COMP001 – Test of elastoplastic behaviors. Simulation in a material point

Summary:

This test implements a simulation of a way of loading in constraints or deformations in a material point, i.e. on a model such as the stress and strain states are homogeneous at any moment. It thus makes it possible to test a certain number of elastoplastic models of behavior, with an aim of checking the robustness of their digital integration, their insensitivity compared to a change of units, the good taking into account of the variables of order whose the coefficients depend on the model, invariance compared to a total rotation applied to the problem, the accuracy of the tangent matrix.

Modeling a: this modeling makes it possible to validate the model VMIS_ISOT_LINE in 3D and C_PLAN.
Modeling b: this modeling makes it possible to validate the model VMIS_ISOT_TRAC in 3D and C_PLAN.
Modeling C: this modeling makes it possible to validate the model VMIS_CINE_LINE in 3D.
Modeling D: this modeling makes it possible to validate the model VMIS_ECMI_LINE in 3D and C_PLAN.
Modeling E: this modeling makes it possible to validate the model VMIS_ECMI_TRAC in 3D and C_PLAN.
Modeling F: this modeling makes it possible to validate the model VMIS_CIN1_CHAB in 3D.
Modeling G: this modeling makes it possible to validate the model VMIS_CIN2_CHAB in 3D.
Modeling H: this modeling makes it possible to validate the model VMIS_ISOT_PUIS in 3D.
Modeling J: this modeling makes it possible to validate the model MOHR_COULOMB in 3D.

1 Problem of reference

1.1 Geometry

Geometry (generated automatically in the macro-order SIMU_POINT_MAT [U4.51.12] is single and simple: it is acted in 3D of a tetrahedron on side 1, and as 2D of a triangle on side 1, with the nodes of which one applies linear relations to obtain a homogeneous stress and strain state.

1.2 Properties of material

The characteristics of materials are defined for each behavior via the order DEFI_MATERIAU. The elastic characteristics and of isotropic work hardening selected are those of standard steel $\mathrm{16MND5}$:

- $E=200\,000\,MPa$,
- $\nu=0.3$,
- $\sigma_y=437\,MPa$.

The other parameters describing the laws were selected starting from the cases test of ASTER. The two following tables summarize the whole of the laws of the code ASTER considered and the parameters associated:

Modeling	elastoplastic laws of code_Aster	parameters selected	criteria retained for the choice of the parameters
With	VMIS_ISOT_LINE	$SY=437\,MPa$, $DSY=2024\mathrm{MPa}$	Data material $\mathrm{16MND5}$
B	VMIS_ISOT_TRAC	traction diagram with $100°C$ $16\,MND5$	Data material $\mathrm{16MND5}$
C	VMIS_CINE_LINE	$SY=437\,MPa$, $DSY=2024\mathrm{MPa}$	Data material $\mathrm{16MND5}$
D	VMIS_ECMI_LINE	$SY=437\,MPa$, $DSY=2024\mathrm{MPa}$ $C_{PRAG}=1486.9$.	Data material $\mathrm{16MND5}$
E	VMIS_ECMI_TRAC	traction diagram with $100°C$ $16\,MND5$ $C_{PRAG}=1486.9$.	Data material $\mathrm{16MND5}$
F	VMIS_CIN1_CHAB	$SY=437.0$; $Rinf=758.0$; $b=2.3$; $Cinf=63767.0$ $Gamma0=341.0$	work hardening: $données16MND5$ other parameters: ssnv101c
G	VMIS_CIN2_CHAB	$SY=437.0$; $Rinf=758.0$; $b=2.3$; $C1inf=63767.0/2.0$ $C2inf=63767.0/2.0$ $Gam1=341.0$ $Gam2=341.0$	Work hardening given $\mathrm{16MND5}$ other parameters ssnv101c Kinematic choice $X1+X2=X$ of VMIS_CIN1_CHAB
H	VMIS_ISOT_PUIS	$SY=437.0$; $APUI=1.3$ $NPUI=3.5$	

J	MOHR_COULOMB	$E=619,3\,MPa$ $\nu=0,3$ $\varphi=33°$ $\psi=27°$ $c_0=1\,MPa$	Sand of Hostun

1.3 Boundary conditions and loadings

1.3.1 Characteristics of the ways of loading

Two ways of loading were defined to treat the cases 3D and 2D plan. They are common to all the laws of behavior. Each one of them respects, the following criteria:

- a cumulated plastic deformation, p , from 4 to 5% on the whole of the way,
- an increase of 1% of p during a portion of the way,

This calibration was carried out on the law `VMIS_ISOT_LINE`, then deferred on the other laws.

