HSNV103 - Thermoplasticity and metallurgy in plane deformations

Summary:

One treats the determination of the mechanical evolution of a right-angled parallépipède in plane deformations subjected to evolutions thermics $T(t)$ and metallurgical $Z(t)$ known and uniform (the metallurgical transformation is of bainitic type).

The elements used are two-dimensional elements in plane deformations and the relation of behavior is the plasticity of von Mises with linear isotropic work hardening (for modeling B, one also takes account of the plasticity of transformation).

The yield stress and the slope of the traction diagram depend on the temperature and the metallurgical composition.

The dilation coefficient α depends on the metallurgical composition.

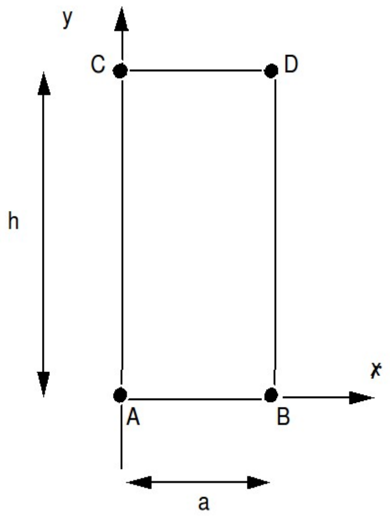
For modeling A (without plasticity of transformation), the reference solution is obtained by the analytical resolution of the problem. For modeling B (with plasticity of transformation), the reference solution is obtained by the digital resolution of the problem by using axisymmetric elements for which one imposes the condition of plane deformations.

Results provided by *Code_Aster* are very satisfactory with errors lower than 0,5%.

1 Problem of reference

1.1 Geometry

Width: $a=0.05\,m$
Height: $h=0.2\,m$



1.2 Properties of materials

$E=200000.10^6$ Pa

$\nu=0.3$

$\alpha_{fbm}=15.\ 10^{-6}\ °C^{-1}$

$\alpha_{aust}=23.5\ 10^{-6}\ °C^{-1}$

$\varepsilon_{ref_{fbm}}=2.52\ 10^{-3}$

$T^{ref}=900°\,C$

$\rho.cp=2\,000\,000$ J.m^{-3} °C^{-1}

$\sigma_y^{aust}=\sigma_o^{aust}+s^{aust}(T-T^o)$

$\sigma_o^{aust}=400.\ 10^6$ Pa

$s^{aust}=0.5\ 10^6$ Pa.°C^{-1}

$\sigma_y^{fbm}=\sigma_o^{fbm}+s^{fbm}(T-T^o)$

$\sigma_o^{fbm}=530.\ 10^6$ Pa

$s^{fbm}=0.5\ 10^6$ Pa.°C^{-1}

$\lambda=9999.9$ W.m^{-1} °C^{-1}

notons $H(t)=\dfrac{\alpha(t)\cdot E(t)}{E(t)-\alpha(t)}$

$H^{aust}=H_o^{aust}+\lambda^{aust}(T-T^o)$

$H_o^{aust}=1250\ 10^6$ Pa

$\lambda^{aust}=-5\ 10^6$ Pa.°C^{-1}

$H^{fbm}=H_o^{fbm}+\lambda^{fbm}(T-T^o)$

$H_o^{fbm}=-\ 50\ 10^6$ Pa

$\lambda^{fbm}=-\ 5\ 10^6$ Pa.°C^{-1}

$k^{fbm}=1.\ 10^{-10}$ Pa^{-1}

$*^{aus}$	=	characteristics relating to the austenitic phase
$*^{fbm}$	=	characteristics relating to the phases ferritic, bainitic and martensitic
α_{fbm}	=	thermal dilation coefficient of the phases ferritic, bainitic and martensitic
α_{aus}	=	dilation coefficient of the austenitic phase
$\varepsilon_{réf\ fbm}$	=	deformation of the phases ferritic, bainitic and martensitic at the temperature of reference, austenite being regarded as not deformed at this temperature: translated the difference in compactness between the cubic crystallographic structures with centered faces (austenite) and cubic centered (ferrite).

