SSNV185 – Crack emerging in a plate 3D of width finished with X-FEM

Summary

The purpose of this test is to validate the method X-FEM [bib1] on an academic case 3D, within the framework of the linear elastic breaking process.

This test brings into play a plate 3D comprising an emerging crack planes at right bottom. Complete calculation as well as the extraction of the stress intensity factors is carried out within the framework of the method X-FEM. The grid is healthy, the crack being represented virtually with level sets.

Several configurations of grid are tested and compared with the analytical solution. With the same dealt problem in a classical way (with a fissured grid) is used as reference in order to compare the precise details of the two methods.

The orders of postprocessing for the visualization of the field of solution displacement on a fissured grid are tested for all modelings calling on X-FEM.

1 Problem of reference

1.1 Geometry

The structure is a plate 3D dimensions $LX=1\,m$, $LY=10\,m$ and $LZ=30\,m$, comprising an emerging plane crack length $a=5\,m$, being located at middle height (see [Figure 1.1-has]).

If with the problem is dealt by a classical method, the crack is with a grid. On the other hand, if method X-FEM is employed, the crack is not with a grid, and the geometry is in fact a healthy plate without crack. The crack will then be introduced by functions of levels (level sets) directly into the file orders using the operator DEFI_FISS_XFEM [U4.82.08]. The level set normal ($LSN=$ distance to the plan of cracking) makes it possible to define the plan of crack and the level set tangent ($LST=$ distance to the bottom of crack) makes it possible to define the position of the bottom of crack.

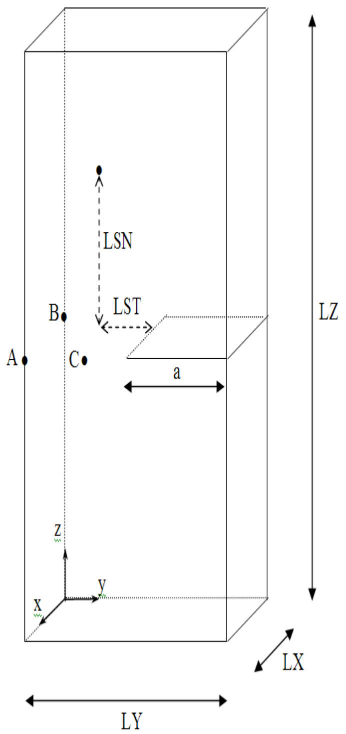

Figure 1.1-a : Geometry of the fissured plate

The points are defined $A(1,0,15)$, $B(0,0,15)$ and $C(1,3,15)$ who will be used to block the rigid modes.

1.2 Properties of material

Young modulus: $E=205\,000\,MPa$ (except contrary mention)
Poisson's ratio: $\nu=0.$

1.3 Bibliographical references

1 MASSIN P., GENIAUT S.: Method X-FEM, Handbook of reference of *Code_Aster*, [R7.02.12]

2 SCREW E.: Calculation of the coefficients of intensity of constraints, Handbook of reference of *Code_Aster*, [R7.02.05]

3 BARTHELEMY B.: Practical notions of the Eyrolles, breaking process, 1980.

4 LABORDE P., APPLE TREE J., FOX Y., SALÜN MR.: “High-order extended finite element method for cracked domains”, International Newspaper for Numerical Methods in Engineering, 64 (3), 354-381, 2005.

5 G. Erdogan, G.C. Sih, “One the ace extension in punts under planes loading and transverse shear”, Newspaper of BASIC Engineering, 85,519-27, 1963.

2 Modeling a: fissures with a grid in traction

In this modeling, the crack is with a grid, and one uses the standard method of the finite elements to carry out calculation. This modeling will be used as reference and will allow the comparison with the method X-FEM .

2.1 Characteristics of the grid

The structure is modelled by a regular grid composed of $5\times30\times50$ HEXA8, respectively along the axes x, y, z (see [Figure 2.1-has]). Two superimposed surfaces are the lips of the crack

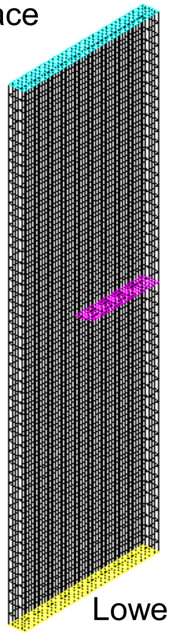

Figure 2.1-a : Fissured grid

2.2 Boundary conditions and loadings

Two types of loading will be studied: a loading of traction on the faces lower and higher of the structure, then a loading which consists in imposing a field of displacement in any node, identical to the field of asymptotic displacement in mode I (solution of Westergaard for an infinite medium [bib2]).

2.2.1 Loading of traction

A pressure distributed is forced on the faces lower and higher of the structure (see [Figure 2.1-has]). The pressure is $p=10^{-6} Pa(\sigma_{zz}=-p)$, which makes it possible to request the crack in mode of opening I pure.
The rigid modes are blocked in the following way:

- The point A is blocked according to the 3 directions: $DX^{N4265}=0$, $DY^{N4265}=0$, $DZ^{N4265}=0$
- The point B is blocked along the axis Oz : $DZ^{N3751}=0$

- The point C is blocked along the axes Ox and Oz : $DX^{N4256}=0$ $DZ^{N4256}=0$

2.2.2 Loading with the asymptotic field in mode I

The asymptotic field in mode I pure, solution of a problem of elastic rupture linear is known in an analytical way [bib2]. In the defined reference mark, this field takes the following shape:

$$u_x=0$$ **éq 2.2.2-1**

$$u_y=-\frac{1+\nu}{E}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}(3-4\nu-\cos\theta)$$ **éq 2.2.2-2**

$$u_z=\frac{1+\nu}{E}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}(3-4\nu-\cos\theta)$$ **éq 2.2.2-3**

This field is imposed on all the nodes of the structure by the means of formulas in the operator `AFFE_CHAR_MECA_F` [U4.44.01]. These formulas utilize the polar coordinates (r,θ) in the local base at the bottom of crack:

$$r=\sqrt{(5-y)^2+(z-15)^2}\,, \qquad \theta=\arctan\left(\frac{z-15}{5-y}\right)$$ **éq 2.2.2-4**

However, it is advisable to treat except for the nodes belonging to the lips of the crack. Indeed, for the nodes of the lower lip, the formula being used to calculate the angle θ (it is not valid would give π whereas theoretically, θ is worth $-\pi$). For the nodes of the lower lip, the value of the angle is thus not calculated by the equation [éq 2.2.2-4] but is directly put at $-\pi$. For the nodes of the upper lip, the formula is nevertheless valid.

2.3 Solutions of the problem

2.3.1 Loading of traction

The stress intensity factor in mode I is given [bib3] by:

$$K_I=\sigma_{zz}\sqrt{\pi a}\, f\left(\frac{a}{LY}\right)$$ **éq 2.3.1-1**

where

$$f\left(\frac{a}{b}\right)=\left(\frac{2b}{\pi a}\tan\frac{\pi a}{2b}\right)^{1/2}\frac{0.752+0.37\left(1-\sin\frac{\pi a}{2b}\right)^3+2.02\frac{a}{b}}{\cos\frac{\pi a}{2b}}$$ **éq 2.3.1-2**

The precision of this formula reaches 0.5% whatever the report $\frac{a}{b}$.

2.3.2 Loading with the asymptotic field in mode I

In the presence of such a loading, the theoretical value is

$K_I = 1$ **éq 2.3 .2 - 1**

2.4 Sizes tested and results

The option `CALC_K_G` of the operator `CALC_G` [U4.82.04] the calculation of the stress intensity factors by the energy method "G-theta allows". This functionality is tested with the loading n°1. This case of loading is used as a basis of comparison for the method X-FEM. When one of the loads is a function or a formula (coming from `AFFE_CHAR_MECA_F` [U4.44.01]), the option becomes `CALC_K_G_F`. This functionality is tested with the loading n°2.

The loading n°1 characterizes a loading of traction, which will be either constant on the faces higher and lower of the structure, or constant or variable on the level of the lips of the crack.
One tests the values of K_I along the bottom of crack, for various crowns of fields theta.
Values of the rays $\inf$ and $\sup$ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
Rinf	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 2.5-has

To test all the nodes of the bottom of crack in only once, the values are tested min and max of K_I on all the nodes of the bottom of crack.

2.4.1 Loading of traction

2.4.1.1 Constant pressure on the faces higher and lower

Identification	Type of reference	Value of reference	% Tolerance
Crown 1 : `MAX (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 1 : `MIN (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 2 : `MAX (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 2 : `MIN (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 3 : `MAX (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 3 : `MIN (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 4 : `MAX (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 4 : `MIN (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 5 : `MAX (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 5 : `MIN (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 6 : `MAX (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 6 : `MIN (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%

2.4.1.2 Constant pressure on the lips

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : `MAX (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 1 : `MIN (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%
Crown 2 : `MAX (K_I)`	'ANALYTICAL'	$1.1202664\ 10^7$	6.5%

Crown 2 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	6.5%

2.4.1.3 Variable pressure on the lips

Identification	Type of reference	Value of reference	% Tolerance
Crown 1, not initial : K_I	'NON_REGRESSION'	5.99 10^6	0.2%
Crown 1, full stop : K_I	'NON_REGRESSION'	4.52 10^6	0.2%
Crown 2, not initial : K_I	'NON_REGRESSION'	5.99 10^6	0.2%
Crown 2, full stop : K_I	'NON_REGRESSION'	4.52 10^6	0.2%
Crown 3, not initial : K_I	'NON_REGRESSION'	5.99 10^6	0.2%
Crown 3, full stop : K_I	'NON_REGRESSION'	4.52 10^6	0.2%
Crown 4, not initial : K_I	'NON_REGRESSION'	5.99 10^6	0.2%
Crown 4, full stop : K_I	'NON_REGRESSION'	4.52 10^6	0.2%
Crown 5, not initial : K_I	'NON_REGRESSION'	5.99 10^6	0.2%
Crown 5, full stop : K_I	'NON_REGRESSION'	4.52 10^6	0.2%
Crown 6, not initial : K_I	'NON_REGRESSION'	5.99 10^6	0.2%
Crown 6, full stop : K_I	'NON_REGRESSION'	4.52 10^6	0.2%

2.4.2 Loading with the asymptotic field in mode I

Identification	Type of reference	Value of reference	% Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1.0	0.2%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1.0	0.2%

2.5 Comments

The 1st loading of this modeling is used as a basis of comparison for the method X-FEM . The 2nd case of loading makes it possible to validate the option CALC_K_G_F for the elements 3D .

3 Modeling b: fissures X-FEM coïncidente in traction

In this modeling, the crack is not with a grid any more, but it is represented by level sets:

$$LSN=z-15 \quad \textbf{éq}\ 3\text{-}1$$
$$LST=LY-a-y \quad \textbf{éq}\ 3\text{-}2$$

3.1 Characteristics of the grid

The structure is modelled by a healthy, regular grid composed of $5\times30\times50$ HEXA8, respectively along the axes X,Y,Z in order to have the same number of elements as for the grid of modeling A (see [Figure 4.1-has]). Thus, the plan of crack is in correspondence with faces ofHEXA8 and bottom of crack with edges ofHEXA8. It is said that it is coïncidente with the grid.

