SSNV209 – Interface in contact rubbing with X-FEM

Summary:

This problem corresponds to a quasi-static analysis of a problem of mechanics with contact and friction. A rectangular plate is subjected to horizontal and vertical compressive forces and is compressed on a plan where it undergoes forces of contact and friction.

It is about a case equivalent to the CAS-test ssnv128 [V6.04.128], but solved with method XFEM.

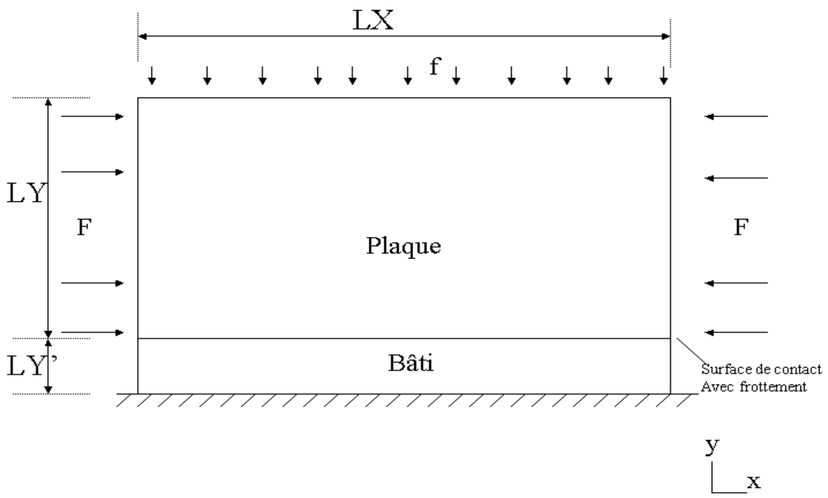
This test brings into play various grids, in $2D$ and $3D$, within the framework of calculations X-FEM [R7.02.12]

It comprises 14 modelings and makes it possible to test:

- modelings $2D$ (QUAD4, TRIA3, QUAD8, TRIA6) and $3D$ (HEXA8),
- a position of crack to the interface or in the middle of the elements,
- the operand XFEM keyword CONTACT
- integration by subelements (resulting from XFEM) of a size depending on a variable of order
- various diagrams of integration for the terms of contact (NODE, SIMPSON, NCOTES)
- formulations with lagranges on the edges and those with lagranges on the nodes (of the initial grid)
- parameters of the generalized formulation (coefficients of regularization and stabilization for the contact and friction),
- the simultaneous calculation of the separated zones, in slipping contact, and adherent contact
- postprocessing, in particular the calculation of the normal reaction RN,
- the algorithm of satisfaction of the condition LBB with the transitions taken off/contact and slipping/adherent,
- method of Lagrangian increased and method penalized for the treatment of contact-friction.

1 Problem of reference in 2D

1.1 Geometry



The structure is a rectangle made up of two of the same plates material, separated by an interface.

Dimensions of the plate, to which the pressures are applied, are:
$LX=80\text{mm}$, $LY=40\,mm$

The second plate, comparable to a frame, has following dimensions:
$LX=80\text{mm}$, $LY'=10\,mm$

The position of the points of reference under the surface of contact (mm) is:

	x	y
A	0	0
B	1.25	0
C	5.	0
D	7.5	0
E	11.25	0

1.2 Material properties

Plate:

Poisson's ratio: 0.2
Young modulus: $1.3\,10^{11}\,N/m^2$

Frame:

Poisson's ratio: 0.0
Young modulus: $1.0\,10^{16}\,N/m^2$

The coefficient of friction under the plan is $\mu=1.0$.

1.3 Boundary conditions and loadings

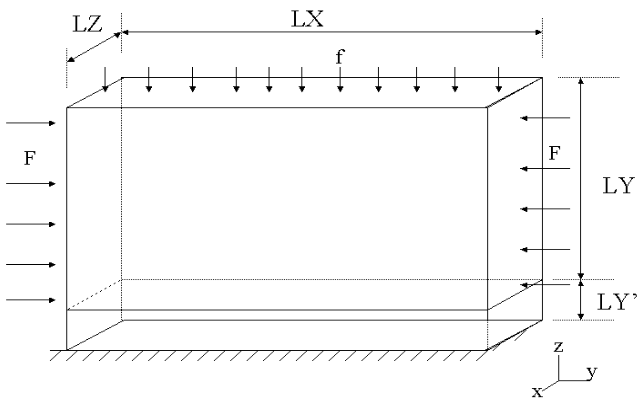
The frame, of the same width than the plate, is blocked by an embedding of its lower face.

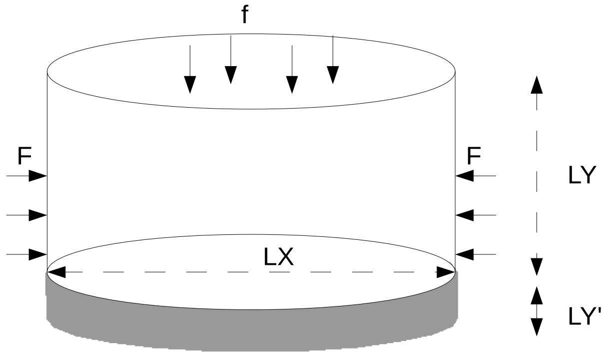
The plate is subjected to two pressures distributed:

- a vertical pressure acting on the face of the top: $f=-5\ daN/mm^2$,
- a horizontal pressure acting on the side faces, for $y>0$, $F=\pm 15\, daN/mm^2$ (according to the principle of compression)

2 Problem of reference in 3D

2.1 Geometry





One takes into account two cases.
The first geometry is obtained by extrusion of the geometry of the problem 2D , the structure is then a right-angled parallelepiped.

The second geometry is obtained by a revolution around the axis y geometry of the problem 2D , the structure is then a cylinder.

In both cases the structure consists of two materials, separated by an interface.
Dimensions of the upper part to which the pressures are applied, are fixed:
$LX=80\text{mm}$, $LY=40\,mm$ and $LZ=1\text{mm}$

The comparable lower part with a frame, has following dimensions:
$LX=80\text{mm}$, $LY'=10\,mm$ and $LZ=1\text{mm}$

The position of the points of reference under the surface of contact (mm) is:

	x	y	z
A	0	0	0
B	1.25	0	0
C	5.	0	0
D	7.5	0	0
E	11.25	0	0

2.2 Material properties

Plate:

Poisson's ratio: 0.2
Young modulus: $1.3\,10^{11}\,N/m^2$

Frame:

Poisson's ratio: 0.0
Young modulus: $1.0\,10^{16}\,N/m^2$
The coefficient of friction under the plan is $\mu=1.0$.

