

## SSND103 - Validation of a bilinear law of behavior on a discrete element (application to the bolted assemblies)

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### Summary:

The objective of this CAS-test is to validate a law of bearing bilinear behavior on a discrete element.

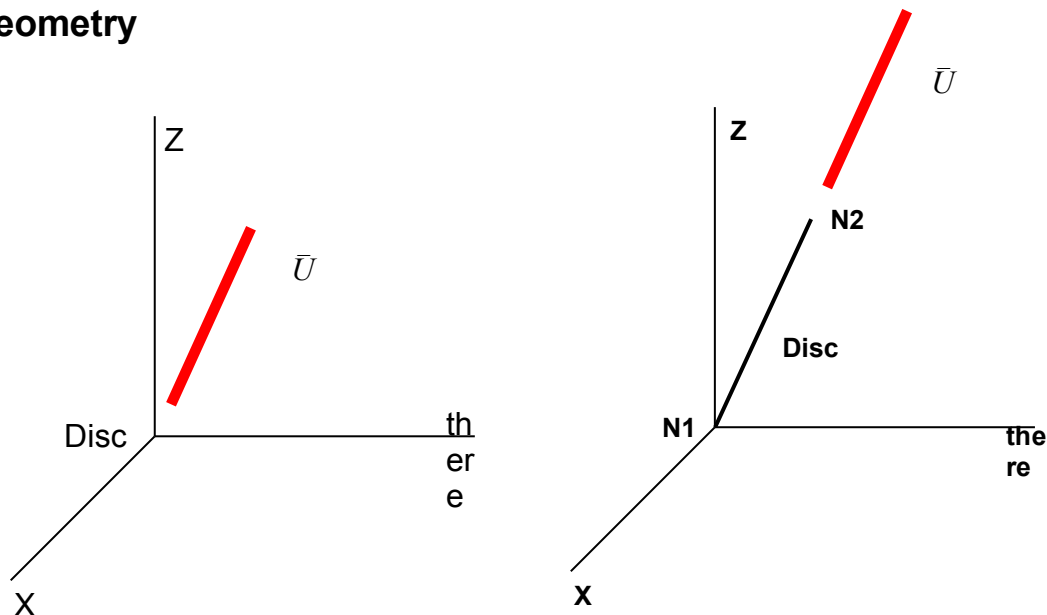
This law of behavior was developed within the framework of a study in which the behavior of the screw of a bolted assembly is modelled by an affected discrete element of the same behavior.

The law of behavior `DIS_BILI_ELAS` demand for arguments two apparent stiffnesses of the screw (fasteners in contact or not) as well as the value of the effort of pretightening imposed on the screw.

It is checked, for various temperatures and various directions of request, that the slopes of the load diagrams obtained, when the discrete one is requested, correspond to the apparent stiffnesses and that the change of incline corresponds to the effort of imposed pretightening.

## 1 Problem of reference

### 1.1 Geometry



*Figure  
has*

*Figure B*

In theory, one seeks to impose a loading (forces some or displacement) on a discrete element represented by a node such as in *Figure has*.

In practice, it is necessary to introduce a condition of blocking on this node in order to avoid any movement of solid body and of giving a physical direction to its stiffness.

This is why the grid will be composed of two nodes, the stiffness being assigned to the segment consisted their junction (cf. *Figure B*).

N.B. : If this segment has a nonworthless length (i.e the two nodes are not confused), its direction fixes the orientation of the discrete one.

- One calls  $N1$  the node to which the conditions of blocking will relate.
- One calls  $N2$  the node on which will be imposed the mechanical loadings.
- One calls *Disc* the discrete element of type `SEG2` which one affects the law of behavior to be validated.

### 1.2 Properties of material

One arbitrarily chooses to assign to the discrete element the data material of an elastic steel:  
 $E = 2.10^{11} Pa$ ,  $\nu = 0.3$ .

### 1.3 Boundary conditions and loadings

One imposes a condition of embedding on the node  $N1$ .

An effort of pretightening is imposed  $F_p$  with the discrete element.