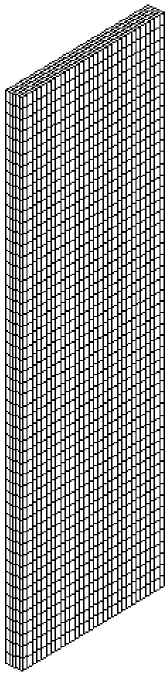

Figure 3.1-a : Healthy grid

3.2 Boundary conditions and loadings

Only one type of loading is studied here: it is about a pressure distributed imposed on the faces lower and higher of the structure (identical to the 1er case of loading of modeling A).
The rigid modes are blocked in the following way:

- The point A is blocked according to the 3 directions: $DX^{N3751}=0$, $DY^{N3751}=0$, $DZ^{N3751}=0$
- The point B is blocked along the axis Oz : $DZ^{N9276}=0$
- The point C is blocked along the axes Ox and Oz : $DX^{N3760}=0$, $DZ^{N3760}=0$

3.3 Sizes tested and results

One tests the values of K_I along the bottom of crack, for various crowns of fields theta.
Values of the rays $\inf$ and $\sup$ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
$R_{\inf}$	2	0,666	1	1	1	2.1
$R_{\sup}$	4	1,666	2	3	4	3.9

Table 3.4-B

To test all the nodes of the bottom of crack in only once, the values are tested min and max of K_I on all the nodes of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.0%

One also tests the min and max values of the parameter G_IRWIN

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 1 : MIN (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 2 : MAX (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 2 : MIN (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 3 : MAX (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 3 : MIN (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 4 : MAX (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 4 : MIN (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 5 : MAX (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 5 : MIN (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 6 : MAX (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%
Crown 6 : MIN (G_IRWIN)	'ANALYTICAL'	612.193573558	2.5%

One tests also the value of K_I product by the operator POST_K1_K2_K3 at the first point of the bottom of crack:

Identification	Type of reference	Value of reference	Tolerance
Crown 1, not initial : K_I	'ANALYTICAL'	1.1202664 10^{7}	5.0%

Moreover, in order to test the orders of postprocessing POST_MAIL_XFEM and POST_CHAM_XFEM one carries out a test to check the exactitude of the files results containing the fissured grid (resulting from the order POST_MAIL_XFEM) and the field of solution displacement (resulting from the order POST_CHAM_XFEM).

One makes a test of nonregression on displacements, which relates to the sum of the absolute values of displacements according to X, Y and Z. The tests of nonregression are made compared to version 10.2.17.

Identification	Type of reference	Value of reference	Tolerance
Somme of the absolute values on DX	'NON_REGRESSION'	5.91783 10^{-4}	10^{-3} %
Somme of the absolute values on DY	'NON_REGRESSION'	1.694474	10^{-3} %
Somme of the absolute values on DZ	'NON_REGRESSION'	1.253088	10^{-3} %

One makes a test of nonregression on the coordinates of the nodes of the grid X-FEM, which relates to the sum of the absolute values of the ordinates according to X, Y and Z. The tests of nonregression are made compared to version 10.2.17.

Identification	Type of reference	Value of reference	Tolerance
Somme of the absolute values on COOR_X	'NON_REGRESSION'	4,743 10^{3}	1,00E-008
Somme of the absolute values on COOR_Y	'NON_REGRESSION'	4,743 10^{4}	1,00E-008
Somme of the absolute values on COOR_Z	'NON_REGRESSION'	1.4229 10^{5}	1,00E-008

3.4 Comments

The results are stable for any selected crown.
With same number of elements, precision of the results got with X-FEM is much better than that obtained in the classical case (less 1% for X-FEM against 6% for a classical method).

4 Modeling C: crack X-FEM non-coïncidente in traction

Modeling identical to modeling B, but the bottom of crack is in the middle of the elements. The crack is thus not coïncidente with the grid.

4.1 Characteristics of the grid

The structure is modelled by a healthy, regular grid composed of $5\times31\times51$ HEXA8, respectively along the axes x, y, z. In this manner, the bottom of crack is in the center of elements and the plan of crack does not correspond any more to faces of elements. The figure 6.1-has represent out of cut Oyz enrichment in a zone near bottom of crack.

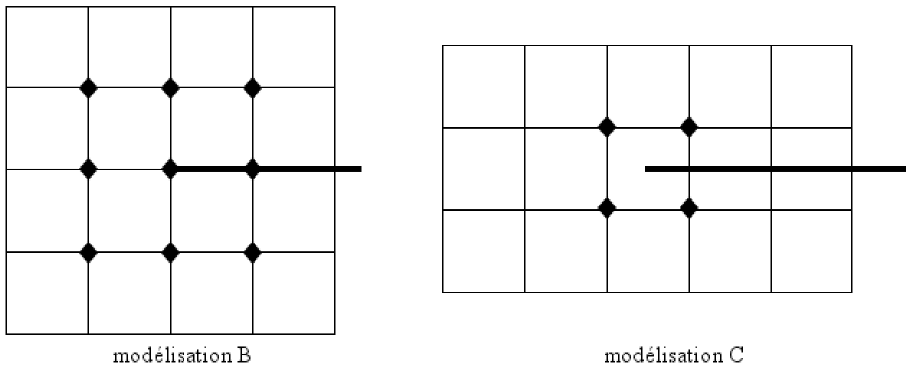

Figure 4.1-a : Various enrichments in bottom of crack

4.2 Boundary conditions and loadings

The loading is identical to that of modeling A :
- either there is a pressure distributed imposed on the faces lower and higher of the structure,
- either there is a constant pressure imposed on the lips of the crack,
- either there is a variable pressure according to X imposed on the lips of the crack.

In order to reproduce the preceding cases, it is necessary to block the same points A, B and C. However, here, there are no nodes in the median plane. To block the rigid modes, it is then necessary to just force relations between the degrees of freedom of the nodes above and below median plane [Figure 6.2-has]:

- the point A is blocked according to the 3 directions: $$\begin{aligned} DX^{N4031}+DX^{N3876} &= 0 \\ DY^{N4031}+DY^{N3876} &= 0 \\ DZ^{N4031}+DZ^{N3876} &= 0 \end{aligned}$$
- the point B is blocked along the axis Oz : $$DZ^{N3886}+DZ^{N4041} = 0$$
- the point C is blocked along the axes Ox and Oz : $$\begin{aligned} DX^{N9768}+DX^{N9767} &= 0 \\ DZ^{N9768}+DZ^{N9767} &= 0 \end{aligned}$$

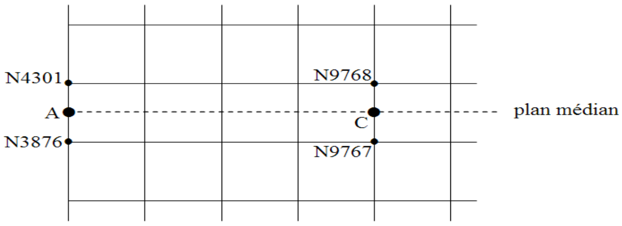

Figure 4.2-a : Conditions of Dirichlet around the median plane

[Figure 6.2 - has] is a schematic sight of the plan Oyz , on which the number of finite elements is not respected. It is simply used to understand the linear relations forced in order to block displacements of the points A and C . For the point B , one acts in the same way.

4.3 Sizes tested and results

One tests the values of K_I along the bottom of crack, for various loadings, various crowns of fields theta. The values of the rays inferior and superior of the torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
Rinf	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 4.4-C

One tests too 2 different smoothings for K_I : a smoothing of the type 'LAGRANGE' and a smoothing of the type 'LAGRANGE_NO_NO' (for this smoothing, only crown 1 is tested).

To test all the nodes of the bottom of crack in only once, one tests the min and max values of K_I on all the nodes of the bottom of crack.

4.3.1 Smoothing 'LAGRANGE'

4.3.1.1 Constant pressure on the faces higher and lower

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%

4.3.1.2 Constant pressure on the lips

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%

One tests also the calculation of G by the order CALC_G, option CALC_G only for the first crown.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (G)	'ANALYTICAL'	612,19	8,00%
Crown 1 : MIN (G)	'ANALYTICAL'	612,19	8,00%

4.3.2 Smoothing 'LAGRANGE_NO_NO'

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	4.0%

One tests also the value of K_I product by the operator POST_K1_K2_K3 at the first point of the bottom of crack starting from the result produced with the loading of type constant pressure on faces higher and lower of the structure.

Identification	Type of reference	Value of reference	Tolerance
Crown 1, not initial : K_I	'ANALYTICAL'	1.1202664 10^7	6.0%

Moreover, in order to test the orders of postprocessing POST_MAIL_XFEM and POST_CHAM_XFEM one carries out a test to check the exactitude of the files results containing the fissured grid (resulting from the order POST_MAIL_XFEM) and the field of solution displacement (resulting from the order POST_CHAM_XFEM). For the grid, one chooses a test of nonregression on the sum of the absolute values of the coordinates of the nodes. For displacement, the test of nonregression relates to the sum of the absolute values of displacements according to X, Y and Z. The tests of nonregression are made compared to version 8.2.14 for the grid and 8.2.8 for displacement.

Identification	Type of reference	Value of reference	Tolerance

Somme of the absolute values on COOR_X	'NON_REGRESSION'	4.8900 10^3	0.0%
Somme of the absolute values on COOR_Y	'NON_REGRESSION'	4.8406 10^4	0.0%
Somme of the absolute values on COOR_Z	'NON_REGRESSION'	1.4670 10^5	0.0%

Identification	Type of reference	Value of reference	Tolerance
Somme of the absolute values on DX	'NON_REGRESSION'	1.25083 10^{-3}	10^{-4} %
Somme of the absolute values on DY	'NON_REGRESSION'	1.79347	10^{-4}%
Somme of the absolute values on DZ	'NON_REGRESSION'	1.53478	10^{-4} %

4.4 Comments

The results are stable for any selected crown.

The precision of the got results is less good than for modeling B. That can be explained by the fact that the zone of enrichment is less wide here.

However, the results remain better than in the classical case.

5 Modeling D: semi-coïncidente crack X-FEM in traction

This modeling is exactly the same one as modeling B, except that the length of the crack is: $a=4.8333$, so that the bottom of crack does not coincide with edges of the elements. On the other hand, the surface of the crack coincides with the faces of the elements (semi-coïncidente crack).

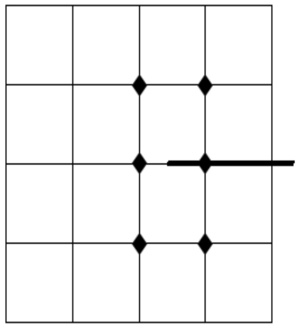

Figure 4.5-5-a : Enrichment in a zone close to the bottom of crack

5.1 Sizes tested and results

One tests the values of K_I along the bottom of crack, for various crowns of fields theta.
The values of the rays inf and sup of the torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
Rinf	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 5.1-D

To test all the nodes of the bottom of crack in only once, one tests the values minimum and maximum of K_I on all the nodes of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1.044774 10^7	3.0%

One tests also the value of K_I product by the operator POST_K1_K2_K3 at the first point of the bottom of crack starting from the result produced with the loading of type constant pressure on faces higher and lower of the structure.

