HSNV120 - Hyperelastic traction of a bar under thermal loading

Summary:

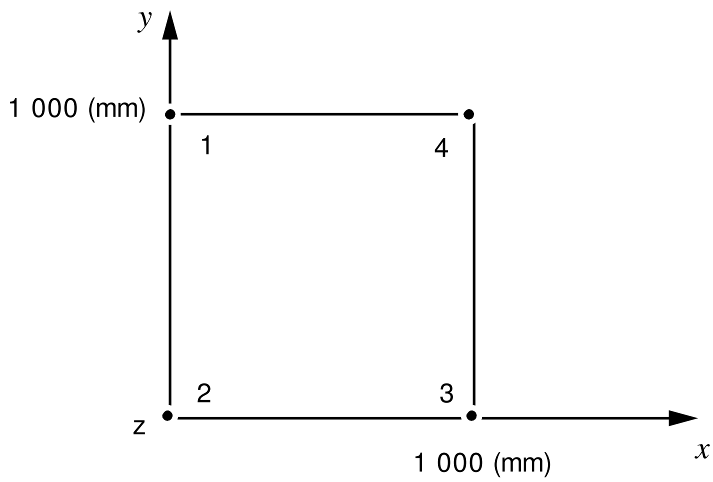
This quasi-static thermomechanical test consists in heating a parallelepipedic bar uniformly, to subject it to an important traction for finally letting it return in a discharged state. One thus validates the kinematics of the great hyperelastic deformations (order STAT_NON_LINE, keyword BEHAVIOR) for a non-linear relation DEC elastic behavior (ELAS_VMIS_LINE and ELAS_VMIS_TRAC) with thermal loading.

The bar is modelled by a voluminal element (HEXA20, modeling A) or quadrangular (QUAD8, assumption of the plane constraints, modeling B).

Results got by *Aster* do not differ from the theoretical solution.

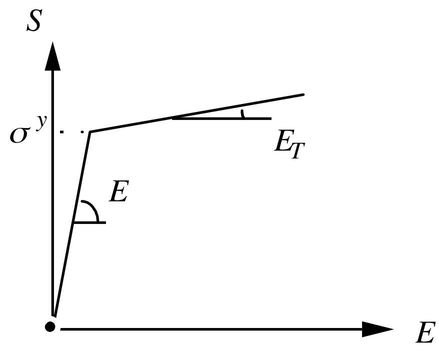
1 Problem of reference

1.1 Geometry



1.2 Material properties

The material obeys a law of isotropic nonlinear behaviour hyperelastic to isotropic linear work hardening.



$E = 2.10^5$ MPa

$E_T = 2.10^3$ MPa

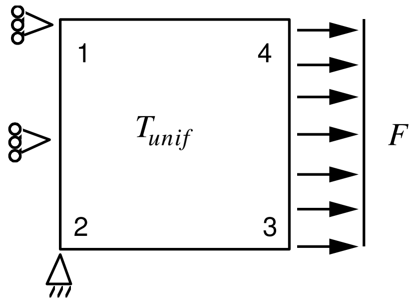
$\sigma^y = 10^3$ MPa

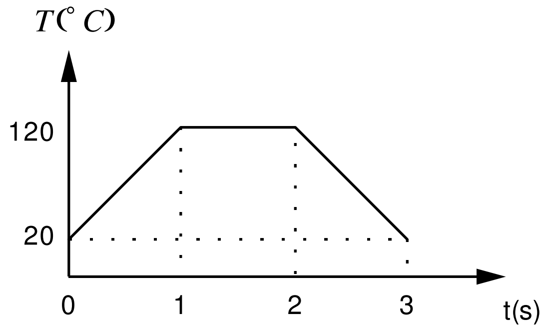
$\nu = 0,3$

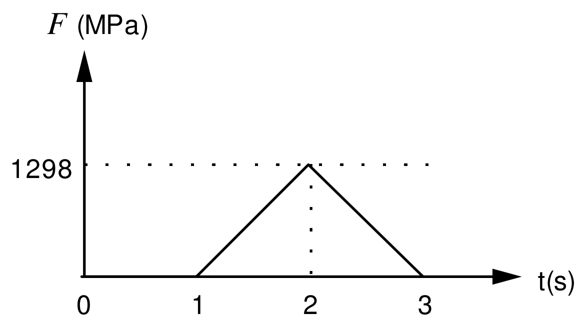
$\alpha = 10^{-4}$ K^{-1}

1.3 Boundary conditions and loadings

The bar blocked in the direction Ox on the face [1.2] is subjected to a uniform temperature T and a tractive effort F distributed on the face [3.4]. The sequences of loading are the following ones:







Temperature of reference: $T_{réf}=20°C$.