2.3 Boundary conditions and loadings

The frame, of the same width than the plate, is blocked by an embedding of its lower face.

For modeling S, one imposes moreover, displacement on Nœuds of the frame under the interface xfem, to zero. Very concretely, the relations following kinematics are imposed on the nodes of the frame (all Heaviside nouveau riches): $DX_{NO}-HX_{NO}=0$, $DY_{NO}-HY_{NO}=0$ and $DZ_{NO}-HZ_{NO}=0$. These relations kinematics do not guarantee a null displacement, on interface XFEM modelling the frame. Consequently, one preserves contrast material of the § 2.2 , while waiting for the improvement of the imposition of the limiting conditions of Dirichlet on elements XFEM (of the developments to come, around the imposition of the limiting conditions of Dirichlet, should allow to solve these problems).

In $3\mathrm{D}$, to avoid a too heavy calculation:

- displacement following the axis Z is blocked only on the higher face,

The plate is subjected to two pressures distributed:

- a vertical pressure acting on the face of the top: $f=-5\ daN/mm^2$,
- horizontal acting on the side faces, for $y>0$, $F=\pm 15\,daN/mm^2$ (according to the principle of compression).

3 Reference solution

3.1 Method of calculating used for the reference solution

The reference solution comes from the results got by modeling A ($2D$) and F ($3D$), namely a modeling D_PLAN in $2D$ and 3D in $3D$ under the keyword FORMULATION= ' CONTINUE' of the operator DEFI_CONTACT.

3.2 Results of reference

Tangential displacements (according to X) at the points $A\ B\ C\ D\ E$ surface of contact.
Normal reaction at the point medium.

4 Modeling A

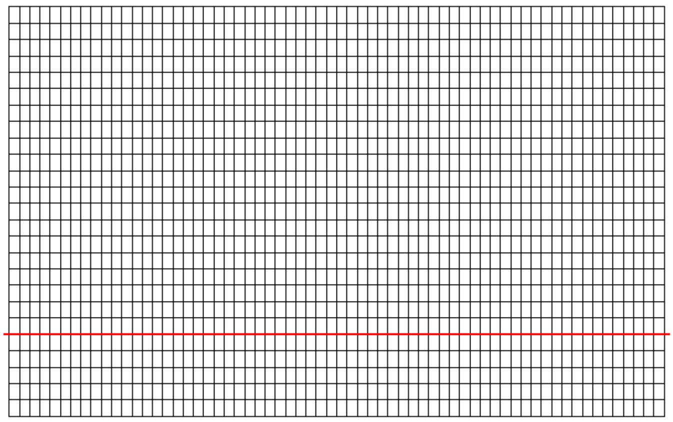
4.1 Characteristics of modeling

Modeling: `D_PLAN` to test the operand `'CONTINUES'` keyword `FORMULATION` under the operator `DEFI_CONTACT` for elements `SEG2` .

The plate and the frame are with a grid with elements `QUAD4`.

4.2 Characteristics of the grid

Many nodes: 975
Many meshs and types: 832 `QUAD4` for the plate and the frame.



4.3 Sizes tested and results

Identification	***Aster***
RN at the point medium	1.04864 E+5
DX at the point A	2.84595 E-5
DX at the point B	2.70793 E-5
DX at the point C	2.27403 E-5
DX at the point D	1.97271 E-5
DX at the point E	1.53641 E-5

4.4 Notice

These results are used as reference for four modelings which follow.

5 Modeling B

5.1 Characteristics of modeling

Modeling: `D_PLAN` to test the operand `'XFEM'` keyword `FORMULATION` under the operator `DEFI_CONTACT` for elements `SEG2` .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of order using the operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\,mm$ higher edge of the plate.

The two materials are introduced thanks to the variable of order `'NEUT1'` under the keyword `AFFE_VARC` order `AFFE_MATERIAU` [U4.43.03]. The values taken by this variable of order are given by a field to the nodes taking the value of the level set normal.

The field of the values of the level set normal is defined using the order `CREA_CHAM` [U4.72.4] and of its keyword `NOM_CHAM=' LNNO'`.

Parameters of material (E and nu , the Poisson's ratio) are functions of the variable of order `'NEUT1'`. The material is defined by the keyword `ELAS_FO` order `DEFI_MATERIAU` [U4.43.01].

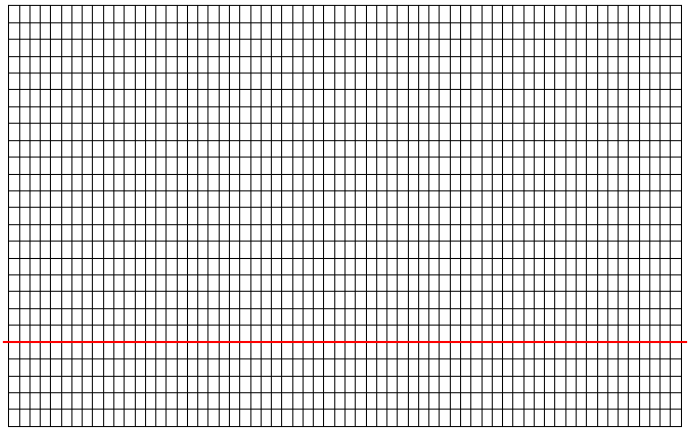
The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located at the interface of the elements.

The plate and the frame are with a grid with elements `QUAD4` 64×25 .

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\,mm$
Frame:
$LX=80\,mm$ and $LY'=10\,mm$

5.2 Characteristics of the grid

Many nodes: 1690
Many meshs and types: 1600 `QUAD4` for the plate and the frame and 178 `SEG2` for the edges.



5.3 Sizes tested and results

Identification	Reference	Tolerance (%)
RN at the point medium	1.04864 E+5	0.1
DX at the point A	2.84595 E-5	2.0
DX at the point B	2.70793 E-5	2.0
DX at the point C	2.27403 E-5	2.0
DX at the point D	1.97271 E-5	2.0
DX at the point E	1.53641 E-5	2.0

6 Modeling C

6.1 Characteristics of modeling

Modeling: `D_PLAN` to test the operand `'XFEM'` keyword `FORMULATION` under the operator `DEFI_CONTACT` for elements `SEG2` .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of order using the operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\,mm$ higher edge of the plate.