The loading suggested varies in a way uncoupled each component from the tensor of the deformations by successive stage. One proposes a cyclic way charges discharge with it by covering the states with traction and compression as well as an inversion with the signs with shearings in order to test a broad range of values.

Schematically, it follows a course on 8 segments $[O-A-B-C-O-C'-B'-A'-O]$ where the second part of way [O-C'-B'-A'-O] is symmetrical compared to the origin of the first $[O-A-B-C-O]$.

1.3.2 Application of the requests

One under investigation brings back material point (by using the macro-order `SIMU_POINT_MAT)` by requesting a homogeneous element of manner while imposing:

- in 3D, 6 components of the tensor of deformation:

$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz}\end{bmatrix}$$

- in 2D three components of the tensor

$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy}\\ \varepsilon_{xy} & \varepsilon_{yy}\end{bmatrix}$$

For a more general writing, the tensor of the deformations imposed will be broken up into a hydrostatic and deviatoric part on bases of shearing:

$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy}\\ \varepsilon_{xy} & \varepsilon_{yy}\end{bmatrix}=p\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+d\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}+\varepsilon_{xy}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\text{ in 2D,}$$

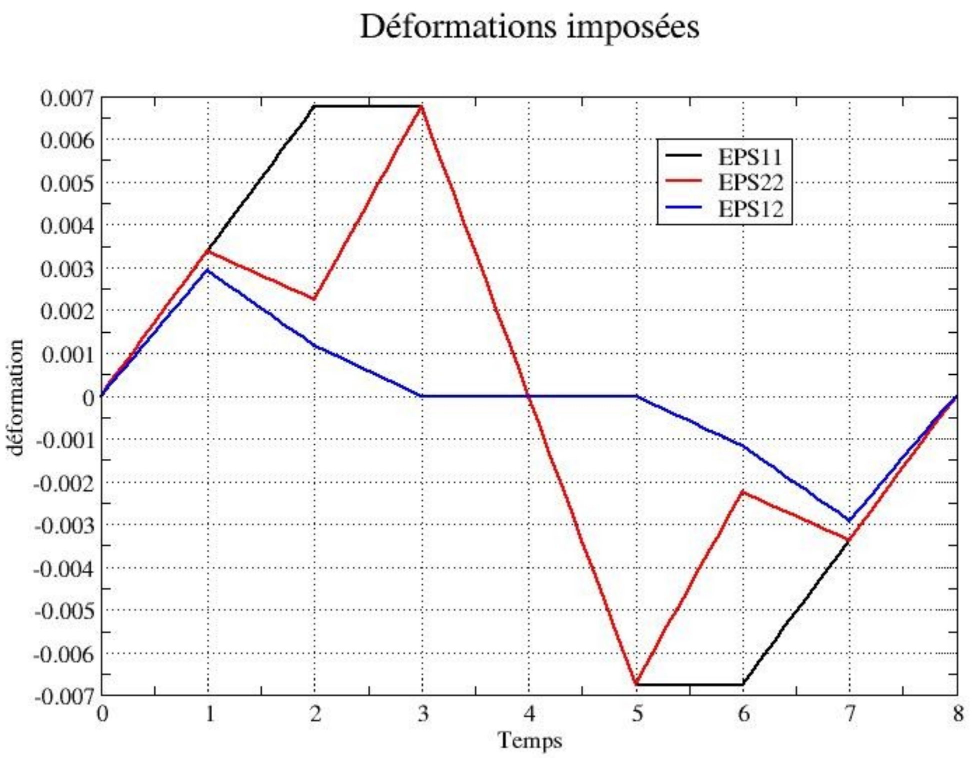
$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz}\end{bmatrix}=p\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}+d_1\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{bmatrix}+d_2\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}+\begin{bmatrix}0 & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & 0 & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & 0\end{bmatrix}\text{ in 3D}$$

