatrix}+a[A]^{-1}[B]\begin{Bmatrix}T^1\\ T^2\end{Bmatrix}=\begin{Bmatrix}0\\ b\end{Bmatrix}$$

Finally one obtains:

$$\begin{bmatrix}1+6a & -6a\\ -6a & 1+6a\end{bmatrix}\{T\}=\begin{Bmatrix}T^1\\ T^2\end{Bmatrix}$$

2.2 Sizes and results of reference

For a time court of 100s, one will have

$$\begin{Bmatrix}T^1\\ T^2\end{Bmatrix}=\begin{Bmatrix}2.10^{-14}\\ 1,045.10^{-7}\end{Bmatrix}$$

One will consider T^1 no one.

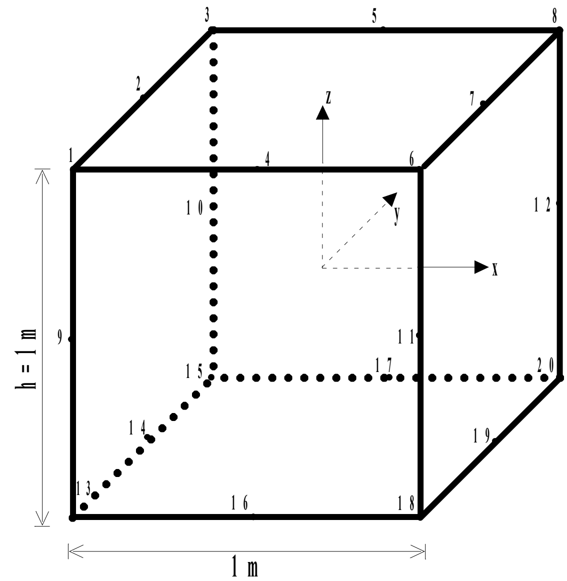
2.3 Uncertainty on the solution

None

3 Modeling A

3.1 Characteristics of modeling

Voluminal modeling `3D_THM`



3.2 Characteristics of the grid

1 mesh `HEXA20`.

3.3 Sizes tested and results

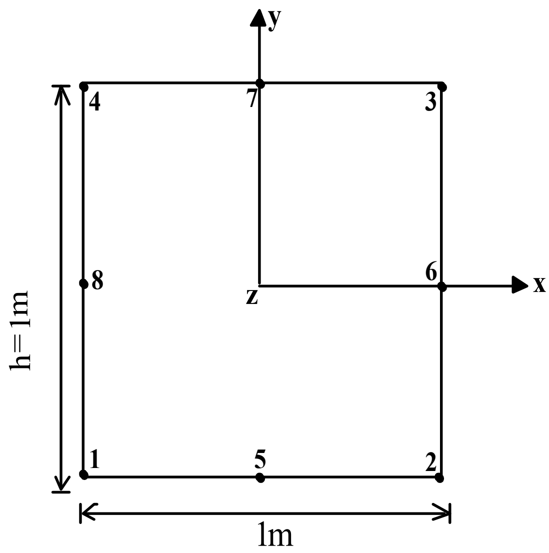
Discretization in time: only one step of time of $100\,s$. The diagram in time is implicit $(\theta=1)$. The results correspond perfectly to the analytical solution.

Node	Type of value	Moment (s)	Reference (analytical)	Tolerance $(\%)$
N1 , N3	*TEMP*	10^2	-1.045×10^{-7}	0,1
N6 , N8	*TEMP*	10^2	-1.045×10^{-7}	0,1

4 Modeling B

4.1 Characteristics of modeling

Plane modeling: `D_PLAN_THM`



4.2 Characteristics of the grid

1 mesh `QUAD8`.

4.3 Sizes tested and results

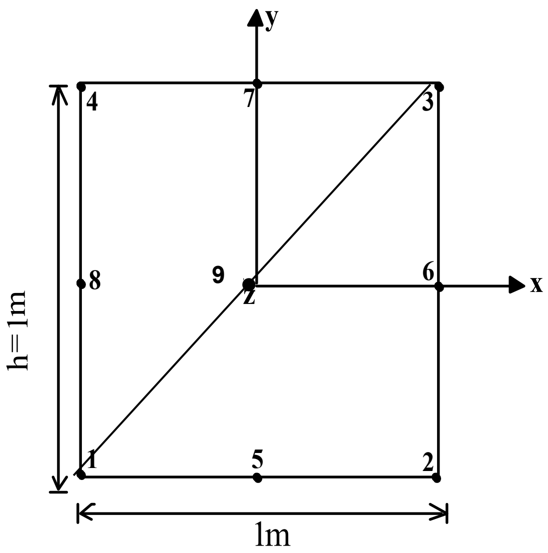
Discretization in time: only one step of time of 100s. The diagram in time is implicit $(\vartheta=1)$. The results correspond perfectly to the analytical solution.

Node	Type of value	Moment (S)	Reference (analytical)	Tolerance $(\%)$
N3	*TEMP*	10^2	-1.045×10^{-7}	0,1
N4	*TEMP*	10^2	-1.045×10^{-7}	0,1

5 Modeling C

5.1 Characteristics of modeling

Plane modeling: D_PLAN_THM



5.2 Characteristics of the grid

2 meshs TRIA6.

5.3 Sizes tested and results

Discretization in time: only one step of time: $10^2 s$. The diagram in time is implicit $(\vartheta=1)$. The results correspond perfectly to the analytical solution.

Node	Type of value	Moment (S)	Reference (analytical)	Tolerance $(\%)$
N3	*TEMP*	10^2	-1.045×10^{-7}	0,1
N4	*TEMP*	10^2	-1.045×10^{-7}	0,1

6 Modeling D

6.1 Characteristics of modeling

Voluminal modeling `3D_THM`.

6.2 Characteristics of the grid

2 mgoes `PENTA15`.

6.3 Sizes tested and results

Discretization in time: only one step of time: $10^2 s$. The diagram in time is implicit $(\theta=1)$. The results correspond perfectly to the analytical solution.

Node	Type of value	Moment (S)	Reference (analytical)	Tolerance $(\%)$
N3	*TEMP*	10^2	-1.045×10^{-7}	$0{,}1$
N4	*TEMP*	10^2	-1.045×10^{-7}	$0{,}1$

7 Summary of the results

The results are in coherence with the analytical solution.