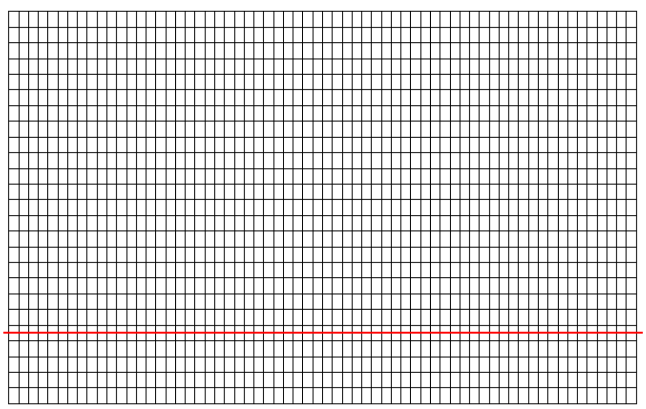
The two materials are introduced as for modeling A. One tests however that the introduction of two materials functions with the operand `BEHAVIOR` in the operator `STAT_NON_LINE.`

The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located in the middle of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\,mm$
Frame:
$LX=80\,mm$ and $LY'=\frac{3}{325}mm$

6.2 Characteristics of the grid

Many nodes: 1625
Many meshs and types: 1536 `QUAD4` for the plate and the frame and 176 `SEG2` for the edges.



Sizes tested and results

Identification	Reference	Tolerance (%)
DX at the point A	2.84595 E-5	2.0
DX at the point B	2.70793 E-5	2.0
DX at the point C	2.27403 E-5	2.0
DX at the point D	1.97271 E-5	2.0
DX at the point E	1.53641 E-5	2.0

7 Modeling D

7.1 Characteristics of modeling

Modeling: D_PLAN to test the operand 'XFEM' keyword FORMULATION under the operator DEFI_CONTACT for elements SEG2 .

The structure is a rectangle, into which a crack will be introduced directly into card-indexing it of order using the operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ higher edge of the plate.

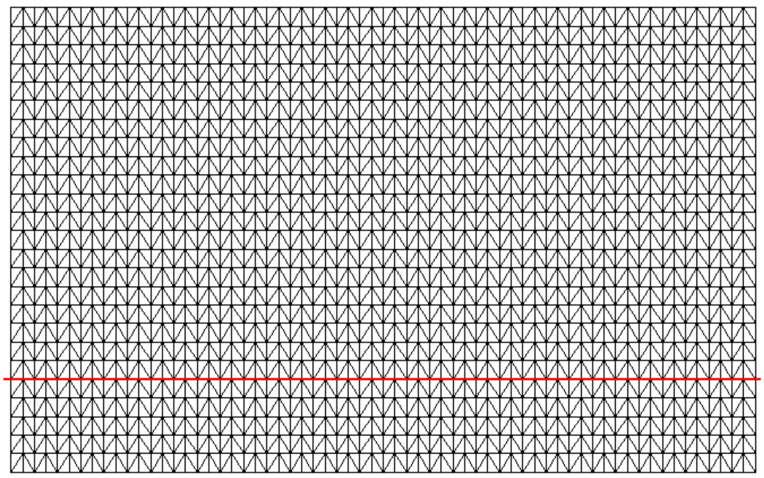
The two materials are introduced as for modeling A.

The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located at the interface of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\text{mm}$
Frame:
$LX=80\,mm$ and $LY'=10\text{mm}$

7.2 Characteristics of the grid

Many nodes: 1690
Many meshs and types: 3200 TRIA3 for the plate and the frame and 178 SEG2 for the edges.



7.3 Sizes tested and results

Identification	Reference	Tolerance (%)
RN at the point medium	1.04864 E+5	10.0
DX at the point A	2.84595 E-5	1.0
DX at the point B	2.70793 E-5	1.0
DX at the point C	2.27403 E-5	1.0
DX at the point D	1.97271 E-5	1.0
DX at the point E	1.53641 E-5	1.0

7.4 Remarks

In this case of the oscillations of the coefficient of Lagrange for the pressure appear. The point on which reaction RN is compared with the reference is a “low” point in the oscillation. The close points have an error of approximately $+8\%$.
Let us note that one can check by decreasing the step of the grid that the condition LBB is respected: the oscillations decrease in amplitude.

8 Modeling E

8.1 Characteristics of modeling

Modeling: D_PLAN to test the operand 'XFEM' keyword FORMULATION under the operator DEFI_CONTACT for elements SEG2 .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of order using the operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ higher edge of the plate.

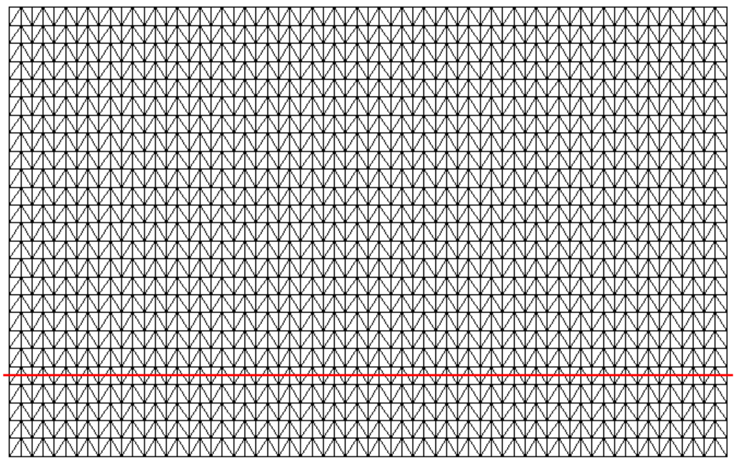
The two materials are introduced as for modeling A.

The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located in the middle of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\text{mm}$
Frame:
$LX=80\,mm$ and $LY'=\frac{3}{325}mm$

8.2 Characteristics of the grid

Many nodes: 1625
Many meshs and types: 3072 TRIA3 for the plate and the frame and 176 SEG2 for the edges.



8.3 Sizes tested and results

In this test, the contact is validated XFEM for a crack non-coïncidente with the grid.