1.3.3 Description of the way of deformation imposed in 2D

The way applied is described in the table below, the values of deformations are gauged with respect to the elastic module:

time	1	2	3	4	5	6	7	8
Not loading	A	B	C	O	C'	B'	A'	O
$\varepsilon_{xx} \times E$	675	1350	1350	0	-1350	-1350	-675	0
$\varepsilon_{yy} \times E$	675	450	1350	0	-1350	-450	-675	0
$\varepsilon_{xy} \times E/(1+\nu)$	450	180	0	0	0	-180	-450	0
P	675	900	1350		-1350	-900	-675	0
D	0	0	450	0	0	-450	0	0

This way is illustrated by the following graph:



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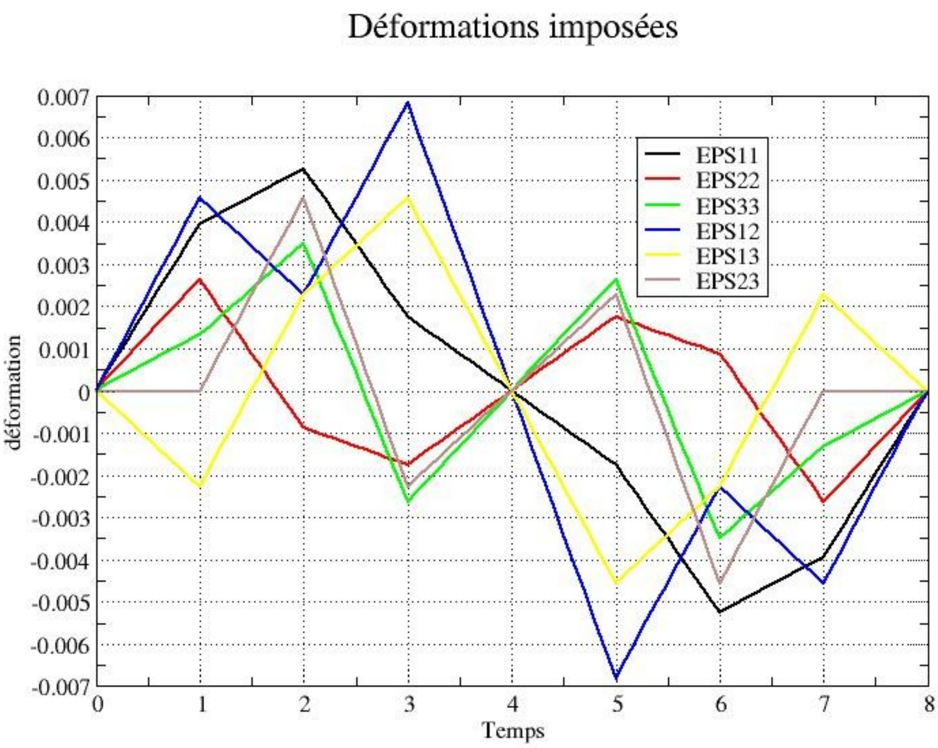
1.3.4 Description of the way of deformation imposed in 3D

The way applied is described in the table below, the values of deformations applied are gauged with respect to the elastic module:

N° segment	1	2	3	4	5	6	7	8
Segment	$0-A$	$A-B$	$B-C$	O	C'	B'	A'	O
$\varepsilon_{xx}\times E$	787.5	1050	350	0	-350	-1050	-787.5	0
$\varepsilon_{yy}\times E$	525.0	-175	-350	0	350	175	525	0
$\varepsilon_{zz}\times E$	262.5	700	-525	0	525	-700	-262.5	0
$\varepsilon_{xy}\times E/(1+\nu)$	700	350	1050	0	-1050	-350	-700	0
$\varepsilon_{xz}\times E/(1+\nu)$	-350	350	700	0	-700	-350	700	0
$\varepsilon_{yz}\times E/(1+\nu)$	0	700	-350	0	350	-700	0	0
P	525	525	-175	0	175	-525	-525	0
$d1$	262.5	525	525	0	-525	-525	-262.5	0
$d2$	262.5	-175	350	0	-350	175	-262.5	0

This way is illustrated by the following graph:



1.4 Initial conditions

Worthless constraints and deformations.