One imposes a loading proportional in displacement to the node  $N2$ .

One imposes a constant field of temperature in the course of time.

It is considered that the discrete element can work only in translation, one affects a modeling to him `DIS_T`.

## 2 Reference solution

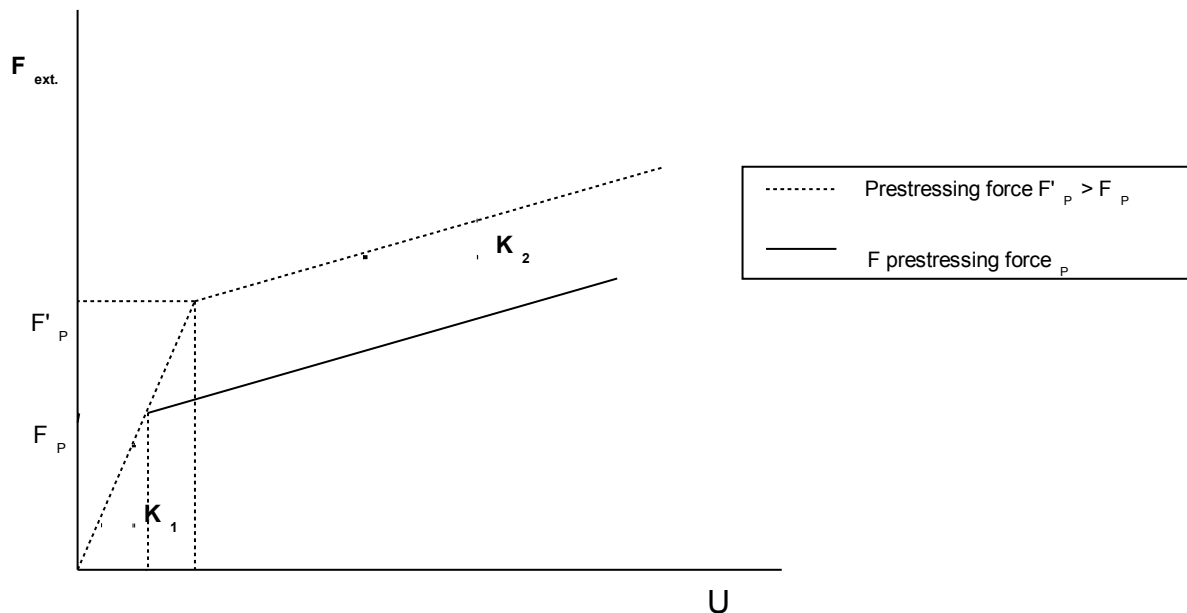
One assigns to the discrete element, via the bilinear law of behavior, two stiffnesses  $K_1$  and  $K_2$  functions of the temperature of the form  $K_i = \alpha_i - \beta_i \dot{T}$ .

One fixes arbitrarily here:

$$\alpha_1 = 2 \alpha_2 = 2 \cdot 10^8 \text{ N.m}^{-1}$$

$$\beta_1 = 2 \beta_2 = 4 \cdot 10^6 \text{ N.m}^{-1}$$

It is checked that, some is displacement  $U$  or effort of constraint  $F_p$  that one imposes, one obtains a nodal reaction  $F$  the discrete one checking the elastic law  $\Delta F = K_i \cdot \Delta U$ , where  $i=1$  if  $F \leq F_p$  and  $i=2$  if  $F > F_p$ .



One raises the nodal reactions of discrete for displacements ( $U_1 < U_2$ ) located on both sides of the change of incline.

Change of incline taking place for  $U_p = F_p / K_1$ , one obtains the reference solutions directly:

$$F_1 = K_1 \cdot U_1 \text{ and } F_2 = F_p + K_2 \cdot (U_2 - U_p)$$

that is to say  $F_2 = F_p \cdot (1 - K_2 / K_1) + K_2 \cdot U_2$

## 3 Modeling A

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### 3.1 Characteristics of modeling

One requests the discrete element in a uniaxial way. One imposes to him:

- a constant temperature  $T=0\text{ }^{\circ}\text{C}$ ,
- a prestressing force  $F_p=5.10^4\text{ N}$  and
- a displacement of axis (0x)  $U_{tot}=10^{-3}\text{ mm}$ .