Identification	Reference	Tolerance (%)
RN at the point medium	1.04864 E+5	5.0
DX at the point A	2.84595 E-5	1.0
DX at the point B	2.70793 E-5	1.0
DX at the point C	2.27403 E-5	1.0

DX at the point D	1.97271 E-5	1.0
DX at the point E	1.53641 E-5	1.0

9 Modeling F

9.1 Characteristics of modeling

Modeling: 3D to test the operand 'CONTINUES' keyword FORMULATION under the operator DEFI_CONTACT for elements HEXA8 .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ described paragraph §4 'Problem of reference in $3D$ '.

The plate and the frame are with a grid with elements HEXA8.

The blocking of the movements of rigid body is done by blocking displacement along the axis Z, that is to say DZ for the 4 nodes with the corners of the higher face. It is necessary to block these modes for the first iteration of contact, during which friction is not activated. Then, friction prevents these movements.

This case is used as reference for the cases $3D$ Following XFEM.

9.2 Characteristics of the grid

Many nodes: 1950
Many meshs and types: 832 HEXA8 for the plate and the frame.
1108 QUAD4 for the elements of edge.

9.3 Sizes tested and results

Identification	*Aster*
DX at the point A	3.06970 E-5
DX at the point B	2.92588 E-5
DX at the point C	2.47360 E-5
DX at the point D	2.15871 E-5
DX at the point E	1.70045 E-5

9.4 Remarks

For calculations in $3D$, boundary conditions $DZ=0$ (to bring back itself to a problem $2D$) are not respected to avoid a too important memory allocation. One imposes a minimum of it to block the movements of rigid body.

These results are used as references for two following modelings.

10 Modeling G

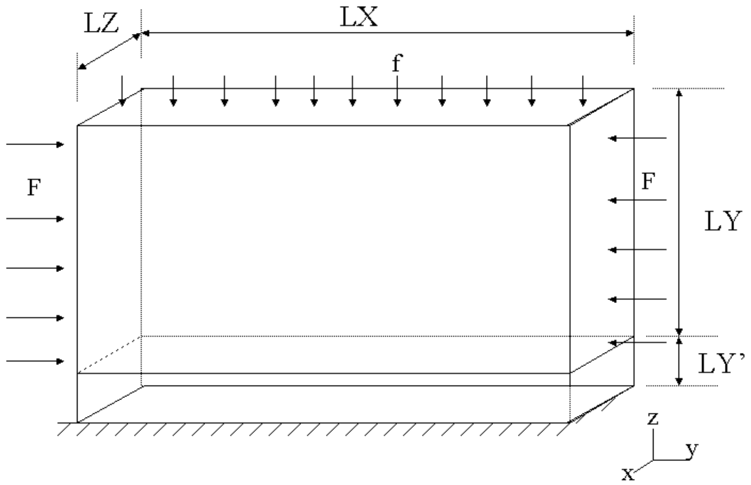
10.1 Characteristics of modeling

Modeling: 3D to test the operand 'XFEM' keyword FORMULATION under the operator DEFI_CONTACT for elements HEXA8 .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ described paragraph §4 'Problem of reference in $3D$ '.

The two materials are introduced as for modeling A.

10.2 Geometry



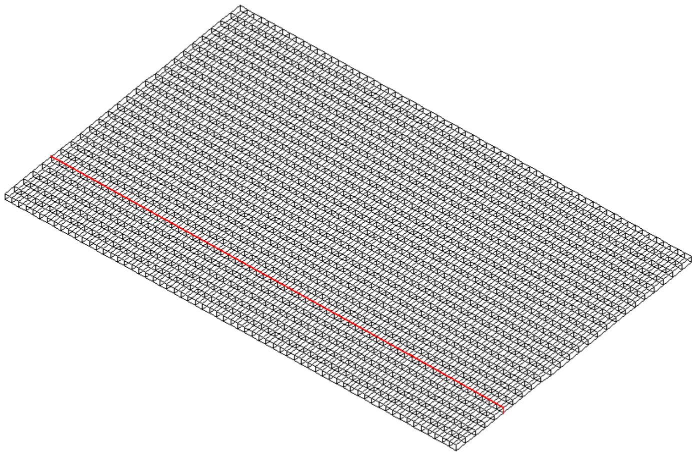
The structure is a healthy parallelepiped, into which a crack will be introduced directly into card-indexing it of order using the operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ higher edge of the plate.

The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located at the interface of the elements.

Dimensions are the following ones:
Plate:
$LX=80\text{mm}$, $LY=40\text{mm}$ and $LZ=1\text{mm}$
Frame:
$LX=80\text{mm}$, $LY'=10\text{mm}$ and $LZ=1\text{mm}$

10.3 Characteristics of the grid

Many nodes: 3380
Many meshs and types: 1600 HEXA8 for the plate and the frame and
1778 QUAD4 for the faces of edges .



10.4 Sizes tested and results

In this test, the contact is validated XFEM for a crack coïncidente with the grid in $3D$.

Identification	Reference	Tolerance (%)
DX at the point A	3.06970 E-5	1.0
DX at the point B	2.92588 E-5	1.0
DX at the point C	2.47360 E-5	1.0
DX at the point D	2.15872 E-5	1.0
DX at the point E	1.70046 E-5	1.0

11 Modeling H

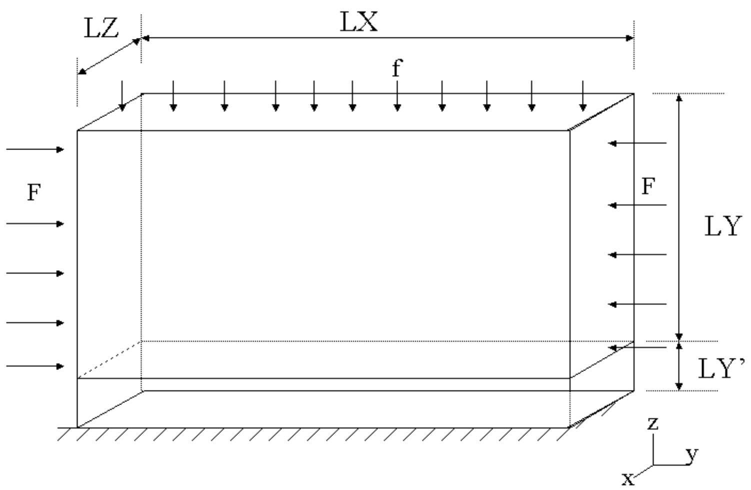
11.1 Characteristics of modeling

Modeling: `3D` to test the operand `'XFEM'` keyword `FORMULATION` under the operator `DEFI_CONTACT` for elements `HEXA8` .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ described paragraph §4 'Problem of reference in $3D$ '.