2 Reference solution

This test proceeds, for each modeling, with an intercomparison between the reference solution (obtained with a step of very fine time), the solution with a fairly coarse discretization, the solution with effect of the temperature (or another variable of order), the solution by changing the system of units (Pa in MPa), and that obtained after rotation or symmetry.

2.1 Definition of the cases tests of robustness

One proposes 3 angles of analysis to test the robustness of the integration of the laws of behavior:
- studies of equivalent problems
- checking of the tangent matrix
- study of the discretization of the step of time

For each one of them, one studies the evolution the relative differences between several calculations using the same law but presenting parameters or different options of calculations. The exploitation relates to the invariants of the tensor of the constraints: trace of the tensor, constraint of Von-Put and the internal variables of scalar nature: generally it is cumulated plasticity.

The total convergence criteria are the values envisaged by default by ASTER. (`RESI_GLOB_RELA=10-6, ITER_GLOB_MAXI=10`). One adopted a usual diagram of Newton for the reactualization of the tangent matrix: with each converged increment (`REAC_INC=1`) and all the 1 iterations (`REAC_ITER=1`).

2.2 Studies of equivalent problems

For a coarse discretization of the ways: 1 pas de time for each segment of the way, the solution obtained for each law is compared with 3 strictly equivalent problems for the state of the material point:
- Tpa , even way with a change of unit, one substitutes them Pa with MPa in the data materials and the possible parameters of the law,
- $Trot$, way by imposing the same tensor $\bar{\varepsilon}$ after a rotation: ${}^{t}R\cdot\bar{\varepsilon}\cdot R$ where R is a matrix of rotation. For the case 2D, the swing angle will be $\alpha=0.9\,radian$, for the configuration 3D, one chose the angles of Euler with the arbitrary values { $\Psi=0.9\,radian$, $\theta=0.7\,radian$, and $\varphi=0.4\,radian$ },
- $Tsym$, way by imposing the tensor $\bar{\varepsilon}$ after a symmetry: permutation of the axes x and y in 2D, permutation of x in y , y in z and z in x in 3D.

For each one of these problems, the solution (invariants of the constraints, cumulated equivalent plastic deformation) must be identical to the basic solution, obtained with the same discretization in time. The value of reference of the variation is thus 0. That means in practice that the found variation must be about the precision machine is approximately $1.\mathrm{E}-15$.

2.3 Test of the tangent matrix

One also tests for each behavior the tangent matrix, by difference with the matrix obtained by disturbance. There still, the value of reference is 0.

2.4 Study of the discretization of the step of time (A2)

One studies the behavior of the integration of the laws according to the discretization. For the same modeling, therefore a given behavior, one studies several different discretizations in time here, while multiplying by 5 the number of steps of the way of loading. In the reference [1], the discretization is pushed up to 3125 increments per segment on the same principle. Here, to limit the duration of the tests, one limits oneself to 3 successive refinements. This led to the following discretization:

Number of interval per segment of loading	1	5	25
Number of total step on the whole of the way	8	40	200
Calculation	$T0$	$T1$	$Tréf$ reference solution

The reference solution, $Tréf$, that is obtained for $N=25$, that is to say 200 pas for the totality of the way. These various solutions make it possible to judge sensitivity to the great steps of time and robustness of integration. To reveal the speed of convergence according to the step of time, one defers here the solutions put forward in [1], until 3125 pas de time for each of the 8 segments of the way of loading.