Identification	Type of reference	Value of reference	Tolerance
Crown 1, not initial : K_I	'ANALYTICAL'	$1.044774\ 10^{7}$	10.0%

Moreover, in order to test the orders of postprocessing POST_MAIL_XFEM and POST_CHAM_XFEM one carries out a test to check the exactitude of the files results containing the fissured grid (resulting from the order POST_MAIL_XFEM) and the field of solution displacement (resulting from the order POST_CHAM_XFEM). One makes a test of nonregression on displacements, which relates to the sum of the absolute values of displacements according to x, y and z. The tests of nonregression are made compared to version 10.2.17.

Identification	Type of reference	Value of reference	Tolerance
Somme of the absolute values on DX	'NON_REGRESSION'	$4.1238\ 10^{-3}$	10^{-4} %
Somme of the absolute values on DY	'NON_REGRESSION'	1.480480	10^{-4} %
Somme of the absolute values on DZ	'NON_REGRESSION'	1.120438	10^{-4} %

One makes a test of nonregression on the coordinates of the nodes of the grid XFEM , which relates to the sum of the absolute values of the ordinates according to X, Y and Z. The tests of nonregression are made compared to version 10.2.17.

Identification	Type of reference	Value of reference	Tolerance
Somme of the absolute values on COOR_X	'NON_REGRESSION'	$4,743\ 10^{3}$	0.0%
Somme of the absolute values on COOR_Y	'NON_REGRESSION'	$4,743\ 10^{4}$	0.0%
Somme of the absolute values on COOR_Z	'NON_REGRESSION'	$1.4229\ 10^{5}$	0.0%

5.2 Comments

These results confirm that the size of the zone of enrichment influences the precision of the results. Here, the zone of enrichment is intermediate between of the same case B and case C, and precision.

6 Modeling E: crack X-FEM in traction – conditions of Dirichlet in mode I

Modeling identical to modeling C, but the loading of traction is the loading n°2 of modeling A

In this modeling, the Young modulus is equal to $100\,MPa$.

6.1 Characteristics of the grid

The structure is modelled by a grid healthy, regular, composed of $3\times 11\times 31$ `HEXA8`, respectively along the axes X, Y, Z (see [Figure 10.1-has]). Such a discretization leads to a configuration of enrichment similar to that of modeling C.

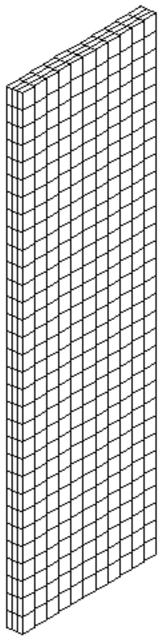

Figure 6.1-6.1-a : Grid

6.2 Boundary conditions and loadings

One wishes to apply the same loading as the loading n°2 of modeling A, i.e. to impose to all the nodes of the grid the asymptotic field of displacement in mode I pure.

For all the classical nodes (not nouveau riches), one then imposes the fields previously definite. For the nodes nouveau riches in bottom of crack, one seeks to impose each degree of freedom enriched. With this intention, one rewrites the analytical expressions of the fields of displacements to impose on the nodes in the base functions of enrichment:

$$u_x=0$$ **éq 6.2-1**

$$u_y=-\frac{1+\nu}{E}\sqrt{\frac{1}{2\pi}}\left(\sqrt{r}\cos\frac{\theta}{2}(2-4\nu)+\sqrt{r}\sin\frac{\theta}{2}\sin\theta\right)$$ **éq 6.2-2**

$$u_z=\frac{1+\nu}{E}\sqrt{\frac{1}{2\pi}}\left(\sqrt{r}\sin\frac{\theta}{2}(4-4\nu)-\sqrt{r}\cos\frac{\theta}{2}\sin\theta\right)$$ **éq 6.2-3**

It is pointed out that the base of the functions of enrichment is the following one:

$$\sqrt{r}\sin\frac{\theta}{2},\sqrt{r}\cos\frac{\theta}{2},\sqrt{r}\sin\frac{\theta}{2}\sin\theta,\sqrt{r}\cos\frac{\theta}{2}\sin\theta$$ **éq 6.2-4**

For the nodes nouveau riches by the Heaviside function, it is also necessary to carry out a first calculation. Let us consider a couple of nodes A and B, and let us note C^+ and C^- points located on the upper lips and lower of the crack, this one cutting a symmetrical element of way. One is in the following configuration:

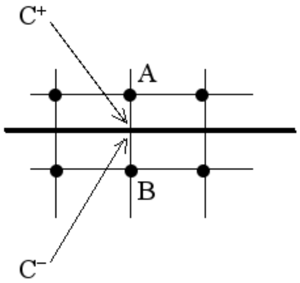

Figure 6.2-a : Heaviside enrichment

Nodes represented by rounds [Figure 10.2 - has] carry classical degrees of freedom a and of the degrees of Heaviside freedom h.
According to the approximation X-FEM, the crack passing in the middle of the elements, displacements are written:

$$\begin{cases} u(A)=a^A+h^A \\ u(B)=a^B-h^B \\ u(C^+)=\dfrac{a^A+h^A}{2}+\dfrac{a^B+h^B}{2} \\ u(C^-)=\dfrac{a^A-h^A}{2}+\dfrac{a^B-h^B}{2} \end{cases}$$ **éq 6.2-5**

By reversing this linear system, one obtains the expressions of the nodal unknown factors according to analytically known displacements:

$$\begin{cases} a^A=\dfrac{u(A)-u(B)}{2}+u(C^-) \\ h^A=\dfrac{u(A)+u(B)}{2}-u(C^-) \\ a^B=\dfrac{u(B)-u(A)}{2}+u(C^+) \\ h^B=\dfrac{-u(B)-u(A)}{2}+u(C^+) \end{cases}$$ **éq 6.2-6**

6.3 Sizes tested and results

One tests the values of K_I along the bottom of crack, for various crowns of fields theta.
Values of the rays inf and $\sup$ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
$Rinf$	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 6.4-E

To test all the nodes of the bottom of crack in only once, one tests the values minimum and maximum of K_I on all the nodes of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : `MAX (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 1 : `MIN (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 2 : `MAX (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 2 : `MIN (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 3 : `MAX (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 3 : `MIN (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 4 : `MAX (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 4 : `MIN (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 5 : `MAX (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 5 : `MIN (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 6 : `MAX (K_I)`	'ANALYTICAL'	1.0	3.0%
Crown 6 : `MIN (K_I)`	'ANALYTICAL'	1.0	3.0%

6.4 Comments

The results are stable for any selected crown.
They make it possible to validate the option `CALC_K_G_F` for the elements $3\mathrm{D}$ X-FEM .

7 Modeling F: crack X-FEM in traction – enrichment geometrical

This modeling is exactly the same one as modeling C. the only difference is as the zone of enrichment in bottom of crack now has a size fixed by the user, it thus is not limited more to only one lay down elements in bottom of crack.

7.1 Enrichment in bottom of crack

The nodes being at a distance from the bottom of crack equal or lower than a certain criterion are enriched by the singular functions. This criterion is selected as in [bib4], equal to a tenth of size of the structure. Here, it is worth $1m$ since LY is worth $10m$.

7.2 Sizes tested and results

One tests the values of K_I along the bottom of crack, for various crowns of fields theta.
Values of the rays inf and $\sup$ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
$Rinf$	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 7.3-F

To test all the nodes of the bottom of crack in only once, one tests the values minimum and maximum of K_I on all the nodes of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	1.8%

One tests also the value of K_I product by the operator POST_K1_K2_K3 at the first point of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1, not initial : K_I	'ANALYTICAL'	1.1202664 10^7	4.0%

7.3 Comments

The results are stable for any selected crown.
The precision of the got results is better than for modeling C. That proves the beneficial influence of the increase the size of the zone of enrichment, with identical grid.
However, if one compares with the precision of modeling B (lower than 1%) one could be astonished not to find better results with the fixed zone. The explanation is in [bib4]. Indeed, on modeling B, the approximation of displacement is exactly in $\sqrt{r}$ on a layer of element around the bottom. On the other hand, in modeling F, the approximation is $\sum_{éléments enrichis} \sqrt{r}$ on the zone of enrichment. In this case (relatively coarse grid), the approximation by a sum of square roots on a wide zone is less good than the approximation by only one square root on a more restricted zone.

However, when one refines sufficiently the grid, the precision obtained with an enrichment on a fixed zone becomes better than that obtained with an enrichment on only one lays down elements.

8 Modeling G: crack X-FEM compression

This modeling is of the same type as modeling C. the only difference is at the level of the sign of the pressure applied to the higher face. In order to request the structure in compression, the pressure is $p=10^6 Pa$.

8.1 Sizes tested and results

8.1.1 Contact pressures

To avoid an iteration of the loop of contact on the active constraints, the activation of the contact is ensured as of the 1era iteration thanks to the keyword `CONTACT_INIT=' OUI'`. The elementary terms of contact are integrated by a digital diagram of Gauss into 12 points per facet of contact.
In order to validate the taking into account of the contact on the lips of the crack, and in particular on the zone of the lips of the crack close to the bottom of crack, the values of contact pressures on the surface of the crack are extracted.

One is interested in the evolution of the contact pressure according to the axis Oy , on two lines (the line in $x=0$ and the line in $x=1$) surface of the crack [Figure 8.2-8.1.1-a]. It would normally be necessary to test the value of the contact pressure in each node of these 2 lines, but to reduce the number of tests, one can simply test the minimum and the maximum of the pressures on each line, which results in carrying out 4 tests.

Identification	Type of reference	Value of reference	Tolerance
Line in $x=0$: MAX (LAGS_C)	'ANALYTICAL'	-10^6	4.0%
Line in $x=0$: MIN (LAGS_C)	'ANALYTICAL'	-10^6	4.0%
Line in $x=1$: MAX (LAGS_C)	'ANALYTICAL'	-10^6	4.0%
Line in $x=1$: MIN (LAGS_C)	'ANALYTICAL'	-10^6	4.0%

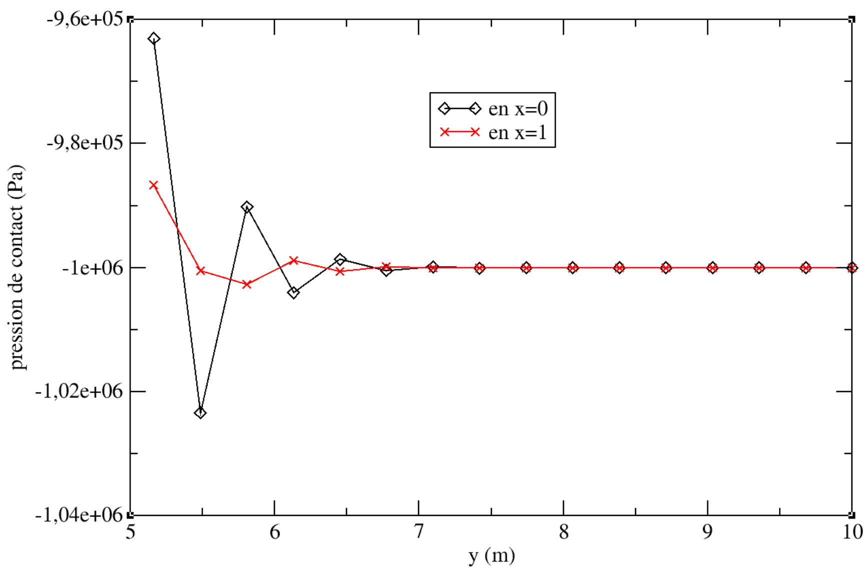

Figure 8.2-8.1.1-a : Evolution of the contact pressure along the crack

8.1.2 Rate of refund of energy room

In compression, the crack does not open, and the rate of refund of energy as well as the stress intensity factors are normally worthless.