The two materials are introduced as for modeling A.

11.2 Geometry



The structure is a healthy parallelepiped, into which a crack will be introduced directly into card-indexing it of order using the operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ higher edge of the plate.

The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located in the middle of the elements.

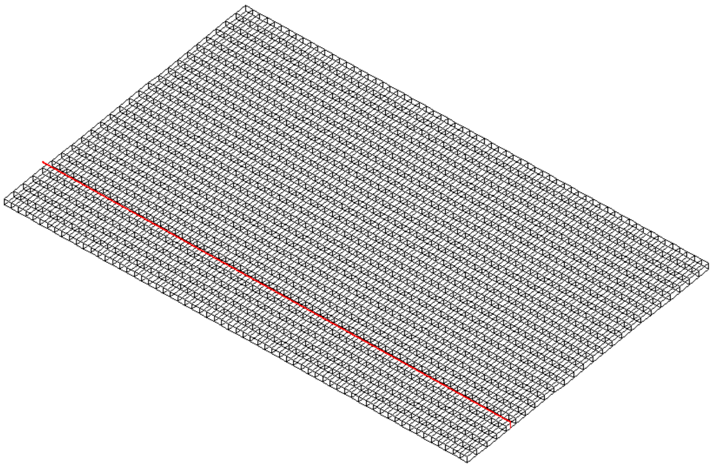
Dimensions are the following ones:
Plate:
$LX=80\text{mm}$, $LY=40\text{mm}$ and $LZ=1\text{mm}$
Frame:
$LX=80\text{mm}$, $LY'=\frac{3}{325}mm$ and $LZ=1\text{mm}$

11.3 Characteristics of the grid

For this case, the grid was reduced in order to limit the computing time. Consequently, the nodes B and E are not any more in the grid and displacements in these points are not calculated.

Many nodes: 858
Many meshs and types: 384 `HEXA8` for the plate and the frame

472 QUAD4 for the faces of edges



11.4 Sizes tested and results

Identification	Reference	Tolerance (%)
DX at the point A	3.06970 E-5	1.0
DX at the point C	2.47360 E-5	1.0
DX at the point D	2.15872 E-5	1.0

12 Modeling I

12.1 Characteristics of modeling

Modeling: $2D$ axisymmetric to test the operand 'CONTINUES' keyword FORMULATION in the operator DEFI_CONTACT for elements TRIA3.

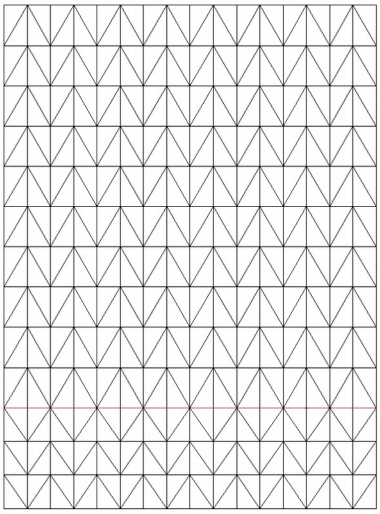
The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ cylinder describes paragraph §4 'Problem of reference in $3D$ '.
The cylindrical symmetry of the problem makes it possible to bring back the problem $3D$ of departure to a problem $2D$ axisymmetric.

The plate and the frame are with a grid with elements TRIA3.

This case is used as reference for the case $3D$ XFEM of modeling J.

12.2 Characteristics of the grid

Many nodes: 255
Many meshs and types: 416 TRIA3 for the plate and the frame.
58 SEG2 for the elements of edge.



12.3 Sizes tested and results

Identification	Aster
DX at the point A	-2.3713E-05
DX at the point C	-1.8445E-05
DX at the point D	-1.568E-05

12.4 Notice

These results are used as references for modeling J.

13 Modeling J

13.1 Characteristics of modeling

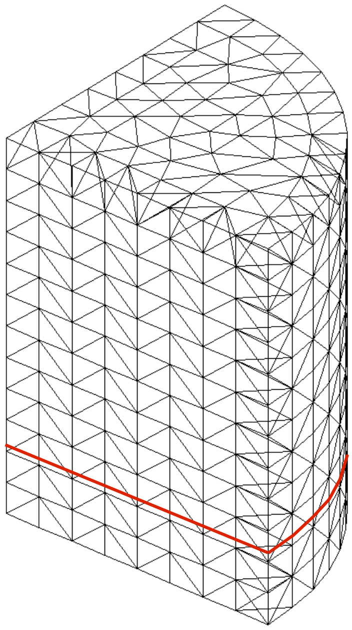
Modeling: $3D$ quarter of cylinder to test the operand 'XFEM' keyword FORMULATION in the operator DEFI_CONTACT for elements TETRA4 .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ cylinder describes paragraph 4 'Problem of reference in $3D$ '.
However the test is carried out on a quarter of cylinder rather than on the whole cylinder. This simplification makes it possible to save time and memory on the cases test of the base.
Limiting conditions of symmetries are applied within cut in order to simulate the presence of the totality of the cylinder.

The plate and the frame are with a grid with elements TETRA4.

13.2 Characteristics of the grid

Many nodes: 873
Many meshs and types: 3613 TETRA4 for the plate and the frame.
918 TRIA3 for the elements of edge.



13.3 Limiting conditions

Let us recall that displacement under X-FEM is the sum of a continuous displacement and a discontinuous displacement. In the case of an interface, bottomless of crack in the following way, the approximation of displacement is written:

$$u^h(x)=\sum_{i\in N_n(x)} a_i \Phi_i(x)+\sum_{j\in N_n(x)\cap K} b_j \Phi_j(x) H(lsn(x))$$

Where:

a_i and b_i are the degrees of freedom of displacement to the node i

Φ_i functions of form associated with the node i .

$N_n(x)$ is the whole of the nodes whose support contains the point x

K is the whole of the nodes whose support is entirely cut by the crack

$H(x)$ is the Heaviside function generalized defined by $H(x)=\begin{cases}-1 & \text{si } x<0\\ +1 & \text{si } x\geq 0\end{cases}$

$lsn(x)$ is the normal value of the level-set at the point x

For more details, to refer to the reference material X-FEM [R7.02.12].

