

## SSNS107 – Cylinder with reinforcements under pressure

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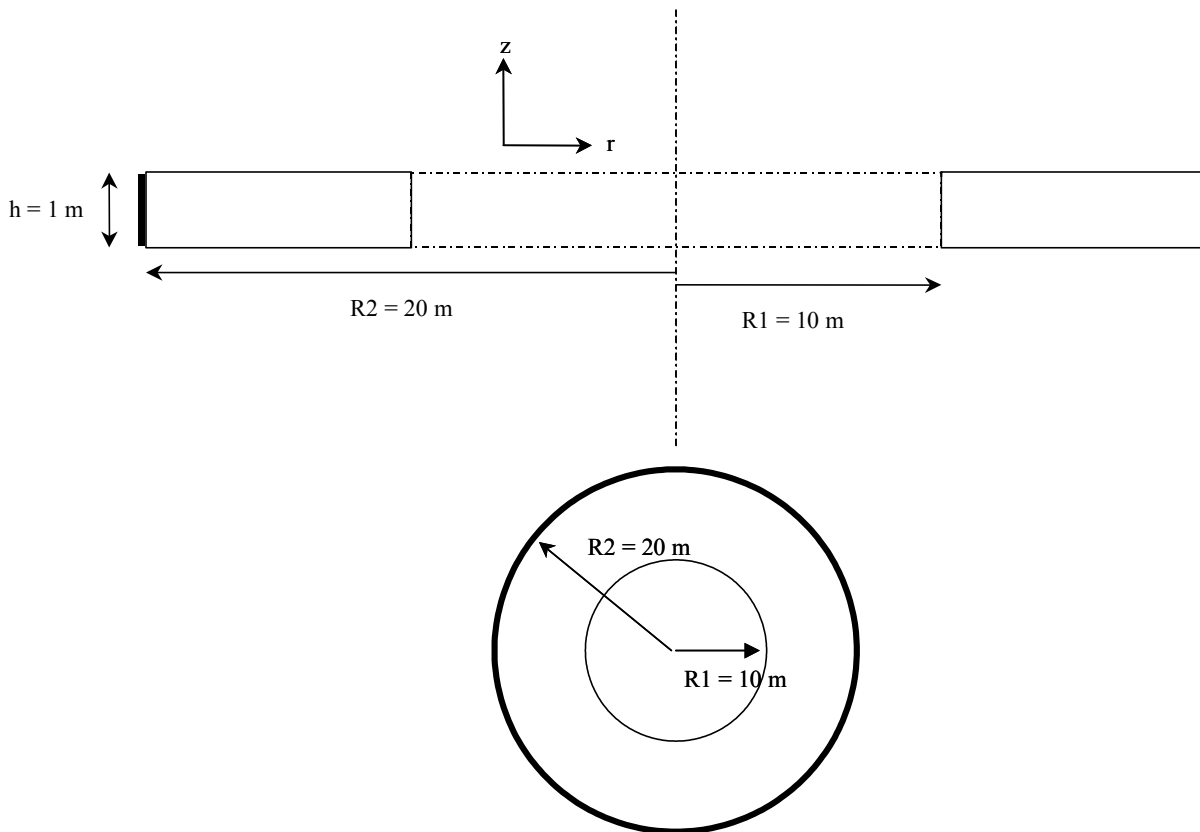
### Summary:

One considers a cylinder with reinforcements under pressure. More exactly, a hollow roll, 3D, are supplemented on its external face by a circumferential reinforcement (skin 2D). This structure is charged by an internal pressure. Calculation is elastic linear. Simulation is compared with an analytical solution in order to validate modelings of reinforcements of the type `GRILLE_MEMBRANE` on a structure.

All the digital values are given in unit IF.

## 1 Problem of reference

### 1.1 Geometry



The cylinder has an interior ray  $R_1 = 10 \text{ m}$ , an external ray  $R_2 = 20 \text{ m}$ , a height  $h = 1 \text{ m}$ . The circumferential reinforcement has a section per unit of length of  $e = 1 \text{ m}^2 / \text{ml}$ .

## 1.2 Properties of materials

The characteristic materials of the concrete (hollow roll) are  $E=2E+10 Pa$  and  $\nu=0.2$  ; those of steel (braces circumferential)  $E=2E+11 Pa$  ,  $\nu=0$  ,  $E_t=2.E+09$  (tangent module),  $\sigma_y=2.E+11 Pa$  (constraint of very large plasticity to remain in the elastic range).

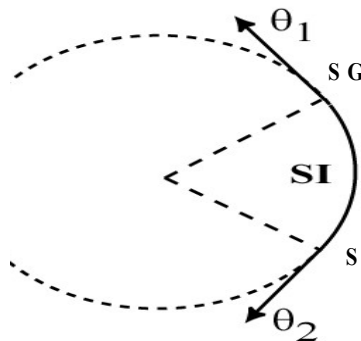
The tablecloth of reinforcement has its vertical principal direction and a section per linear meter of 1 m .

## 1.3 Boundary conditions and loading

One is interested only in one angular section of the cylinder.