TRC to model a metallurgical evolution of bainitic type, on all the structure, of the form:

$$Z_{fbm}=\begin{cases} 0. & si\ \ t\leq\tau_1 & \tau_1=60\,s \\ \dfrac{t-\tau_1}{\tau_2-\tau_1} & si\ \ \tau_1\leq t<\tau_2 & \tau_2=112\,s \\ 1. & si\ \ t\geq\tau_2 & \end{cases}$$

Law of plasticity of transformation: $\dot{\varepsilon}^{pt}=K^{fbm}F(Z_{fbm})\dot{Z}_{fbm}$

with $F(Z_{fbm})=Z_{fbm}(Z-Z_{fbm})$

Notations: $T(\tau_1)=T_1$
$T(\tau_2)=T_2$

1.3 Boundary conditions and loadings

- $u_Y=0$ on the side AB ; $u_X=0$ in A .
- $T=T^0+\mu t$, $\mu=-5\,°C.s^{-1}$ on all the structure.
- The loading on the structure is due with the phenomena of thermal and metallurgical dilation constrained in the direction z by the condition of plane deformations.

1.4 Initial conditions

$T^0=900\,°C=T^{ref}$

2 Reference solution (for modeling A)

2.1 Method of calculating used for the reference solution

Before transformation, thermoelastic solution until in t_1 such as:

$$\sigma_{zz}=-E\,\varepsilon^{th}=\sigma_y \quad\Leftrightarrow\quad T-T^0=\frac{-\sigma_y^{o\gamma}}{E\,\alpha+s}=76.92\,°C$$
$$\Leftrightarrow\quad t_1=15.38\,s$$

thus for $t\le t_1$: $\sigma_{zz}=-E\,\alpha_\gamma(T-T^0)$

Before transformation, and for $t\ge t_1$, thermoelastoplastic solution such as:

$$\varepsilon_{zz}=0 \text{ and } \sigma_{zz}=R\,\varepsilon_{zz}^p+\sigma_y$$

from where $\varepsilon_{xx}^p=\dfrac{-\sigma_y(T)-E\,\alpha_\gamma(T-T^0)}{E+R(T)}$ and $\sigma_{zz}=-E(\varepsilon_{zz}^p+\alpha_\gamma(T-T^0))$

During the transformation, one remains in load as long as $\dot{\varepsilon}^{th}<0$

$$\dot{\varepsilon}^{th}=0 \quad\Leftrightarrow\quad T=\frac{(\alpha_\alpha-\alpha_\gamma)T^0-\varepsilon_{réf\ fbm}+\alpha_\alpha T_1-\alpha_\gamma T_2}{2(\alpha_\alpha-\alpha_\gamma)}=538.82\,°C$$
$$\Leftrightarrow\quad t=t_2=72.23\,s$$

for $T>538.82\,°C$

there is thus a thermoelastoplastic soution with phase shift:

$$\varepsilon_{zz}^p=\frac{-\sigma_y(T,Z)-E\left[\alpha(Z)(T-T^0)+Z\,\varepsilon_{réf\ fbm}\right]}{E+R(T,Z)} \text{ and } \sigma_{zz}=-E(\varepsilon_{zz}^p+\alpha(Z)(T-T^0)+Z\,\varepsilon_{réf\ fbm})$$

and for $T<538.82\,°C$

there is thus a thermoelastic soution with phase shift:

$$\sigma_{zz}=-E\left[\alpha(Z)(T-T^0)+Z\,\varepsilon_{réf\ fbm}+\varepsilon_{zz}^p(t_2)\right]$$

After the transformation, one replastifie when: $\sigma_{zz}=R(T,Z)\,\varepsilon_{zz}^p+\sigma_y(T,Z)$

with $\varepsilon_{zz}=\dfrac{\sigma_{zz}}{E}+\alpha_\alpha(T-T^0)+\varepsilon_{réf\ fbm}=0$

that is to say $T=267.06\,°C$

and one thus has for $T<267.06\,°C$:

$$\varepsilon_{zz}^p=\frac{E\left[-\varepsilon_{réf\ fbm}-\alpha_\alpha(T-T^0)\right]-\sigma_y(T,Z)}{R(Z,T)+E} \text{ and } \sigma_{zz}=R(T,Z)\,\varepsilon_{zz}^p+\sigma_y(T,Z)$$