In this test, one imposes conditions of symmetry on the side faces of the quarter of cylinder. With X-FEM, these conditions do not result in imposing $DX=0$ or $DY=0$ on these faces. Indeed, displacement under X-FEM being the sum of a continuous displacement and a discontinuous displacement, it is possible to have a null displacement in all the nodes of the grid but a displacement interpolated not no one in the middle of an enriched mesh.
So that horizontal displacement according to the axis x either no one just with the top of the level set (the reasoning is similar for a displacement according to the axis y) while having imposed $DX=0$ on all the nodes, it is necessary and sufficient to add the relation $a_x+b_x=0$ for all the nodes located under the level set in the meshs crossed by the crack.
One does not add a similar relation to impose on zero displacement right below the level set because the frame is extremely rigid in front of the rest of the structure and thus does not become deformed.
This relation is added thanks to the keyword `LIAISON_GROUP` of the operator `AFFE_CHAR_MECA`.

This case is treated in X-FEM great slips, contrary to all preceding modelings X-FEM. It is used to validate the approach to solve the conflict between the algorithm of the condition LBB and changes of status of frictions (member/slipping) proposed in documentation [R4.03.53].
The taking into account of the great slips is physically not useful but that is used to test the new algorithms for the treatment of friction. It is pointed out that in this case there is no reactualization of the thresholds of friction and that the total matrix is not symmetrical.

13.4 Sizes tested and results

Identification	Reference	Tolerance (%)
DX at the point A	-2.3713E-05	1.0
DX at the point C	-1.8445E-05	1.0
DX at the point D	-1.568E-05	5.0

13.5 Remarks

The position of the point D for approach X-FEM is not located exactly in $X=7.5\,mm$, that explains the variation more important for this point compared to the points A and B between the axisymmetric approach FEM and approach X-FEM quarter of cylinder.

Case X-FEM is cut out in a grid not structured. There is thus no control on the action of the algorithm of the condition LBB imposing relations of equalities on the Lagrangian ones of contact and friction. This test makes it possible to validate the theoretical choices described in Doc. [R4.03.53].

Another difficulty relates to the directions of slips which are radial and consequently not parallels. This test also makes it possible to validate the strategy of the formulation of Lagrangian to the nodes tops which makes it possible not to impose more a linear relation into hard (the algorithm of the condition LBB impose nothing any more but relations of equality).

14 Modeling K

14.1 Characteristics of modeling

Modeling: `D_PLAN_XFEM_CONT` to test the operand `'XFEM'` keyword `FORMULATION` of the operator `DEFI_CONTACT` for elements `SEG2` .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of order using the operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ higher edge of the plate.

The two materials are introduced as for modeling A.

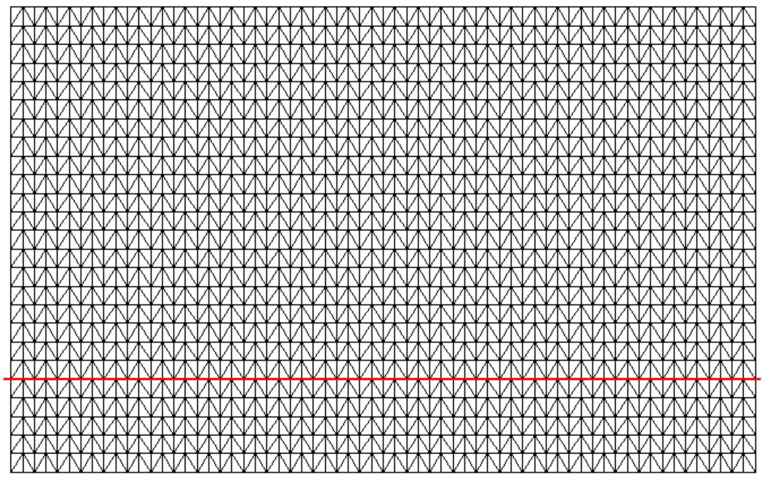
The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located at the interface of the elements.

The penalized method is used for the treatment of contact-friction.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\text{mm}$
Frame:
$LX=80\,mm$ and $LY=10\text{mm}$

14.2 Characteristics of the grid

Many nodes: 1690
Many meshs and types: 3200 `TRIA3` for the plate and the frame and 178 `SEG2` for the edges.



14.3 Sizes tested and results

Identification	**Reference**	**Tolerance (%)**
RN at the point medium	1.04864 E+5	10.0
DX at the point A	2.84595 E-5	1.0
DX at the point B	2.70793 E-5	1.0
DX at the point C	2.27403 E-5	1.0
DX at the point D	1.97271 E-5	1.0

DX at the point E	1.53641 E-5	1.0

15 Modeling L

15.1 Characteristics of modeling

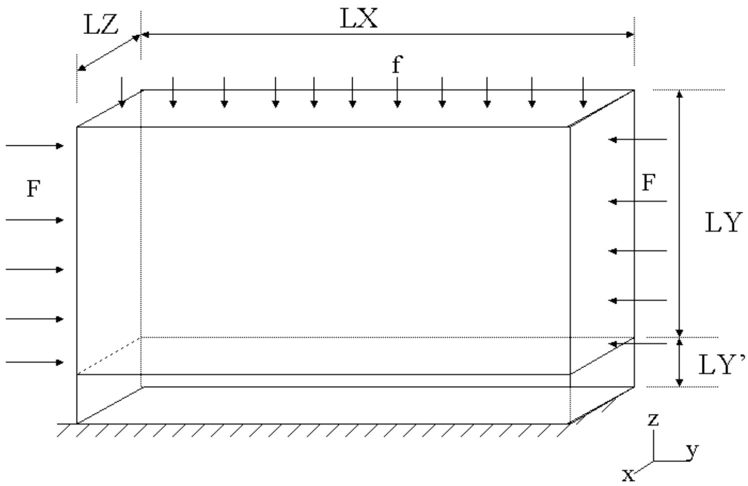
Modeling: `3D` to test the operand `'XFEM'` keyword `FORMULATION` under the operator `DEFI_CONTACT` for elements `HEXA8` .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ described paragraph 4 'Problem of reference in $3D$ '.

The two materials are introduced as for modeling A.

The penalized method is used for the treatment of contact-friction.

15.2 Geometry



The structure is a healthy parallelepiped, into which a crack will be introduced directly into card-indexing it of order using the operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ higher edge of the plate.