### 3.2 Characteristics of the grid

The grid consists of two nodes connected by an element SEG2 of axis  $Ox$ .

### 3.3 Sizes tested and results

One chooses  $U_1=2.10^{-4}\text{ mm}$  and  $U_2=8.10^{-4}\text{ mm}$ .

After digital application, the reference solutions are:

$$F_1=4.10^4\text{ N}$$

$$F_2=1,05.10^5\text{ N}$$

Nodes	$U$	Reference
$N1$	$2.10^{-4}\text{ mm}$	$-4.10^4\text{ N}$
$N2$	$2.10^{-4}\text{ mm}$	$4.10^4\text{ N}$
$N1$	$8.10^{-4}\text{ mm}$	$-1,05.10^5\text{ N}$
$N2$	$8.10^{-4}\text{ mm}$	$1,05.10^5\text{ N}$

## 4 Modeling B

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### 4.1 Characteristics of modeling

Modeling B is in all points the same one as A except the temperature taken constant equalizes with  $25^{\circ}C$

### 4.2 Characteristics of the grid

The grid consists of two nodes connected by an element `SEG2` of axis  $Ox$ .

### 4.3 Sizes tested and results

One chooses  $U_1=2.10^{-4}mm$  and  $U_2=8.10^{-4}mm$ .

After digital application, the reference solutions are:

$$F_1=2.10^4 N$$

$$F_2=6,5.10^4 N$$

Nodes	$U$	Reference
$N1$	$2.10^{-4}mm$	$-2.10^4 N$
$N2$	$2.10^{-4}mm$	$2.10^4 N$
$N1$	$8.10^{-4}mm$	$-6,5.10^4 N$
$N2$	$8.10^{-4}mm$	$6,5.10^4 N$

## 5 Modeling C

### 5.1 Characteristics of modeling

Modeling C is in all points the same one as A except as regards direction of the request. One does not request any more the discrete one solely along his axis, but in the direction of the first trisecting one. One chooses, in addition,  $K_{ix} = K_{iy} = 2 \cdot K_{iz} = K_i$ , where  $K_i$  is given to the §2.

### 5.2 Characteristics of the grid

The grid consists of two nodes connected by an element SEG2 of axis  $Ox$ .

### 5.3 Sizes tested and results

One chooses  $U_1 = 2 \cdot 10^{-4} \text{ mm}$  and  $U_2 = 8 \cdot 10^{-4} \text{ mm}$ .  
After digital application, the reference solutions are:

$$F_{1x} = 4 \cdot 10^4 \text{ N}$$

$$F_{1y} = 4 \cdot 10^4 \text{ N}$$

$$F_{1z} = 2 \cdot 10^4 \text{ N}$$

$$F_{2x} = 10,5 \cdot 10^4 \text{ N}$$

$$F_{2y} = 10,5 \cdot 10^4 \text{ N}$$

$$F_{2z} = 6,5 \cdot 10^4 \text{ N}$$

Node	$U$	Components	Reference
N2	$2 \cdot 10^{-4} \text{ mm}$	$F_{1x}$	$4 \cdot 10^4 \text{ N}$
		$F_{1y}$	$4 \cdot 10^4 \text{ N}$
		$F_{1z}$	$2 \cdot 10^4 \text{ N}$
N2	$8 \cdot 10^{-4} \text{ mm}$	$F_{2x}$	$10,5 \cdot 10^4 \text{ N}$
		$F_{2y}$	$10,5 \cdot 10^4 \text{ N}$
		$F_{2z}$	$6,5 \cdot 10^4 \text{ N}$

## 6 Summary of the results

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The law of behavior `DIS_BILI_ELAS` give results perfectly in conformity with those resulting from the analytical expressions, that the stiffness is function of the temperature or differentiated according to the directions from space.