2 Reference solution

2.1 Method of calculating used for the reference solution

The field of displacement is sought U in the form:

$$\mathbf{U}(x,y,z) = \begin{pmatrix} ux \\ vy \\ vz \end{pmatrix}$$

The gradient of the transformation, the deformation and its mechanical share are then:

$$\mathbf{F} = \begin{bmatrix} 1+u & 0 & 0 \\ 0 & 1+v & 0 \\ 0 & 0 & 1+v \end{bmatrix}$$

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\,\mathbf{F} - \mathbf{1}\right) = \begin{bmatrix} \frac{u(u+2)}{2} & 0 & 0 \\ 0 & \frac{v(v+2)}{2} & 0 \\ 0 & 0 & \frac{v(v+2)}{2} \end{bmatrix}$$

$$\mathbf{E^m} = \mathbf{E} - \alpha\,\Delta T\,\mathbf{1} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}$$

with:

$$\begin{cases} a = \frac{u(u+2)}{2} - \alpha\,\Delta T \\ b = \frac{v(v+2)}{2} - \alpha\,\Delta T \end{cases}$$

Note:

$(E^m)_{\text{eq}} = |a-b| = a-b$ (it is supposed that $a>b$)

The relation of behavior is written:

$$\begin{cases} S_{xx} = K(a+2b) + \frac{2}{3} G(a-b) \\ S_{yy} = S_{zz} = K(a+2b) - \frac{1}{3} G(a-b) \end{cases}$$

with:

$$3K = \frac{E}{1-2\nu} \quad \text{module de compressibilité}$$

To determine G by taking account of linear work hardening, one introduces:

- the modulus of rigidity: $2\mu = \frac{E}{1+\nu}$
- the module of work hardening: $R' = \frac{E\,E_T}{E-E_T}$,

The “internal variable pseudonym” p is worth then:

$$p = \frac{2\mu\left(\mathbf{E}^{\mathbf{m}}\right)_{eq} - \sigma^y}{R'+3\mu} = \frac{2\mu(a-b)-\sigma^y}{R'+3\mu}$$

Finally, G is written:

$$G = \frac{\sigma^y + R'\,p}{a-b}$$

By taking account of the boundary conditions:

$$\begin{cases} S_{xx} = \frac{F}{1+u} & \text{(charge morte)} \\ S_{yy} = 0 & \text{(bord libre)} \end{cases}$$

The system to be solved is written:

$$\begin{cases} K(a+2b) + \frac{2}{3}\left[\sigma^y + R'\frac{2\mu(a-b)-\sigma^y}{R'+3\mu}\right] = \frac{F}{1+u} \\ K(a+2b) - \frac{1}{3}\left[\sigma^y + R'\frac{2\mu(a-b)-\sigma^y}{R'+3\mu}\right] = 0 \end{cases}$$

He is also written:

$$\begin{cases} 3K(a+2b) &= \dfrac{F}{1+u} \\ 2\mu(a-b) &= \dfrac{F}{1+u}\left(1+\dfrac{3\mu}{R'}\right) - \sigma^y \dfrac{3\mu}{R'} \end{cases}$$

With F fixed, it is thus about a nonlinear system in u and v, since a is quadratic in u and b quadratic in v.

Nevertheless, one can choose to fix u (thus a) and to solve a linear system in F and b (from which one deduces p and v):

- $a = \dfrac{u(u+2)}{2} - \alpha\,\Delta T$
- $\begin{cases} \dfrac{1}{1+u} F - 6\,K\,b &= 3\,K\,a \\ \left(1+\dfrac{3\mu}{R'}\right) \dfrac{1}{1+u} F + 2\mu\,b &= 2\mu\,a + \sigma^y \dfrac{3\mu}{R'} \end{cases}$
- $p = \dfrac{2\mu\,(a-b) - \sigma^y}{R' + 3\mu}$
- $v = \sqrt{1+2(b+a\,\Delta T)} - 1$

It then remains to express the constraint of Cauchy:

$$\boldsymbol{\sigma} = \frac{1}{\mathrm{Det}(\mathbf{F})} \mathbf{F} \cdot \mathbf{S} \cdot \mathbf{F}^T$$