The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located in the middle of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$, $LY=40\,mm$ and $LZ=1\,mm$
Frame:
$LX=80\,mm$, $LY'=\frac{1}{19}LY$ and $LZ=1\,mm$

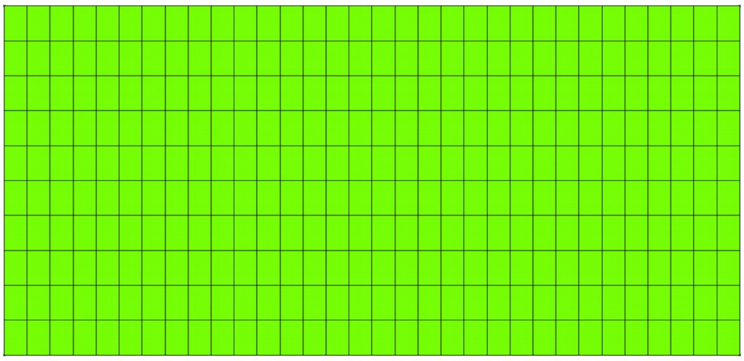
15.3 Characteristics of the grid

For this case, the grid was reduced in order to limit the computing time. Consequently, nodes B and E are not any more in the grid and displacements in these points are not calculated.

Moreover, Δy size of the elements in the direction y was selected so that the interface between the plate and the frame passes in the middle of an element that the frame measures half of an element following the direction y. One has as follows:

$$LY'=\frac{\Delta y}{2}$$

Many nodes: 726
Many meshs and types: 320 `HEXA8` for the plate and the frame
404 `QUAD4` for the faces of edges



15.4 Limiting conditions

Let us recall that displacement under X-FEM is the sum of a continuous displacement and a discontinuous displacement. In the case of an interface, bottomless of crack in the following way, the approximation of displacement is written:

$$u^h(x)=\sum_{i\in N_n(x)} a_i\Phi_i(x)-\sum_{i\in\Omega^+(x)\cap K} b_i\Phi_i(x)2\chi_-(x)+\sum_{i\in\Omega^-(x)\cap K} b_i\Phi_i(x)2\chi_+(x)$$

Where:

a_i and b_i are the degrees of freedom of displacement to the node i
Φ_i functions of form associated with the node i.
$\chi_\pm(x)$ functions characteristic on the sides + and – crack,
$\Omega^\pm(x)$ half spaces + and – determined by the crack,
$N_n(x)$ is the whole of the nodes whose support contains the point x,
K is the whole of the nodes whose support is entirely cut by the crack

For more details, to refer to the reference material X-FEM [R7.02.12].

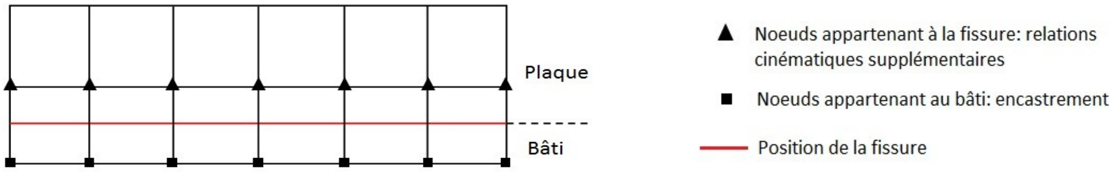
The rigid frame, of the same width than the plate, is blocked entirely by an embedding. DDL of displacement of nodes frame according to X and Y are put at 0:

$$DX_{NBATI}=0 \text{ and } DY_{NBATI}=0$$

Additional equations are written on the DDL of the nodes of the elements cut by the crack, so that displacement is null everywhere on the lower part of these elements corresponding to the frame. Very concretely the relations kinematics which impose that in any point built under the interface displacement is quite null are the following ones:

$$DX_{NPLAQUE}-2HX_{NPLAQUE}=0 \qquad DY_{NPLAQUE}-2HY_{NPLAQUE}=0$$

The following figure shows the nodes which carry these relations kinematics.



15.5 Sizes tested and results

Identification	Reference	Tolerance (%)
DX at the point A	3.06970 E-5	1.0
DX at the point C	2.47360 E-5	1.0
DX at the point D	2.15872 E-5	1.0

16 Modeling M

16.1 Characteristics of modeling

Modeling: `D_PLAN_XFEM_CONT` to test the operand `'XFEM'` keyword `FORMULATION` under the operator `DEFI_CONTACT` for elements `SEG3` .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of order using the operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY = 40\text{mm}$ higher edge of the plate.

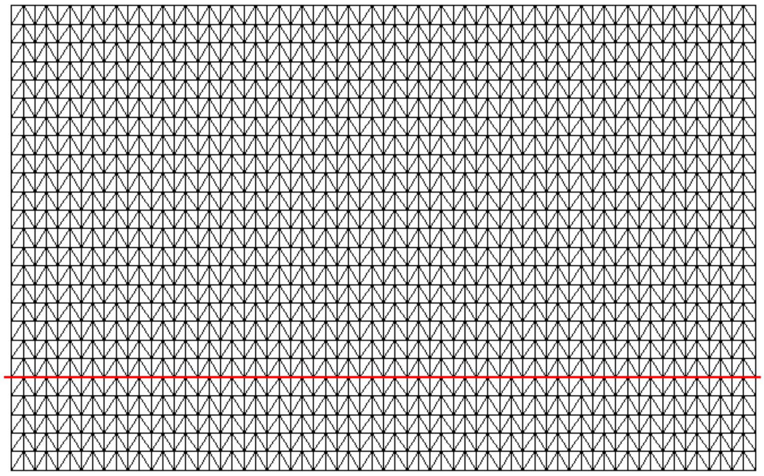
The two materials are introduced as for modeling A.

The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located at the interface of the elements.

Dimensions are the following ones:
Plate:
$LX = 80\,mm$ and $LY = 40\text{mm}$
Frame:
$LX = 80\,mm$ and $LY' = 10\text{mm}$

16.2 Characteristics of the grid

Many nodes: 1690
Many meshs and types: 3200 `TRIA6` for the plate and the frame and 178 `SEG3` for the edges.



16.3 Sizes tested and results

Identification	Reference	Tolerance (%)
DX at the point A	2.8617 E-5	1.0
DX at the point B	2.7229 E-5	1.0
DX at the point C	2.2855 E-5	1.0
DX at the point D	1.9805 E-5	1.0
DX at the point E	1.5411 E-5	1.0

The values of reference result from quadratic modeling FEM.