2.4.1 VMIS_LINE

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.70e-02	1.38e-02	3.37e-03	6.82e-04	1.14e-04	0.00e+00
$VMIS$	4.34e-03	1.86e-03	4.72e-04	9.72e-05	1.64e-05	0.00e+00
$TRAC$	1.19e-01	6.89e-02	1.70e-02	3.45e-03	5.80e-04	0.00e+00

2.4.2 VMIS_ISOT_TRAC

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.58e-02	1.34e-02	3.26e-03	6.60e-04	1.11e-04	0.00e+00
$VMIS$	6.38e-03	2.38e-03	5.81e-04	1.18e-04	2.00e-05	0.00e+00
$TRAC$	1.20e-01	7.69e-02	1.67e-02	3.30e-03	5.53e-04	0.00e+00

2.4.3 VMIS_CINE_LINE

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$VMIS$	3.91e-03	1.05e-03	2.16e-04	4.15e-05	6.91e-06	0.00e+00
$TRAC$	7.48e-14	7.44e-14	7.44e-14	7.69e-14	5.87e-14	0.00e+00

2.4.4 VMIS_ECMI_LINE

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.71e-02	1.39e-02	3.40e-03	6.88e-04	1.15e-04	0.00e+00
$VMIS$	3.63e-03	2.00e-03	4.79e-04	9.53e-05	1.61e-05	0.00e+00
$TRAC$	1.64e-01	9.16e-02	2.18e-02	4.33e-03	7.30e-04	0.00e+00

2.4.5 VMIS_ECMI_TRAC

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.70e-02	1.38e-02	3.38e-03	6.84e-04	1.15e-04	0.00e+00
$VMIS$	2.36e-03	1.18e-03	2.98e-04	6.00e-05	1.01e-05	0.00e+00
$TRAC$	1.46e-01	8.51e-02	2.13e-02	4.31e-03	7.22e-04	0.00e+00

2.4.6 VMIS_CIN1_CHAB

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.32e-02	1.12e-02	2.57e-03	5.10e-04	8.52e-05	0.00e+00
$VMIS$	9.04e-02	3.24e-02	7.45e-03	1.48e-03	2.49e-04	0.00e+00
$TRAC$	3.34e-14	3.31e-14	3.27e-14	3.48e-14	3.86e-14	0.00e+00

2.4.7 VMIS_CIN2_CHAB

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.32e-02	1.12e-02	2.57e-03	5.10e-04	8.52e-05	0.00e+00
$VMIS$	9.04e-02	3.24e-02	7.45e-03	1.48e-03	2.49e-04	0.00e+00
$TRAC$	3.72e-14	3.69e-14	3.69e-14	3.83e-14	4.75e-14	0.00e+00

2.4.8 MOHR_COULOMB

The internal variables have the following meanings:

- $V1$: plastic voluminal deformation $\varepsilon_v^p = \frac{1}{3} trace(\boldsymbol{\varepsilon}^p)$
- $V2$: plastic voluminal deformation $|\varepsilon_d^p| = \sqrt{\frac{2}{3}(\boldsymbol{\varepsilon}^p - \varepsilon_v^p \boldsymbol{I}):(\boldsymbol{\varepsilon}^p - \varepsilon_v^p \boldsymbol{I})}$
- $V3$: indicator of plasticity

Variation $(\%)$	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1$	1.34	0.57	0.16	0.03	0.006	0
$V2$	1.34	0.57	0.16	0.03	0.006	0
$V3$	0	0	0	0	0	0
$VMIS$	6.41	2.34	0.57	0.11	0.02	0
$TRAC$	2.06	0.87	0.24	0.05	0.009	0

2.5 REFERENCES

1) P.LEVASSEUR: “Application Third party maintenance of the code _Aster” Checking of the robustness and the reliability of the integration of laws of behavior in ASTER. Report PRINCIPIA RET.693.127.01 December 2006.

3 Modeling A

3.1 Characteristics of modeling

The behavior tested is VMIS_ISOT_LINE , in 3D and C_PLAN .

3.2 Sizes tested and results

Modeling C_PLAN :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	3.4	1	0
VMIS	0	0	0	0.4	0.1	0
TRACE	0	0	0	0	0	0

Modeling 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	2.5	0.9	0
$VMIS$	0	0	0	0.4	0.2	0
$TRACE$	0	0	0	0	0	0

Tangent matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	2.E-9

4 Modeling B

4.1 Characteristics of modeling

The behavior tested is VMIS_ISOT_TRAC , in 3D and C_PLAN .