That is to say here:

$$\begin{cases} \sigma_{xx} &= \dfrac{1+u}{(1+v)^2} S_{xx} \\ \sigma_{yy} &= \sigma_{zz} = 0 \end{cases}$$

As for the force exerted on the face [3.4], because of assumption of died loads, she is written simply:

$$\begin{cases} \mathsf{F}_x &= F\ S_o \qquad \text{où } S_o : \text{ surface initiale de la face } [3,4] \\ \mathsf{F}_y &= 0 \\ \mathsf{F}_z &= 0 \end{cases}$$

2.2 Results of reference

One will adopt like results of reference displacements, the constraint of Cauchy and the force exerted on the face $[3,4]$ (in 3D only):

At time $t=2\ s$ ($\Delta T=100\,°C$, traction F)

In fact, one seeks F such as lengthening:

$$u = 0{,}1$$

- $K=166\,666\ MPa$ $\mu=76\,923\ MPa$ $R'=2\,020\ MPa$
- $a=0.095$
- $\begin{cases} 0.90909\ F - 10^6\ b = 47\,500 \\ 104.76\ F + 153.85\ 10^3\ b = 128.85\ 10^3 \end{cases}$

$$\Rightarrow \begin{cases} F = 1\,298 \text{ MPa} \\ b = -0.046 \end{cases}$$

- $p = 8.91\ 10^{-2}$
- $v = -3.70\ 10^{-2}$
- $\sigma_{xx} = 1\,399.66 \text{ MPa}$ $\sigma_{xy} = 0$ $F_x = 1\,298\ 10^9 \text{ N}$
  $\sigma_{yy} = 0$ $\sigma_{xz} = 0$ $F_y = 0$
  $\sigma_{zz} = 0$ $\sigma_{yz} = 0$ $F_z = 0$

At time $t=3\ s$ $(\Delta T=0,\ F=0)$

The bar returned in its initial state:

$$\begin{cases} \mathbf{U} = 0 \\ \boldsymbol{\sigma} = 0 \\ p = 0 \end{cases}$$

2.3 Uncertainty on the solution

The solution is analytical. With the rounding errors near, one can consider it exact.

2.4 Bibliographical references

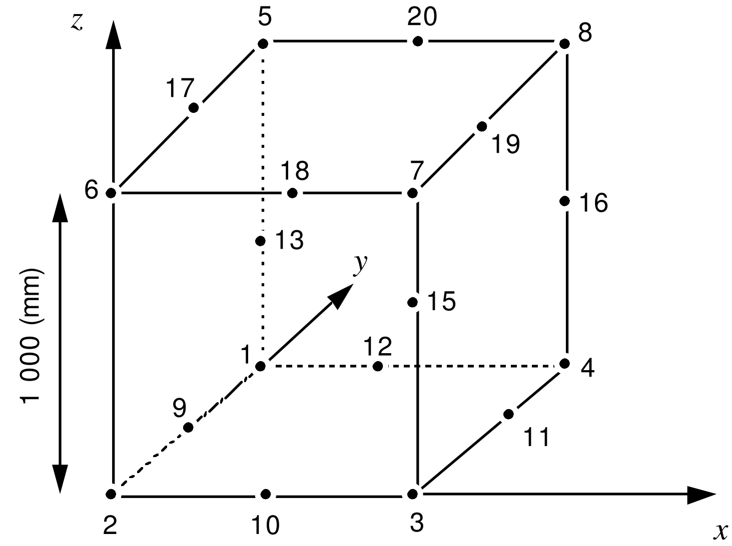
One will be able to refer to:

1) E. LORENTZ: A nonlinear relation of behavior hyperelastic - Note interns EDF DER HI-74/95/011/0

3 Modeling A

3.1 Characteristics of modeling

Voluminal modeling: 1 mesh `HEXA20`
1 mesh `QUAD8`



Boundary conditions:

$N2$: $U_x = U_y = U_z = 0$

$N1$: $U_x = U_z = 0$

$N6$: $U_x = U_y = 0$

$N9$, $N13$, $N14$, $N5$, $N17$: $U_x = 0$

Load: Traction on the face $[3\ 4\ 8\ 7\ 11\ 16\ 19\ 15]$

3.2 Characteristics of the grid

Many nodes: 20

Many meshs: 2

1 `HEXA20`
1 `QUAD8`

3.3 Sizes tested and results

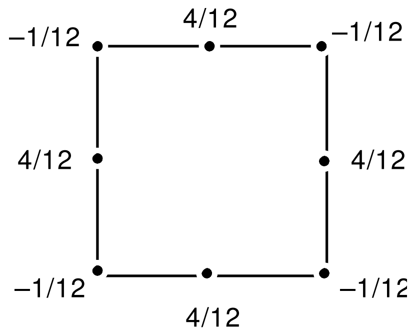
Identification	Reference
$t=2$ Displacement DX ($N8$)	100
$t=2$ Displacement DY ($N8$)	– 37
$t=2$ Displacement DZ ($N8$)	– 37
$t=2$ Constraints $SIGXX$ ($PG1$)	1399.66
$t=2$ Constraints $SIGYY$ ($PG1$)	11013.986
$t=2$ Constraints $SIGZZ$ ($PG1$)	0
$t=2$ Constraints $SIGXY$ ($PG1$)	0
$t=2$ Constraints $SIGXZ$ ($PG1$)	0
$t=2$ Constraints $SIGYZ$ ($PG1$)	0
$t=2$ Variable p $VARI$ ($PG1$)	8.9110^{-2}
$t=3$ Displacement DX ($N8$)	0
$t=3$ Displacement DY ($N8$)	0
$t=3$ Displacement DZ ($N8$)	0
$t=3$ Constraints $SIGXX$ ($PG1$)	0
$t=3$ Constraints $SIGYY$ ($PG1$)	0
$t=3$ Constraints $SIGZZ$ ($PG1$)	0
$t=3$ Constraints $SIGXY$ ($PG1$)	0
$t=3$ Constraints $SIGXZ$ ($PG1$)	0
$t=3$ Constraints $SIGYZ$ ($PG1$)	0
$t=3$ Variable p $VARI$ ($PG1$)	0
$t=2$ Nodal force DX ($N8$)	-1.081710^{8}
$t=2$ Nodal force DY ($N8$)	0
$t=2$ Nodal force DZ ($N8$)	0

3.4 Remarks

Calculation of the nodal force:

The force applied to the face $[3,4]$, F_x, is distributed between the various nodes according to following weighting:

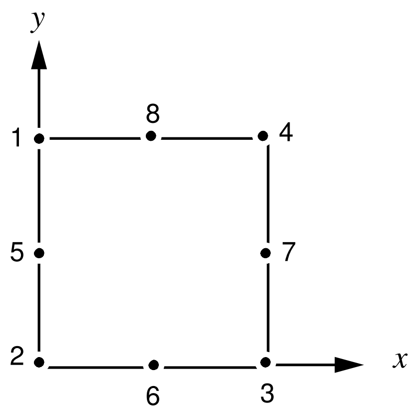
- nodes tops: $-1/12\,\mathsf{F}_x$
- nodes mediums: $4/12\,\mathsf{F}_x$



4 Modeling B

4.1 Characteristics of modeling

Modeling 2D plane constraints: 1 mesh QUAD8
1 mesh SEG3



Boundary conditions:

$N2$: $U_x = 0$ $U_y = 0$
$N1$: $U_x = 0$
$N5$: $U_x = 0$

Loading:

Traction on the face $[3\,4\,7]$ (mesh SEG3)

4.2 Characteristics of the grid

Many nodes: 8

Many meshs: 2

1 QUAD8
1 SEG3

4.3 Sizes tested and results

Identification	Reference
$t=2$ Displacement DX ($N4$)	100
$t=2$ Displacement DY ($N4$)	– 37
$t=2$ Constraints $SIGXX$ ($PG1$)	1399.66
$t=2$ Constraints $SIGYY$ ($PG1$)	0
$t=2$ Constraints $SIGXY$ ($PG1$)	0
$t=2$ Variable p $VARI$ ($PG1$)	$8.91 10^{-2}$
$t=3$ Displacement DX ($N4$)	0
$t=3$ Displacement DY ($N4$)	0
$t=3$ Constraints $SIGXX$ ($PG1$)	0
$t=3$ Constraints $SIGYY$ ($PG1$)	0
$t=3$ Constraints $SIGXY$ ($PG1$)	0
$t=3$ Variable p $VARI$ ($PG1$)	0

5 Summary of the results

The digital and analytical results coincide remarkably. One can however be astonished by the execution time manifestly longer for modeling in plane constraints ($123{,}8\,s$) that for 3D ($47{,}2\,s$). The difference is explained by a discretization in time much finer for the plane constraints, related to problems of convergence (the algorithm of resolution of the nonlinear scalar equation in p is still rudimentary).