17 Modeling NR

17.1 Characteristics of modeling

Modeling: D_PLAN_XFEM_CONT to test the operand 'XFEM' keyword FORMULATION under the operator DEFI_CONTACT for elements SEG3.

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of order using the operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\mathrm{mm}$ higher edge of the plate.

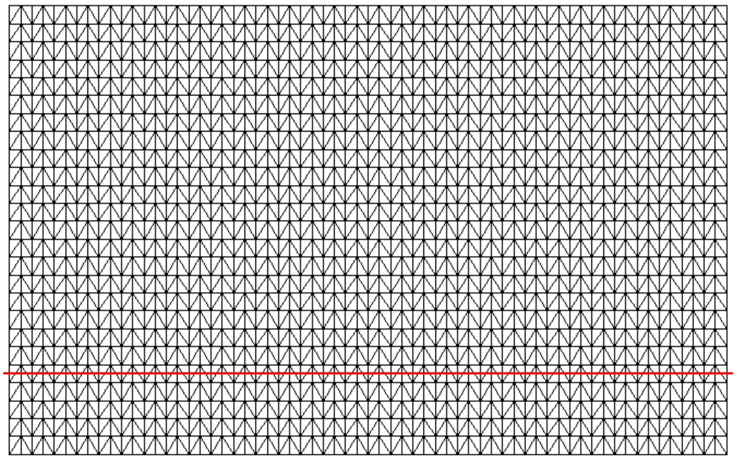
The two materials are introduced as for modeling A.

The bottom of crack is located apart from the structure, which allows a total separation. This modeling is such as the crack is located in the middle of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\mathrm{mm}$
Frame:
$LX=80\,mm$ and $LY'=\frac{3}{325}mm$

17.2 Characteristics of the grid

Many nodes: 1625
Many meshs and types: 3072 TRIA6 for the plate and the frame and 176 SEG3 for the edges.



17.3 Sizes tested and results

Identification	Reference	Tolerance (%)
DX at the point A	2.8617 E-5	1.0
DX at the point B	2.7229 E-5	1.0
DX at the point C	2.2855 E-5	1.0
DX at the point D	1.9805 E-5	1.0
DX at the point E	1.5411 E-5	1.0

The values of reference result from quadratic modeling FEM.

18 Modeling P

18.1 Characteristics of modeling

It is the same modeling that modeling J, one is always in great slips, but with a penalized formulation this time. One uses a formulation with the nodes.

18.2 Characteristics of the grid

It is the same grid as that of modeling J.

18.3 Sizes tested and results

The values of reference and the tolerances are identical to modeling J.

Identification	**Reference**	**Tolerance (%)**
DX at the point A	-2.3713E-05	1.0
DX at the point C	-1.8445E-05	1.0
DX at the point D	-1.568E-05	5.0

19 Modeling R

19.1 Characteristics of modeling

We take again characteristics identical to modeling H, put except for the grid which is quadratic. The objective is thus to test the operand ' XFEM ' keyword FORMULATION under the operator DEFI_CONTACT , with friction (FROTTEMENT=' COULOMB') for a modeling 3D , but with a discretization P2P1 (CONTACT=' P2P1' informed in MODI_MODELE_XFEM), which differentiates this modeling from modeling H.
For recall, the geometry, the boundary conditions and the loadings are those defined in section 12.2 , corresponding to the problem of reference 3D described paragraph §5 'Problem of reference in 3D '.

19.2 Characteristics of the grid

The grid is that of the modeling H which one made quadratic. As for modeling H, Nœuds B and E are not any more in the grid and displacements in these points are not calculated.

19.3 Sizes tested and results

One tests the results with the references of modeling F. the error is of a few percent (1% max) taking into account the difference in interpolation of the displacement, which is quadratic for this modeling and linear for the reference.

Identification	Reference	Tolerance (%)
DX at the point A	-2.3713E-05	1.0
DX at the point C	-1.8445E-05	1.0
DX at the point D	-1.568E-05	1.0

20 Modeling S

20.1 Characteristics of modeling

We take again characteristics identical to modeling R, but by dealing with the problem in great slips. To recapitulate, the objective is thus to test the operand 'XFEM' keyword FORMULATION under the operator DEFI_CONTACT, with friction (FROTTEMENT=' COULOMB') for a modeling 3D, with a discretization P2P1 (CONTACT=' P2P1'well informed in MODI_MODELE_XFEM), but in great slips (REAC_GEOM=' CONTRÔLE' informed in DEFI_CONTACT) what differentiates this modeling from modeling R.
For recall, the geometry, the boundary conditions and the loadings are those defined in section 12.2, corresponding to the problem of reference 3D described paragraph §5 'Problem of reference in 3D'.

20.2 Characteristics of the grid

The grid is that of the modeling H which one made quadratic. As for modeling H, Nœuds B and E are not any more in the grid and displacements in these points are not calculated.

20.3 Sizes tested and results

One tests the results with the references of modeling F. the error is of a few percent (3% max) taking into account the difference in interpolation of the displacement, which is quadratic for this modeling and linear for the reference.

Identification	Reference	Tolerance (%)
DX at the point A	-2.3713E-05	2.0
DX at the point C	-1.8445E-05	3.0
DX at the point D	-1.568E-05	3.0

21 Summary of the results

The case test ssnv209 makes it possible to observe the three states of the conditions of contact, namely separation, the slipping contact and the adhering contact. It puts also concerned modelings $2D$ and $3D$ with elements of various natures (quadrangles, triangles, parallelepipeds) within the framework of X-FEM.

This case test makes it possible to highlight satisfactory results for modelings $2D$ and $3D$, in particular the satisfaction of the condition LBB for the modeling of contact-rubbing with X-FEM.

The goals of this test are achieved:

1) with a relative error compared to the reference solution of less 1% for displacements,
2) whether the crack is in the middle of an element or with the interface,
3) that modeling is in $2D$ or in $3D$,
4) that the formulation places the unknown factors of contact at the edges of the elements or the nodes,
5) with several diagram of integration for the terms of contact `(NODE, SIMPSON, NCOTES)`,
6) by using the coefficients (friction and pressure) of the generalized formulation `(COEF_REGU_CONT, COEF_STAB_CONT, COEF_REGU_FROT, COEF_STAB_FROT)`,
7) by using the methods of Lagrangian increased and penalization for the treatment of contact-friction.