To ensure itself some, one tests the values of G room along the bottom of crack, for various crowns of fields theta (the values of the rays of the tori are those used for modeling C) by taking into account a smoothing of the type 'LAGRANGE' .

To test all the nodes of the bottom of crack in only once, the values are tested min and max of G on all the nodes of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 1 : MIN (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 2 : MAX (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 2 : MIN (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 3 : MAX (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 3 : MIN (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 4 : MAX (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 4 : MIN (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 5 : MAX (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 5 : MIN (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 6 : MAX (G)	'ANALYTICAL'	0.0	5.0 10^{-4}
Crown 6 : MIN (G)	'ANALYTICAL'	0.0	5.0 10^{-4}

One tests also the value of K_I product by the operator POST_K1_K2_K3 at the first point of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1, not initial : K_I	'NON_DEFINI'	0.0	10^{-6}

8.2 Comments

Contact pressures close to the bottom of crack present light disturbances, which are reduced as one moves away from the bottom of crack. That is due to the enrichment of displacement with singular functions. Indeed, the elements containing the bottom of crack are enriched by singular functions, but these not-polynomial functions are not integrable exactly by a classical diagram of Gauss. A light inaccuracy on displacement, and thus on the jump of displacement through the crack causes a light disturbance on contact pressures close to the bottom of crack. Far from the bottom, this enrichment is not present, and displacements as well as contact pressures are perfectly concordant with the analytical solution.

The values of the rate of refund of energy room are quasi-worthless, and this for any crown of field of theta. However, it is noted that the values of K_I are nonworthless and 10 times less large than those obtained for modeling C in opening. However, theoretically, the bilinear form $g(u,v)$ give 0 if u is a field of displacement without singularity (like here the field of displacement in compression) and v the asymptotic field in mode I .

9 Modeling H: crack X-FEM inclined in traction

In this modeling, the crack is not with a grid, it is represented by level sets. The crack passes by the points to $y=0$ and $z=15$, its length is $2{,}5$, it is tilted of an angle of $45°$:

$$LSN=(z-L).\cos\alpha-y.sin\,\alpha$$

$$LST=(z-L-a.sin\alpha).\sin\alpha+(y-a.cos\,\alpha).\cos\alpha$$

$$\text{with}\quad\begin{cases}L=15\\a=2.5\\\alpha=45°\end{cases}$$

In this modeling, the Young modulus is equal to $205000\,MPa$.

9.1 Characteristics of the grid

The structure is modelled by a healthy, regular grid of $5\times31\times51$ HEXA8, respectively along the axes X, Y, Z [Figure 9.1-a]. The bottom of crack is in the middle of elements, as in modeling C, but with a tilted crack.

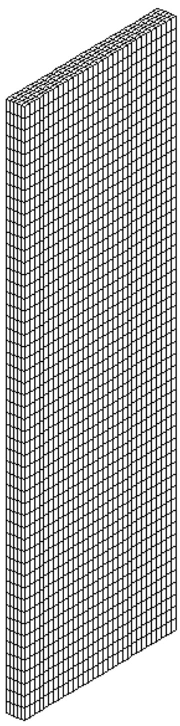

Figure 9.1-a : Grid 5*31*51 HEXA8

9.2 Boundary conditions and loadings

The loading applied is the same one as the loading n°1 of modeling A, i.e. a uniform request in traction by an imposed pressure $\sigma=-10^6 Pa$ on the lower and higher faces. The crack being tilted, one will introduce a loading in mixed mode K_I and K_{II}

For a leaning crack of an angle α compared to the normal plan with the direction of request in pure traction, the coefficients of stress concentration in bottom of crack are:

$K_{1(\alpha)}=K_{1(0)}.\cos(\alpha)$

$K_{2(\alpha)}=K_{1(0)}.\sin(\alpha)$

where $K_{1(0)}$ is the coefficient resulting from a request in K_I pure ($\alpha=0$).

Thus, whatever the intensity of the loading (within the limits of application of the theories of the linear elastic mechanics of rupture LEFM), direction of propagation of crack depending only on the report K_I/K_{II} is [bib5]:

$$\beta=2\arctan\left[\frac{1}{4}.\left(\frac{K_I}{K_{II}}-sign(K_{II}).\sqrt{\left(\frac{K_I}{K_{II}}\right)^2+8}\right)\right] \quad \text{with} \quad \frac{K_I}{K_{II}}=\frac{\cos(\alpha)}{\sin(\alpha)}$$

Therefore, if $\alpha=45°$: $\beta=-53.13°=-0.927295 rad$

9.3 Sizes tested and results

One tests the values of β along the bottom of crack for various crowns of fields theta.
According to the geometry, the higher ray of integration must remain lower than $a*\cos\alpha$. The lower and higher rays of the torus are thus parameterized according to the values of a and α. Values of the reports $rayon/(a*\cos\alpha)$ brought back between 0 and 1 are:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
$R_{\text{inf}}/(a*\cos\alpha)$	0.1	0.2	0.4	0.1	0.3	0.4
$R_{\text{sup}}/(a*\cos\alpha)$	1	1	1	0.7	0.7	0.8

Table 9.4-G : Crowns of the field theta

What gives us with $a=2,5$ and $\alpha=45°$:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
R_{inf}	0.1768	0.3536	0.7071	0.1768	0.5303	0.7071
R_{sup}	1.7678	1.7678	1.7678	1.2374	1.2374	1.4142

Table 9.4-H : Crowns of the field theta

To test all the nodes of the bottom of crack in only once, one tests the minimal and maximum values of β on all the points of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (β)	'ANALYTICAL'	-0.9273	6.1%

Crown 1 : MIN (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 2 : MAX (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 2 : MIN (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 3 : MAX (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 3 : MIN (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 4 : MAX (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 4 : MIN (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 5 : MAX (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 5 : MIN (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 6 : MAX (β)	'ANALYTICAL'	-0.9273	6.1%
Crown 6 : MIN (β)	'ANALYTICAL'	-0.9273	6.1%

9.4 Comments

The results are stable whatever the crown, but it are however higher than the expected results. That comes owing to the fact that the reports K_I / K_{II} are not rigorously equal to 1.

The objective of the test being primarily to validate the calculation of the angle β of propagation in CALC_G compared to the values of K_I and K_{II} obtained, the test is conclusive.

10 Modeling I: Crack X-FEM - Forces surface on edges (mode I)

This modeling is identical to modeling E, but the loading of Dirichlet is replaced here by a loading of Neumann. Only mode I is solicited.

10.1 Characteristics of the grid

The grid is identical to that of modeling E.

10.2 Boundary conditions and loadings

One wishes to apply the same loading as that of modeling E, i.e. a loading in mode I pure. Instead of imposing the value of displacement, one imposes surface efforts on the faces of the structure, corresponding to the analytical expressions of the constraints in mode I.

It is pointed out that the mode I pure (i.e. $K_I=1$, $K_{II}=0$ and $K_{III}=0$), the state of stress is the following:

$$\sigma_{11}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_{12}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\sigma_{22}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

In our case, the definite reference mark is such as:

$$e_1=-e_y$$
$$e_2=e_z$$
$$e_3=-e_x$$

The surface forces to apply are then the following ones:

- on the left side face (of outgoing normal $-e_y$): $\begin{cases} FY=-\sigma_{yy}=-\sigma_{11} \\ FZ=-\sigma_{yz}=\sigma_{12} \end{cases}$
- on the right side face (of outgoing normal e_y): $\begin{cases} FY=\sigma_{yy}=\sigma_{11} \\ FZ=\sigma_{yz}=-\sigma_{12} \end{cases}$
- on the higher face (of outgoing normal e_z): $\begin{cases} FY=\sigma_{yz}=-\sigma_{12} \\ FZ=\sigma_{zz}=\sigma_{22} \end{cases}$
- on the lower face (of outgoing normal $-e_z$): $\begin{cases} FY=-\sigma_{yz}=\sigma_{12} \\ FZ=-\sigma_{zz}=-\sigma_{22} \end{cases}$

The rigid modes are blocked thanks to the points A , B and C [Figure 1.1-a]:

- the point A is blocked according to the 3 directions,
- the point B is blocked along the axis Oz ,
- the point C is blocked along the axes Ox and Oz .

In fact, like the number of elements along the axis Oz is odd, the points A , B and C do not coincide with nodes of the grid. One thus uses nodes located just above and below these points to impose equivalent linear relations:

Are $A1$ (respectively $B1$ and $C1$) the node right below point A (B and C).
Are $A2$ (respectively $B2$ and $C2$) the node right above point A (B and C).

One writes the 6 following linear relations to block the rigid modes:

$$DX^{A1}+DX^{A2}=0$$
$$DY^{A1}+DY^{A2}=0$$
$$DZ^{A1}+DZ^{A2}=0$$

$$DZ^{B1}+DZ^{B2}=0$$

$$DX^{C1}+DX^{C2}=0$$
$$DZ^{C1}+DZ^{C2}=0$$

10.3 Sizes tested and results

One tests the values of K_I along the bottom of crack, for various crowns of fields theta.
The values of the rays lower and higher of the torus are the following ones:

	Crown 1	Crown 2	Crown 3
$Rinf$	0,666	1	1
Rsup	1,666	2	3

Table 10.4-E

To test all the nodes of the bottom of crack in only once, the values are tested min and max of K_I on all the nodes of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1	6.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1	6.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1	6.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1	6.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1	6.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1	6.0%

10.4 Comments

The results are stable for any selected crown.
They make it possible to validate the option CALC_K_G_F for the elements 3D X-FEM .

11 Modeling J: Crack X-FEM - Propagation in the plan with the method simplex

The crack is that of modeling B.

The Young modulus is equal to $205000\,MPa$.

11.1 Characteristics of the grid

The structure is modelled by a healthy, regular grid of $2\times20\times51$ HEXA8, respectively along the axes x,y,z [Figure 11.1-a]. The bottom of crack is inside an element as in modeling C [Figure 4.1-a].

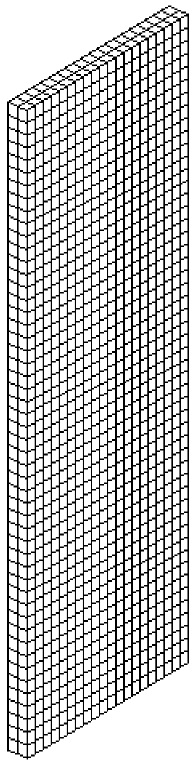

Figure 11.1-a : Grid 2*20*51 HEXA8

11.2 Boundary conditions and loadings

The loading applied is the same one as the loading n°1 of modeling A, i.e. a uniform request in traction by an imposed pressure $\sigma=-10^6\,Pa$ on the lower and higher faces. The crack is requested in mode K_I pure.