The following conditions are imposed:

- 1) a normal displacement no one on the side faces of the angular section of cylinder (  $SG$  and  $SD$  on the schematic figure) (ortho-radial displacement no one)
- 2) a vertical displacement no one on the low face of the section of the cylinder (simple support)
- 3) a pressure imposed of  $10^6 Pa$  on the inner face of the section of the cylinder (  $SI$  ) on the schematic figure) (pressure interns)



Schematic designation of surfaces of the cylinder

## 2 Reference solutions

For the solution without reinforcement, and seen symmetries of the problem, displacement can be written in the form:

$$U(r, z) = g(r)u_r + h(z)u_z$$

With:

$$g(r) = (a + 3b) \frac{r}{2} + \frac{c}{r}$$

$$h(z) = -az$$

$$\varepsilon_{rr}(r) = \frac{1}{2}(a + 3b) - \frac{c}{r^2}$$

$$\varepsilon_{\theta\theta}(r) = \frac{1}{2}(a + 3b) + \frac{c}{r^2}$$

$$\varepsilon_{zz}(r) = -a$$

$$\sigma_{rr}(r) = A - 2\mu \frac{c}{r^2}$$

$$\sigma_{\theta\theta}(r) = A + 2\mu \frac{c}{r^2}$$

$$\sigma_{zz}(r) = B$$

The boundary conditions then make it possible to clarify the constants:

$$\sigma_{rr}(R_1) = -P_1$$

$$\sigma_{rr}(R_2) = -P_2$$

$$\sigma_{zz} = 0$$

with  $R_2$  the ray on the level of the interface enters the concrete and the circumferential tablecloth of reinforcements.

$$A = \frac{P_1 R_1^2 - P_2 R_2^2}{R_2^2 - R_1^2}$$

$$B = 0$$

$$2\mu c = R_1^2 R_2^2 \frac{P_1 - P_2}{R_2^2 - R_1^2}$$

$$b = \frac{2(P_1 R_1^2 - P_2 R_2^2)}{3(3\lambda + 2\mu)(R_2^2 - R_1^2)}$$

$$a = \frac{\lambda(P_1 R_1^2 - P_2 R_2^2)}{\mu(3\lambda + 2\mu)(R_2^2 - R_1^2)}$$

One gives the solution here with  $P_2$  nonworthless because that will be useful in the continuation.

If there is circumferential reinforcement on the external face of the cylinder, this reinforcement puts pressure. It is thus necessary to determine this pressure (function of displacement), then to apply the preceding results.

One applies the "formula of the ironmongers" (presence of reinforcements):

$$P = \sigma_{\theta\theta}^{armature} \frac{e}{2R_2}$$

To connect the constraint to solution displacement, one passes by the deformation (deformation of the reinforcements being deformation of the cylinder):

$$P = \frac{E^{armature} e}{2R_2} \left[ \frac{P_1 R_1^2 - P_2 R_2^2}{(3\lambda + 2\mu)(R_2^2 - R_1^2)} \right] \left[ \frac{\lambda}{2\mu} + 1 \right] + \frac{R_1^2}{R_2^2 - R_1^2} \frac{P_1 - P_2}{2\mu}$$

By applying the preceding results with  $P_2 = P$ , the sought solution is obtained.

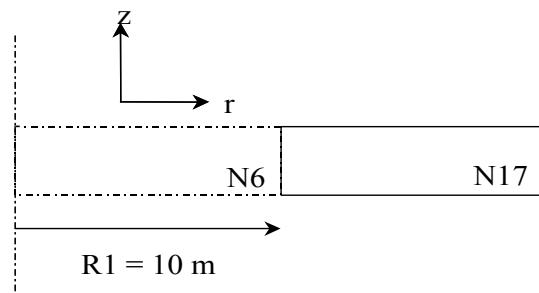
## 3 Modeling A

Modeling relates to only one angular sector of the cylinder, an element in the section, an element in the height, 100 elements enters  $R_1$  and  $R_2$

### 3.1 Values tested

One tests the values of radial displacements of points located on the interior face of the concrete cylinder ( $N6$  on figure Ci below) and the outside of the concrete cylinder ( $NI7$  on the figure below) and one compared to the analytical solution.

The cylinder is with a grid with elements TRIA3.



Node	Reference
$N6$	6.76923E-04
$NI7$	3.8462E-04

## 4 Modeling B

Modeling B is identical to modeling A. Seuls the elements used for the grid change (QUAD4 instead of TRIA3)

The results of modeling B are the following:

Node	Reference
$N6$	6.76923E-04
$NI7$	3.8462E-04

## 5 Modeling C

Modeling C is identical to modeling A. Seuls the elements used for the grid change (quadratic elements instead of linear)

The results of modeling C are the following:

Node	Reference
$N6$	6.76923E-04
$NI7$	3.8462E-04

## 6 Modeling D

Modeling D is identical to modeling B. Only the elements used for the grid change (quadratic elements instead of linear)

The results of modeling D are the following:

Node	Reference
<i>N6</i>	6.76923E-04
<i>NI7</i>	3.8462E-04

## 7 Synthesis

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Various modelings of this case test validate the behavior `GRILLE_MEMBRANE` for a structure supplements (cylinder with reinforcement)