2.2 Results of reference

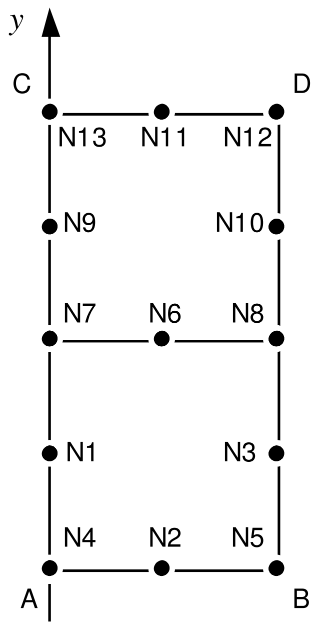
With $t=16\,s$:	χ	p	ε_{xx}	σ_{zz}
With $t=60\,s$:	χ	p	ε_{xx}	σ_{zz}
With $t=72\,s$:	χ	p		
With $t=112\,s$:	χ	p		σ_{zz}
With $t=176\,s$:	χ		ε_{xx}	σ_{zz}

2.3 Bibliography

1) DONORE A.M. - WAECKEL F. - Influence of structure transformations in the elastoplastic laws of behavior Notes HI-74/93/024.

3 Modeling A

3.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

3.2 Characteristics of the grid

Many nodes: 13.

Many meshs and types: 2 meshs QUAD8, 6 meshs SEG3.

4 Results of modeling A

4.1 Values tested

One tests the structural parameters of data results:

Identification	Reference
INST for NUME_ORDRE= 176	176.0
ITER_GLOB for NUME_ORDRE=176	2

Identification		Reference
ε_{xx}	$t=16\,s$	$-2.4599\ 10^{-3}$
χ	$t=16\,s$	1
σ	$t=16\,s$	$360.13\ 10^{6}$
p	$t=16\,s$	$7.9345\ 10^{-5}$
ε_{xx}	$t=60\,s$	$-1.0309\ 10^{-2}$
p	$t=60\,s$	$5.7213\ 10^{-3}$
σ	$t=60\,s$	$265.73\ 10^{6}$
p	$t=72\,s$	$5.8420\ 10^{-3}$
χ	$t=112\,s$	0
σ	$t=112\,s$	$12.82\ 10^{6}$
p	$t=112\,s$	$5.8421\ 10^{-3}$
ε_{xx}	$t=176\,s$	$-1.5886\ 10^{-2}$
χ	$t=176\,s$	1
σ	$t=176\,s$	$133.55\ 10^{6}$

4.2 Remarks

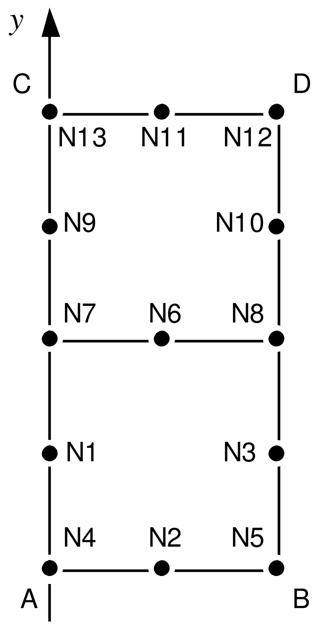
In this modeling:

$$\varepsilon_{zz}^{pt}(T,Z)=0$$

The error on the plastic deformation cumulated at 72 seconds comes by way of the mistake made on the digital description of the metallurgical transformation which is, at this moment, from approximately 0.5%.

5 Modeling B

5.1 Characteristics of modeling



$A=N4$, $B=N5$, $C=N13$, $D=N12$.

5.2 Characteristics of the grid

Many nodes: 13.

Many meshs and types: 2 meshs QUAD8, 6 meshs SEG3.

6 Results of modeling B

6.1 Values tested

Identification		Reference
σ	$t=60\,s$	265.73 10^{6}
p	$t=60\,s$	5.7213 10^{-3}
ε_{yy}	$t=89\,s$	– 1.0325 10^{-2}
p	$t=89\,s$	5.7213 10^{-3}
σ	$t=89\,s$	– 13,545 10^{6}
ε_{yy}	$t=112\,s$	– 8.9197 10^{-3}
σ	$t=112\,s$	101.39 10^{6}
p	$t=112\,s$	5.7213 10^{-3}
ε_{yy}	$t=176\,s$	– 1.5884 10^{-2}
p	$t=176\,s$	9.3610 10^{-2}
σ	$t=176\,s$	130.72 10^{6}

6.2 Remarks

In this modeling, one takes into account the term due to the plasticity of transformation:

$$\dot{\varepsilon}^{pt}(Z,T)\neq 0 \text{ when } \dot{Z}\neq 0$$

The reference solution is obtained by the digital resolution of the problem with axisymmetric elements for which one imposes the condition of plane deformations.

7 Summary of the results

Results found with *Code_Aster* are very satisfactory, with percentages of error lower than 0.5%.