Advance of the crack imposed on each call to PROPA_FISS is the following one: $\Delta a=0.5$

Lengths of crack to each call to PROPA_FISS are thus:

Initial state: $a_0=4.9$
Iteration 1: $a_1=5.4$
Iteration 2: $a_2=5.9$

The stress intensity factors are given by: $K_I = -P \cdot \sqrt{\pi a} \cdot f\left(\frac{a}{b}\right)$

with:

$$f\left(\frac{a}{b}\right) = \left(\frac{2b}{\pi a}\tan\frac{\pi a}{2b}\right)^{0.5} \cdot \frac{0.752 + 0.37 \cdot \left(1 - \sin\frac{\pi a}{2b}\right)^3 + 2.02\frac{a}{b}}{\cos\frac{\pi a}{2b}}$$

$$b = 10$$
$$P = 10^6$$

From where:

$$K_{I0} = 1.07418.10^7$$
$$K_{I1} = 1.33139.10^7$$
$$K_{I2} = 1.67342.10^7$$

11.3 Sizes tested and results

One tests the values of K_I along the bottom of crack, for the crown $[1.;4.]$.
To test all the points of the bottom of crack in only once, one tests the minimal and maximum values of K_I on all the points of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Initial state : MAX (K_I)	'ANALYTICAL'	1.07417689 10^7	5.0%
Initial state : MIN (K_I)	'ANALYTICAL'	1.07417689 10^7	5.0%
Iteration 1 : MAX (K_I)	'ANALYTICAL'	1.33138926 10^7	5.0%
Iteration 1 : MIN (K_I)	'ANALYTICAL'	1.33138926 10^7	5.0%

11.4 Comments

The values obtained are relatively close to the expected values. However, it is noted that the value of K_I calculated by Aster is lower than the analytical solution of reference. That probably comes from values of K_{II} quasi-worthless instead of being strictly equal to 0. Moreover, one observes that by propagating on very little step of time (here 3) these values of K_{II} undesirable are all the more present (report $K_I/K_{II}=6000$ at the initial state, 2600 after the 1era iteration. That does not imply inevitably that the crack is propagated "badly", because a deviation of the crack of its initial plan can be caught up with by one K_{II} who rectifies his trajectory.
First approach of the propagation of crack XFEM in Aster allows nevertheless to validate the complete process of propagation of level sets.

12 Modeling K: Crack X-FEM – multi-cracking in traction

12.1 Geometry

In this modeling two cracks are present in the structure [Figure 12.1-a] in order to test the features of multi-crackings in the case of a structure presenting of the emerging cracks multiple.

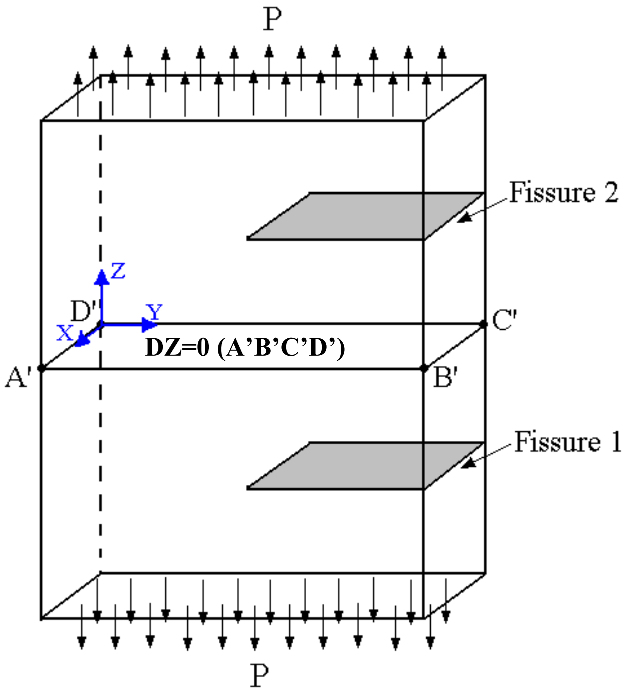

Figure 12.1-a : Diagram of modeling K

The geometry of the initial structure (modeling A) was modified by doubling the height of the plate $3D$. The plan $z=0$ thus became symmetry plane. There is thus a plate $3D$ for which two cracks identical and symmetrical compared to the plan $z=0$ are introduced by functions of levels in the same way as for modelings presented before.

12.2 Characteristics of the grid

The structure is modelled by a healthy, regular grid composed of $5\times31\times102$ HEXA8, respectively along the axes x, y, z . One observes that compared to modeling C, the number of elements according to the direction z doubled, in agreement with the geometrical modification mentioned above. In this manner, the funds of cracks are, as for modeling C, in the center of elements and the plan of crack does not correspond to faces of elements [Figure 4.1-a].

12.3 Boundary conditions and loadings

As for modeling C, a pressure distributed is imposed on the faces lower and higher of the structure what has like effect the opening of the two cracks in mode I, in a symmetrical way. With this intention boundary conditions are imposed on the nodes belonging to the median plane ($z=0$). Thus displacement following the direction z (degree of freedom DZ) is blocked for all these nodes. Moreover, to prevent the movements of the rigid body, two other displacements were blocked for the nodes corresponding to the points A' and B' Figure 12.1-a.

12.4 Sizes tested and results

One tests the values of K_I along the bottom of crack, for the two cracks and various crowns of fields theta of which values of the rays inf and $\sup$ torus same as those are considered for modeling C [].
One will be interested only in crowns 2 and 3.

Smoothing is used `'LAGRANGE'` and to test all the nodes of the bottom of crack in only once, the values are tested min and max of K_I on all the nodes of the bottom of crack.

For crack 1:

Identification	Type of reference	Value of reference	Tolerance
Crown 2 : MAX (K_I)	'ANALYTICAL'	$5.081166724\ 10^6$	10.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	$5.081166724\ 10^6$	10.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	$5.081166724\ 10^6$	10.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	$5.081166724\ 10^6$	10.0%

For crack 2:

Identification	Type of reference	Value of reference	Tolerance
Crown 2 : MAX (K_I)	'ANALYTICAL'	$5.081166724\ 10^6$	10.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	$5.081166724\ 10^6$	10.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	$5.081166724\ 10^6$	10.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	$5.081166724\ 10^6$	10.0%

12.5 Comments

One observes that the results for the two cracks are identical, which one expected because of symmetrical geometrical and kinematic. In more one finds the same values digital as those found for modeling C (only one fissures positioned same manner compared to the grid).

13 Modeling L: Crack X-FEM - Forces surface on edges (modes II and III)

Even modeling that modeling I, but in mode II and III.

In this modeling, the Young modulus is equal to $100\,MPa$.

13.1 Characteristics of the grid

The grid used is that of modeling I.

13.2 Boundary conditions and loadings

13.2.1 Mode II

One wishes to simulate the opening of the crack in mode II pure. For that, one applies to all the faces of the structure a loading in surface forces corresponding to the analytical expressions of the constraints in mode II.

It is pointed out that the mode II pure (i.e. $K_I=0$, $K_{II}=1$ and $K_{III}=0$), the state of stress is the following:

$$\sigma_{11}=\frac{-1}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$
$$\sigma_{12}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad \text{éq 13.2.1-1}$$
$$\sigma_{13}=\frac{1}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

In our case, the definite reference mark is such as:

$$e_1=-e_y$$
$$e_2=e_z \quad \text{éq 13.2.1-2}$$
$$e_3=-e_x$$

The surface forces to apply are then the following ones:

- on the left side face (of outgoing normal $-e_y$): $\begin{cases} FY=-\sigma_{yy}=-\sigma_{11} \\ FZ=-\sigma_{yz}=\sigma_{12} \end{cases}$
- on the right side face (of outgoing normal e_y): $\begin{cases} FY=\sigma_{yy}=\sigma_{11} \\ FZ=\sigma_{yz}=-\sigma_{12} \end{cases}$
- on the higher face (of outgoing normal e_z): $\begin{cases} FY=\sigma_{yz}=-\sigma_{12} \\ FZ=\sigma_{zz}=\sigma_{22} \end{cases}$
- on the lower face (of outgoing normal $-e_z$): $\begin{cases} FY=-\sigma_{yz}=\sigma_{12} \\ FZ=-\sigma_{zz}=-\sigma_{22} \end{cases}$

The rigid modes are blocked thanks to the points A, B and C [Figure 1.1-a]:

- the point A is blocked according to the 3 directions,

- the point B is blocked along the axis Oz ,
- the point C is blocked along the axes Ox and Oz .

In fact, like the number of elements along the axis Oz is odd, the points A , B and C do not coincide with nodes of the grid. One thus uses nodes located just above and below these points to impose equivalent linear relations:

Are $A1$ (respectively $B1$ and $C1$) the node right below point A (B and C).
Are $A2$ (respectively $B2$ and $C2$) the node right above point A (B and C).

One writes the 6 following linear relations to block the rigid modes:

$$DX^{A1}+DX^{A2}=0$$
$$DY^{A1}+DY^{A2}=0$$
$$DZ^{A1}+DZ^{A2}=0$$

$$DZ^{B1}+DZ^{B2}=0$$

$$DX^{C1}+DX^{C2}=0$$
$$DZ^{C1}+DZ^{C2}=0$$

13.2.2 Mode III

One wishes to simulate the opening of the crack in mode III pure. For that, one applies to all the faces of the structure a loading in surface forces corresponding to the analytical expressions of the constraints in mode III .

It is pointed out that the mode III pure (i.e. $K_I=0$, $K_{II}=0$ and $K_{III}=1$), the state of stress is the following:

$$\sigma_{13}=\frac{-1}{\sqrt{2\pi r}}\sin\frac{\theta}{2} \qquad \sigma_{23}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}$$

éq 16 - 3

In our case, the definite reference mark is such as:

$$e_1=-e_y \qquad e_2=e_z \qquad e_3=-e_x$$

éq 16 - 4

The surface forces to apply are then the following ones:

- on the left side face (of outgoing normal $-e_y$): $FX=-\sigma_{xy}=-\sigma_{13}$
- on the right side face (of outgoing normal e_y): $FX=\sigma_{xy}=\sigma_{13}$
- on the higher face (of outgoing normal e_z): $FX=\sigma_{xz}=-\sigma_{23}$
- on the lower face (of outgoing normal $-e_z$): $FX=-\sigma_{xz}=\sigma_{23}$
- on the frontal face (of outgoing normal e_x): $\begin{cases} FY=\sigma_{xy}=\sigma_{13} \\ FZ=\sigma_{xz}=-\sigma_{23} \end{cases}$

- on the face postpones (of outgoing normal $-e_x$): $\begin{cases} FY = -\sigma_{xy} = -\sigma_{13} \\ FZ = -\sigma_{xz} = \sigma_{23} \end{cases}$

The rigid modes are blocked thanks to the points A , B and C [Figure 1.1-a]:
- the point A is blocked according to the 3 directions,
- the point B is blocked along the axis Oz ,
- the point C is blocked along the axes Ox and Oz .

In fact, as the number of elements along axis OZ is odd, the points A , B and C do not coincide with nodes of the grid. One thus uses nodes located just above and below these points to impose equivalent linear relations:
Are $A1$ (respectively $B1$ and $C1$) the node right below point A (B and C).
Are $A2$ (respectively $B2$ and $C2$) the node right above point A (B and C).