4.2 Sizes tested and results

Modeling C_PLAN :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	3.3	1	0
$VMIS$	0	0	0	0.6	0.2	0
$TRACE$	0	0	0	0	0	0

Modeling 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	2.5	0.9	0
$VMIS$	0	0	0	0.3	0.1	0
$TRACE$	0	0	0	0	0	0

Tangent matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	1.6 E-9

5 Modeling C

5.1 Characteristics of modeling

The behavior tested is VMIS_CINE_LINE , in 3D.

5.2 Sizes tested and results

Modeling 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$VMIS$	0	0	0	0.4	0.08	0
$TRACE$	0	0	0	0	0	0

Tangent matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	7.7 E-10

6 Modeling D

6.1 Characteristics of modeling

The behavior tested is VMIS_ECMI_LINE , in 3D and C_PLAN .

6.2 Sizes tested and results

Modeling C_PLAN :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	3.4	1.1	0
$VMIS$	0	0	0	0.3	0.2	0
$TRACE$	0	0	0	0	0	0

Modeling 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	2.4	0.9	0
$VMIS$	0	0	0	0.4	0.1	0
$TRACE$	0	0	0	0	0	0

Tangent matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	1. E-9

7 Modeling E

7.1 Characteristics of modeling

The behavior tested is VMIS_ECMI_TRAC , in 3D and C_PLAN .

7.2 Sizes tested and results

Modeling C_PLAN :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	3.4	1.1	0
$VMIS$	0	0	0	0.2	0.08	0
$TRACE$	0	0	0	0	0	0

Modeling 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	2.5	0.9	0
$VMIS$	0	0	0	0.4	0.09	0
$TRACE$	0	0	0	0	0	0

Tangent matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	2.6 E-9

8 Modeling F

8.1 Characteristics of modeling

The behavior tested is VMIS_CIN1_CHAB , in 3D.

8.2 Sizes tested and results

Modeling 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	3.1	0.9	0
$VMIS$	0	0	0	8	2	0
$TRACE$	0	0	0	0	0	0

Tangent matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	0,031

9 Modeling G

9.1 Characteristics of modeling

The behavior tested is VMIS_CIN2_CHAB , in 3D.

9.2 Sizes tested and results

Modeling 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0	3.1	0.9	0
$VMIS$	0	0	0	8	2.5	0
$TRACE$	0	0	0	0	0	0

Tangent matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	0,031

10 Modeling J

10.1 Characteristics of modeling

The behavior tested is MOHR_COULOMB , in 3D.

10.2 Sizes tested and results

The internal variables have the following meanings:

- $V1$: plastic voluminal deformation $\varepsilon_v^p=\frac{1}{3}trace(\boldsymbol{\varepsilon}^p)$
- $V2$: plastic voluminal deformation $|\varepsilon_d^p|=\sqrt{\frac{2}{3}(\boldsymbol{\varepsilon}^p-\varepsilon_v^p\boldsymbol{I}):(\boldsymbol{\varepsilon}^p-\varepsilon_v^p\boldsymbol{I})}$
- $V3$: indicator of plasticity

Modeling 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1$	0	0	0	1.19	0.4	0
$V2$	0	0	0	1.19	0.4	0
$V3$	0	0	0	0	0	0
$VMIS$	0	0	0	5.88	1.79	0
$TRACE$	0	0	0	1.82	0.63	0

Tangent matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	0

11 Synthesis

For the whole of the elastoplastic behaviors tested in modelings A with G, the results are satisfactory:

- the results are valid during a physical change of unit of the problem (Pa in Mpa), or following a rotation or a symmetry of the loading
- the results converge correctly with the step of time, and the diagrams of integration (implicit for the studied elastoplastic behaviors) are robust, since they make it possible to use great steps of time
- the tangent matrices are correct because similar to the matrices tangent calculated by disturbance.