One writes the 6 following linear relations to block the rigid modes:

$$DX^{A1} + DX^{A2} = 0$$
$$DY^{A1} + DY^{A2} = 0$$
$$DZ^{A1} + DZ^{A2} = 0$$

$$DZ^{B1} + DZ^{B2} = 0$$

$$DX^{C1} + DX^{C2} = 0$$
$$DZ^{C1} + DZ^{C2} = 0$$

13.3 Sizes tested and results

The goal of this modeling is to test the imposition of surface forces of the elements of edge X-FEM containing the bottom of crack. Those intervene at the time of the imposition of FY and FZ on the faces frontal and back of the structure.

One tests the values of K_I , K_{II} and K_{III} along the bottom of crack, for various crowns of fields theta.
Values of the rays inf and $\sup$ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
Rinf	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 12.4-I

To test all the nodes of the bottom of crack in only once, one tests the values minimum and maximum of K_I on all the nodes of the bottom of crack.

13.3.1 Tests of K_I, K_{II} and K_{III} mode II

13.3.1.1 Values of K_I

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 1 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MIN (K_I)	'ANALYTICAL'	0.0	0.02

13.3.1.2 Values of K_{II}

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1,0	1.5%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1,0	1.5%

13.3.1.3 Values of K_{III}

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 1 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MIN (K_I)	'ANALYTICAL'	0.0	0.02

13.3.2 Tests of K_I, K_{II} and K_{III} mode III

13.3.2.1 Values of K_I

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 1 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MIN (K_I)	'ANALYTICAL'	0.0	0.02

13.3.2.2 Values of K_{II}

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 1 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MIN (K_I)	'ANALYTICAL'	0.0	0.02

13.3.2.3 Values of K_{III}

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1,0	8.0%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1,0	8.0%

13.3.3 Tests on the coordinates of the nodes

One tests the min/max of the coordinates of the nodes of the upper lip belonging to the face of right-hand side.

Identification	Type of reference	Value of reference	Tolerance
MAX (COOR_Y)	'ANALYTICAL'	10,0	1e-7%
MIN (COOR_Y)	'ANALYTICAL'	10,0	1e-7%
MAX (COOR_Z)	'ANALYTICAL'	15,0	1e-7%
MIN (COOR_Z)	'ANALYTICAL'	15,0	1e-7%

13.3.4 Notice on the visualization of the field of displacement

Notice concerning the visualization of the field of displacement with the operators POST_MAIL_XFEM and POST_CHAM_XFEM. When one visualizes the field of total displacement $\sqrt{u_x^2+u_y^2+u_z^2}$, it is necessary to pay attention to what could be interpreted like a “bug” in visualization [Figure 13.3.4-a left], but which is in fact completely normal. One sees a discontinuity of displacement between a classical element and the two elements X-FEM with dimensions. This is explained by the fact why, here, only u_x is nonnull; what means that total displacement that one visualizes is worth in fact $\sqrt{u_x^2+u_y^2+u_z^2}=|u_x|$. This function is not linear: there is thus a difference between a linear approximation (on the classical element) and a linear approximation per pieces (on the two elements X-FEM).
This “bug” disappears when a linear size is visualized, for example u_x [Figure 13.3.4-a right-hand side].

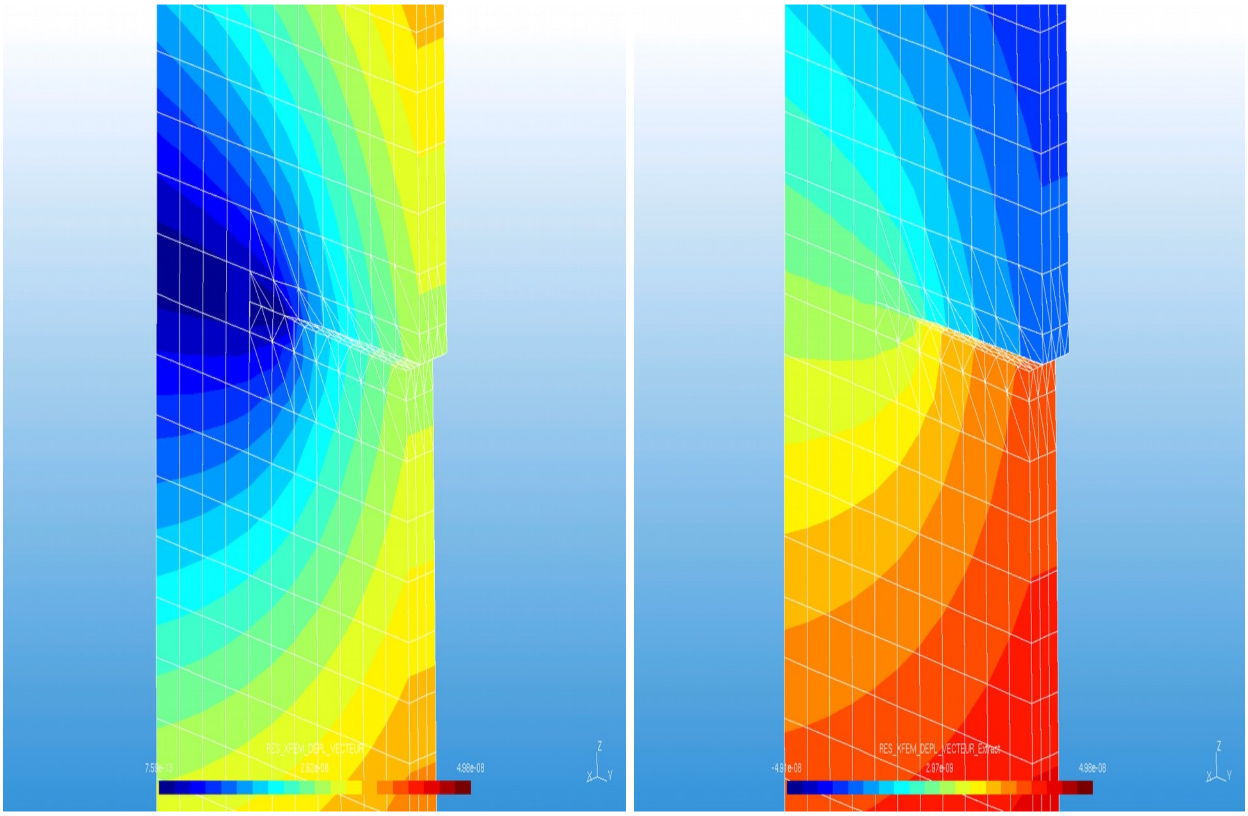

Figure 13.3.4-a : visualization of displacement (magnitude on the left, component DX on the right)

13.4 Comments

The results are stable for any selected crown.

14 Modeling M: Crack X-FEM - surface Forces on edges (modes II and III) – geometrical enrichment

This modeling is exactly the same one as modeling L. the only difference is as the zone of enrichment in bottom of crack now has a size fixed by the user, it thus is not limited more to only one lay down elements in bottom of crack.

14.1 Enrichment in bottom of crack

The nodes being at a distance from the bottom of crack equal or lower than a certain criterion are enriched by the singular functions. This criterion is selected equal to the double of the usual criterion [bib4]. Here, it is worth $2\,m$.

14.2 Sizes tested and results

One tests the values of K_I , K_{II} and K_{III} along the bottom of crack, for various crowns of fields theta.
Values of the rays inf and $\sup$ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
Rinf	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 13.3-J

To test all the nodes of the bottom of crack in only once, the values are tested min and max of K_I on all the nodes of the bottom of crack.

14.2.1 Values of K_I

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 1 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MIN (K_I)	'ANALYTICAL'	0.0	0.02

14.2.2 Values of K_{II}

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 1 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 2 : MAX (K_I)	'ANALYTICAL'	0.0	0.02

Crown 2 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 3 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 4 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 5 : MIN (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MAX (K_I)	'ANALYTICAL'	0.0	0.02
Crown 6 : MIN (K_I)	'ANALYTICAL'	0.0	0.02

14.2.3 Values of K_{III}

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1,0	3.0%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1,0	3.0%

14.3 Comments

The results are stable for any selected crown.
These results highlight the contribution of the keyword RAYON_ENRI. Without this keyword (modeling L), mistakes made on K_{III} are it order of 10% . While informing this keyword, this error is brought back to less 5% . However, that has a cost. The number of degrees of freedom nouveau riches is increased, which weighs down calculations. For example, time spent in STAT_NON_LINE doubled between modelings L and M).

14.4 Notice

In it CAS-test, the keyword FISS_XFEM order DEFI_GROUP is used in all the cases in order to make sure of its good performance.

15 Modeling NR: Crack X-FEM in traction - quadratic elements

This modeling is exactly the same one as modeling C. the only difference is that the finite elements used are quadratic elements instead of linear elements.

15.1 Sizes tested and results

One tests the values of K_I along the bottom of crack, for various crowns of fields theta, resulting from the order CALC_G.
Values of the rays inf and $\sup$ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
Rinf	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 14.2-K

To test all the nodes of the bottom of crack in only once, the values are tested min and max of K_I on all the nodes of the bottom of crack.

15.1.1 Values of K_I for the smoothing of the type 'LAGRANGE'

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	1.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	2.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	1.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	2.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	1.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	2.0%
Crown 4 : MAX (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	1.0%
Crown 4 : MIN (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	2.0%
Crown 5 : MAX (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	1.0%
Crown 5 : MIN (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	2.0%
Crown 6 : MAX (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	1.0%
Crown 6 : MIN (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	2.0%

15.1.2 Values of K_I for the smoothing of the type 'LAGRANGE_NO_NO'

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	1.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	$1.1202664\ 10^{7}$	2.0%

15.2 Values tested resulting from the order POST_K1_K2_K3

The order is used POST_K1_K2_K3 to determine the value of K1. The maximum curvilinear X-coordinate is 1.5, and the number of cut nodes is 6.

One tests the value of K1 for the first point of the bottom.

Identification	Type of reference	Value of reference	Tolerance
K1	'ANALYTICAL'	$1.1202664\ 10^{7}$	3.5%

15.3 Values tested resulting from the order POST_MAIL_XFEM

The order is used POST_MAIL_XFEM to generate the fissured grid.
One test the value of the sum of the absolute values of the ordinates according to X, Y, Z. It is a test of not-regression compared to the values obtained with version 10.2.17.

Identification	Type of reference	Value of reference	Tolerance
SOMM_ABS (COOR_X)	'NON_REGRESSION'	$4,992\ 10^{3}$	1e- 8%
SOMM_ABS (COOR_Y)	'NON_REGRESSION'	$4,992\ 10^{4}$	1e- 8%
SOMM_ABS (COOR_Z)	'NON_REGRESSION'	$1.4976\ 10^{5}$	1e- 8%

15.4 Values tested resulting from the order POST_CHAM_XFEM

The order is used POST_CHAM_XFEM to generate the adequate fields of result to the grid fissured previously created.
One test the value of the sum of the absolute values of displacements of the nodes of the grid thus created. It is a test of not-regression compared to the values obtained with version 10.2.17.

Identification	Type of reference	Value of reference	Tolerance
SOMM_ABS (DX)	'NON_REGRESSION'	$8,312\ 10^{-3}$	10^{-4} %
SOMM_ABS (DY)	'NON_REGRESSION'	6,588	10^{-4} %
SOMM_ABS (DZ)	'NON_REGRESSION'	4,616	10^{-4} %

16 Modeling O: Crack with a grid - voluminal Forces

In this modeling, the crack is with a grid, and one uses the standard method of the finite elements to carry out calculation. This modeling will be used as reference and will allow the comparison with the method X-FEM in modeling P.

16.1 Characteristics of the grid

The structure is modelled by a regular grid composed of $5\times30\times50$ HEXA8, respectively along the axes X, Y, Z (see [Figure 2.1-has]). Two superimposed surfaces are the lips of the crack.

16.2 Boundary conditions and loadings

Two types of voluminal loadings (leading to the same result) are studied here:

- A voluminal loading imposed on all the structure with FORCE_INTERNE $FX=0$, $FY=0$, $FZ=-78000$
- UN loading of gravity with the keyword GRAVITY $=(10,0,0,-1)$, acceleration 10 in the direction $-z$ (it is pointed out that the density of the structure is equal to 7800).

For each of the two loadings, the higher face of the structure is embedded.

16.3 Sizes tested and results

One tests the values of K_I, K_{II} and K_{III} along the bottom of crack, for various crowns of fields theta. Values of the rays inf and $\sup$ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
$Rinf$	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 15.55 -L

To test all the nodes of the bottom of crack in only once, one tests the min and max values of K_I on all the nodes of the bottom of crack. One has only the results of the first loading (with FORCE_INTERNE), those of the second are perfectly identical. The tests presented here are tests of nonregression.

Values of K_I

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 1 : MIN (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 2 : MAX (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 2 : MIN (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 3 : MAX (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 3 : MIN (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 4 : MAX (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 4 : MIN (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 5 : MAX (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 5 : MIN (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%
Crown 6 : MAX (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%

Crown 6 : MIN (K_I)	'NON_REGRESSION'	1.23 10^7	0.3%

16.4 Comments

The results are stable for any selected crown. A median value of $1.23D+07$ is taken as reference for calculations with a crack $XFEM$ in following modeling.

17 Modeling P: Crack X-FEM - Voluminal forces

Even modeling that modeling B, but with the loadings of modeling O.
Modeling O is used as reference solution.

17.1 Characteristics of the grid

The structure is modelled by a healthy, regular grid composed of $5\times30\times50$ HEXA8, respectively along the axes X, Y, Z in order to have the same number of elements as for the grid of modeling A (see [Figure 4.1-has]). Thus, the plan of crack is in correspondence with faces ofHEXA8 and bottom of crack with edges ofHEXA8. (see [Figure 3.1-a]).
This grid is identical to that of modeling B.

17.2 Boundary conditions and loadings

Two types of voluminal loadings (leading to the same result) are studied here:
- a loading n°1: voluminal loading imposed on all the structure with FORCE_INTERNE $FX=0$, $FY=0$, $FZ=-78000$
- a loading n°2: UN loading of gravity with the keyword GRAVITY acceleration 10 in the direction $-z$ (it is pointed out that the density of the structure is equal to 7800).

For each of the two loadings, the higher face of the structure is embedded.

17.3 Enrichment in bottom of crack

The nodes being at a distance from the bottom of crack equal or lower than a certain criterion are enriched by the singular functions. One uses topological enrichment by default.

17.4 Results of modeling P

One tests the values of K_I, K_{II} and K_{III} along the bottom of crack, for various crowns of fields theta. Values of the rays inf and ¿ torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
$Rinf$	2	0,666	1	1	1	2.1
Rsup	4	1,666	2	3	4	3.9

Table 17.5 -m

To test all the nodes of the bottom of crack in only once, one tests the values minimum and maximum of K_I on all the nodes of the bottom of crack. One has only the results of the first loading (with FORCE_INTERNE), those of the second are perfectly identical. The tests presented here are tests compared to a digital reference solution with crack with a grid (modeling O).

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 1 : MIN (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 2 : MAX (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 2 : MIN (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 3 : MAX (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 3 : MIN (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%

Crown 4 : MAX (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 4 : MIN (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 5 : MAX (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 5 : MIN (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 6 : MAX (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%
Crown 6 : MIN (K_I)	'AUTRE_ASTER'	1.23 10^7	6.0%

One also tests the values of G (more precisely the maximum of G along the bottom of crack) obtained by the order CALC_G, option CALC_G, for the voluminal loading as well as the loading of gravity. One restricts oneself with the 1st crown of integration. One compared to the value of G obtained by the order CALC_G, option CALC_K_G.

Identification	Type of reference	Value of reference	Tolerance
Crown 1, loading n°1 : MAX (G)	'AUTRE_ASTER'	826,143	0.1%
Crown 1, loading n°2 : MAX (G)	'AUTRE_ASTER'	826,143	0.1%

One tests also the value of K_I product by the operator POST_K1_K2_K3 at the first point of the bottom of crack with the loading n°1:

Identification	Type of reference	Value of reference	Tolerance
Crown 1, not initial : K_I	'NON_REGRESSION'	1.3416697 10^7	0.1%

The order is used POST_MAIL_XFEM to generate the fissured grid.
One test the sum of the absolute values of the ordinates according to X, Y, Z nodes of the fissured grid. It is a test of not-regression compared to the values obtained with version 10.2.17.

Identification	Type of reference	Reference	Tolerance
SOMM_ABS (COOR_X)	'NON_REGRESSION'	4,743 10^3	1e-8%
SOMM_ABS (COOR_Y)	'NON_REGRESSION'	4,743 10^4	1e-8%
SOMM_ABS (COOR_Z)	'NON_REGRESSION'	1.4229 10^5	1e-8%

The order is used POST_CHAM_XFEM to generate the adequate fields of result to the grid fissured previously created. One test the sum of the absolute values of displacements of the nodes of the grid thus created. It is a test of not-regression compared to the values obtained with version 10.2.17.

Identification	Type of reference	Reference	Tolerance
SOMM_ABS (DX)	'NON_REGRESSION'	3.88717 10^{-4}	1e-6%
SOMM_ABS (DY)	'NON_REGRESSION'	2.3488575	1e-6%
SOMM_ABS (DZ)	'NON_REGRESSION'	2.310308	1e-6%

17.5 Comments

The results are stable for any selected crown. The error is about 5% what is satisfactory. It is noticed however that it is only one comparison with a digital method (crack with a grid) which in addition, if one has an analytical solution, provides results less precise than the method XFEM .

18 Modeling Q: Crack X-FEM - Propagation in the plan with the method upwind

In this modeling, one takes again modeling J and one uses the method upwind of the operator PROPA_FISS to solve the equations of propagations. No auxiliary grid is used because the grid of the structure is very regular and it can thus be used directly.

The Young modulus is equal to $205000\,MPa$.

18.1 Characteristics of the grid

The structure is modelled by a healthy, regular grid of $2\times20\times51$ HEXA8, respectively along the axes X,Y,Z [Figure 18.1-a]. The bottom of crack is inside an element as in modeling C [Figure 4.1-a].

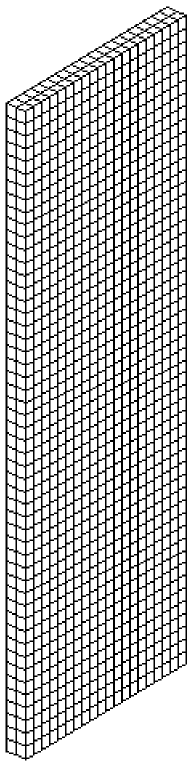

Figure 18.1-a : Grid 2*20*51 HEXA8

18.2 Boundary conditions and loadings

The loading applied is the same one as the loading n°1 of modeling A, i.e. a uniform request in traction by an imposed pressure $P=-10^6 Pa$ on the lower and higher faces. The crack is requested in mode K_I pure.
Advance of the crack imposed on each call to PROPA_FISS is the following one: $\Delta a=0.5\mathrm{m}$

Lengths of crack to each call to PROPA_FISS are thus:

Initial state: $a_0=4.9\,m$
Iteration 1: $a_1=5.4\,m$

Iteration 2: $a_2=5.9\,m$

The stress intensity factors are given by: $K_I=-P\cdot\sqrt{\pi a}\cdot f\left(\frac{a}{b}\right)$

with:

$$f\left(\frac{a}{b}\right)=\left(\frac{2b}{\pi a}\tan\frac{\pi a}{2b}\right)^{0.5}\cdot\frac{0.752+0.37\cdot\left(1-\sin\frac{\pi a}{2b}\right)^3+2.02\frac{a}{b}}{\cos\frac{\pi a}{2b}}$$

$b=10\text{m}$

$P=10^6\,Pa$

From where:

$$K_{I0}=10.7418\,MPa\cdot\sqrt{m}$$
$$K_{I1}=13.3139\,MPa\cdot\sqrt{m}$$
$$K_{I2}=16.7342\,MPa\cdot\sqrt{m}$$

18.3 Sizes tested and results

One tests the values of K_I along the bottom of crack, for the crown $[1.;4.]$.
To test all the points of the bottom of crack in only once, one tests the minimal and maximum values of K_I on all the points of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
Initial state : MAX (K_I)	'ANALYTICAL'	10.7417689	5.0%
Initial state : MIN (K_I)	'ANALYTICAL'	10.7417689	5.0%
Iteration 1 : MAX (K_I)	'ANALYTICAL'	13.3138925	5.0%
Iteration 1 : MIN (K_I)	'ANALYTICAL'	13.3138925	5.0%

18.4 Comments

The values obtained are relatively close to the expected values. However, it is noted that the value of K_I calculated by Aster is lower than the analytical solution of reference. That is related to the grid used which is not very refined.

19 Modeling R: Crack X-FEM in traction - Grid with pyramids

This modeling is exactly the same one as modeling C. the only difference is that, prior to mechanical calculation, one calls Lobster to refine certain meshs HEXA8. This process generates meshs PYRA5.

19.1 Sizes tested and results

One tests the values of K_I along the bottom of crack, for various crowns of fields theta.
The values of the rays inferior and superior of the torus are the following ones:

	Crown 1	Crown 2	Crown 3	Crown 4	Crown 5	Crown 6
R_{inf}	2	0,666	1	1	1	2.1
R_{sup}	4	1,666	2	3	4	3.9

Table 7.3-F

To test all the nodes of the bottom of crack in only once, the values are tested min and max of K_I on all the nodes of the bottom of crack.

Smoothing 'LAGRANGE' is used.

Identification	Type of reference	Value of reference	Tolerance
Crown 1 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 1 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 2 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 2 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 3 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 3 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 4 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 4 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 5 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 5 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 6 : MAX (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%
Crown 6 : MIN (K_I)	'ANALYTICAL'	1.1202664 10^7	2.0%

The calculation of K_I by extrapolation of the jumps of displacements also carried out by the order POST_K1_K2_K3. One tests the value of K1 at the first point of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
K_I at the first point of the bottom of crack	'ANALYTICAL'	1.1202664 10^7	4.0%

20 Modeling S: cohesive crack, calculation of KI are equivalent

This modeling is of the same type that modeling b: the crack right and is requested in traction, it coincides with the faces of the grid HEXA8, the initial crack is with a grid. Compared to modeling B, one introduces cohesive zones into the prolongation of the crack. This prolongation is represented by level-sets, so that discontinuity is taken into account by a modeling XFEM, as for modeling B. The cohesive law CZM_LIN_MIX is introduced into this model XFEM by the order DEFI_CONTACT.

One applies then the loading envisaged. The cohesive parameters are selected so that this causes to open some cohesive elements in the vicinity of the point of initial crack:

- Not to have a complete rupture, but simply a decoherence close to the point of initial crack, one takes $G_c > G$, while preserving the same order of magnitude for the two values. In our case, $G_c = 1097\,N.m^{-1}$ and $G = 612\,N.m^{-1}$.
- For all the same observing a decoherence in the vicinity of the point, size characteristic of the cohesive zone $l_c = \frac{E\,G_c}{(1-\nu^2)\sigma_c^2}$ is selected of kind to cover some elements while remaining small in front of the size of the structure $h \le l_c \le LX$. In this case test, to reduce the computing time, one taken $l_c = 0.25\,m \sim h$, which leads to $\sigma_c = 30\,MPa$.

20.1 Sizes tested and results

20.1.1 Factors of intensity of the constraints equivalents

Analytical values of reference for K_I are those of modeling A, i.e. those of *initial crack* . In the presence of cohesive zones, the calculation of K_I does not base itself on a calculation in a crown centered on the bottom of crack, but on surface integrals on the cohesive zone. *It is thus not necessary to specify rays of crown* .

One tests the value of K_I along the face. To test all the nodes of the bottom of crack in only once, the values are tested min and max of K_I on all the nodes of the bottom of crack.

Smoothing 'LAGRANGE' is used.

Identification	Type of reference	Value of reference	Tolerance
MAX (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%
MIN (K_I)	'ANALYTICAL'	1.1202664 10^{7}	4.0%

20.1.2 Rate of refund of energy room

One tests the values of G room along the bottom of crack, always by taking into account a smoothing of the type 'LAGRANGE'. The value of reference is that of modeling B.

To test all the nodes of the bottom of crack in only once, the values are tested min and max of G on all the nodes of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance

MAX (G)	'ANALYTICAL'	612.193573558	9.0%
MIN (G)	'ANALYTICAL'	612.193573558	9.0%

20.2 Comments

Values of K_I and G are approached by excess with this method (the computed values are higher than the analytical references). Indeed, by the opening of cohesive elements in the prolongation of the initial crack, one actually calculates quantities corresponding to a crack slightly longer than the objective, from where the approximation by excess. For this test, IL has there a variation of some percent: we noticed that the precision increases by choosing a size cohesive much lower than that of preexistent crack, which requires a rather fine grid at a peak. Having for objective a case test of a few seconds, we did not refine the grid for this modeling.

21 Modeling T: crack X-FEM compression

This modeling is identical to modeling G, only the grid and the sizes tested change.

21.1 Characteristics of the grid

The structure is modelled by a healthy, regular grid of $5\times15\times25$ HEXA8, respectively along the axes x, y, z . The bottom of crack is inside an element as in modeling C [Figure 4.1-a].

21.2 Boundary conditions and loadings

The loading applied is the same one as that of modeling G

21.3 Sizes tested and results

21.3.1 Contact pressures

One is interested in the evolution of the contact pressure according to the axis Oy , on two lines (the line in $x=0$ and the line in $x=1$) surface of the crack [Figure 8.2-8.1.1-a]. It would normally be necessary to test the value of the contact pressure in each node of these 2 lines, but to reduce the number of tests, one can simply test the minimum and the maximum of the pressures on each line, which results in carrying out 4 tests.

Identification	Type of reference	Value of reference	Tolerance
Line in $x=0$: MAX (LAGS_C)	'ANALYTICAL'	-10^6	4.0%
Line in $x=0$: MIN (LAGS_C)	'ANALYTICAL'	-10^6	4.0%
Line in $x=1$: MAX (LAGS_C)	'ANALYTICAL'	-10^6	4.0%
Line in $x=1$: MIN (LAGS_C)	'ANALYTICAL'	-10^6	4.0%

21.3.2 Rate of refund of energy room

One tests the value of G product by the operator POST_K1_K2_K3 at the first point of the bottom of crack.

Identification	Type of reference	Value of reference	Tolerance
G	'ANALYTICAL'	0.0	10^{-6}

21.3.3 Energy of the structure

The tensor of the constraints analytical solution is:

$$\boldsymbol{\sigma}=-p\,\boldsymbol{e}_z\otimes\boldsymbol{e}_z .$$

S hears $\Omega=[0,LX]\times[0,LY]\times[0,LZ]$ the field occupied by the solid. The energy of the structure is defined by:

$$E^e=\frac{1}{2}\int_\Omega \boldsymbol{\sigma}:\boldsymbol{\varepsilon}\,dV .$$

Since $\nu=0$, one has $\varepsilon=\frac{1}{E}\sigma$. From where:

$$\sigma:\varepsilon=\frac{p^2}{E}.$$

One thus has, for p expressed in MPa:

$$E^e=\frac{1}{2}\frac{p^2}{E^2}|\Omega|=\frac{1}{2}\frac{p^2}{E}LX\ LY\ LZ\approx 7{,}31707317073\times 10^2\ \text{MJ}.$$

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E E	'ANALYTICAL'	7.31707317073 10^2	0.1%

21.3.4 L normalizes² displacement

The field of analytical solution displacement is:

$$\boldsymbol{u}=\frac{-p}{E}\left(z-\frac{LZ}{2}\right)\boldsymbol{e}_z.$$

The standard L^2 displacement is defined by:

$$\|\boldsymbol{u}\|_{L^2}^2=\int_\Omega \|\boldsymbol{u}\|^2 dV.$$

One a:

$$\|\boldsymbol{u}\|^2=\frac{p^2}{E^2}\left(z-\frac{LZ}{2}\right)^2,$$

for any point of Ω . One thus has:

$$\|\boldsymbol{u}\|_{L^2}^2=LX\ LY\frac{p^2}{E^2}\int_0^{LZ}\left(z-\frac{LZ}{2}\right)^2 dV=LX\ LY\frac{p^2}{E^2}\frac{LZ^3}{12}.$$

That is to say:

$$\|\boldsymbol{u}\|_{L^2}=\frac{p}{E}\frac{1}{2}\sqrt{LX\ LY\frac{LZ^3}{3}}\approx 7{,}31707317073\times 10^{-4}\ \text{m}^{\frac{5}{2}}.$$

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ²	'ANALYTICAL'	7.31707317073 10^{-4}	0.1%

22 Modeling U: crack X-FEM compression

This modeling is of the same type as modeling T, one is placed this time in the case as of plane deformations (D_PLAN).

That amounts considering the borderline case where $LX \rightarrow \infty$.

22.1 Characteristics of the grid

The structure is modelled by a healthy, regular grid of 31×51 QUAD4, respectively along the axes x, y . The bottom of crack is inside an element as in modeling T [Figure 4.1-a].

22.2 Boundary conditions and loadings

The loading applied is the same one as that of modeling T

22.3 Sizes tested and results

22.3.1 Rate of refund of energy room

One tests the value of G product by the operator POST_K1_K2_K3.

Identification	Type of reference	Value of reference	Tolerance
G	'ANALYTICAL'	0.0	10^{-6}

22.3.2 Energy of the structure

The tensor of the constraints analytical solution is:

$$\boldsymbol{\sigma} = -p\, \boldsymbol{e}_y \otimes \boldsymbol{e}_y .$$

S hears $\Omega = [0, LY] \times [0, LZ]$ the field occupied by the solid. The energy of the structure is defined by:

$$E^e = \frac{1}{2} \int_\Omega \boldsymbol{\sigma} : \boldsymbol{\varepsilon}\, dS .$$

Since $\nu = 0$, one has $\boldsymbol{\varepsilon} = \frac{1}{E} \boldsymbol{\sigma}$. From where:

$$\boldsymbol{\sigma} : \boldsymbol{\varepsilon} = \frac{p^2}{E} .$$

One thus has, for p expressed in MPa:

$$E^e = \frac{1}{2} \frac{p^2}{E} |\Omega| = \frac{1}{2} \frac{p^2}{E} LY\, LZ \approx 7{,}31707317073 \times 10^2 \text{ MJ} \times \text{m}^{-1} .$$

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E E	'ANALYTICAL'	$7.31707317073\ 10^2$	0.1%

22.3.3 L normalizes² displacement

The field of analytical solution displacement is:

$$\boldsymbol{u}=\frac{-p}{E}\left(y-\frac{LZ}{2}\right)\boldsymbol{e}_y .$$

The standard L^2 displacement is defined by:

$$\|\boldsymbol{u}\|_{L^2}^2=\int_\Omega \|\boldsymbol{u}\|^2 dS .$$

One a:

$$\|\boldsymbol{u}\|^2=\frac{p^2}{E^2}\left(y-\frac{LZ}{2}\right)^2,$$

for any point of Ω . One thus has:

$$\|\boldsymbol{u}\|_{L^2}^2=LY\frac{p^2}{E^2}\int_0^{LZ}\left(y-\frac{LZ}{2}\right)^2 dS=LY\frac{p^2}{E^2}\frac{LZ^3}{12} .$$

That is to say:

$$\|\boldsymbol{u}\|_{L^2}=\frac{p}{E}\frac{1}{2}\sqrt{LY\frac{LZ^3}{3}}\approx 7{,}31707317073\times 10^{-4}\ \mathrm{m}^2 .$$

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator `POST_ERREUR`.

Identification	Type of reference	Value of reference	Tolerance
`L normalizes` ²	'ANALYTICAL'	$7.31707317073\ 10^{-4}$	0.1%

23 Modeling V: crack X-FEM compression

This modeling is of the same type as modeling U, one places this time them in the case of plane constraints (C_PLAN).

That amounts considering the borderline case where $LX \rightarrow 0$.

23.1 Characteristics of the grid

The grid used is the same one as that of modeling U.

23.2 Boundary conditions and loadings

The loading applied is the same one as that of modeling U

23.3 Sizes tested and results

23.3.1 Rate of refund of energy room

One tests the value of G product by the operator POST_K1_K2_K3.

Identification	Type of reference	Value of reference	Tolerance
G	'ANALYTICAL'	0.0	10^{-6}

23.3.2 Energy of the structure

The analytical value of the energy of the structure is the same one as that obtained for modeling U. One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E^E	'ANALYTICAL'	$7.31707317073\ 10^{2}$	0.1%

23.3.3 L normalizes² displacement

The analytical value of the standard L^2 displacement is the same one as that obtained for modeling U. One tests the value of $\|u\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes ²	'ANALYTICAL'	$7.31707317073\ 10^{-4}$	0.1%

24 Modeling W: crack X-FEM compression

This modeling is of the same type as modeling U, one is placed this time in the case axisymmetric (AXIS).

That amounts considering either the case of a crack emerging in a plate, but the case of a circular crack in a cylinder of ray LY and height LZ .

24.1 Characteristics of the grid

The grid used is the same one as that of modeling U.

24.2 Boundary conditions and loadings

The loading applied is the same one as that of modeling U

24.3 Sizes tested and results

24.3.1 Rate of refund of energy room

One tests the value of G product by the operator POST_K1_K2_K3.

Identification	Type of reference	Value of reference	Tolerance
G	'ANALYTICAL'	0.0	10^{-6}

24.3.2 Energy of the structure

The tensor of the constraints analytical solution is:

$$\boldsymbol{\sigma}=-p\,\boldsymbol{e}_z\otimes\boldsymbol{e}_z\,.$$

S hears Ω the field occupied by the solid. The energy of the structure is defined by:

$$E^e=\frac{1}{4\pi}\int_\Omega \boldsymbol{\sigma}:\boldsymbol{\varepsilon}\,dV\,.$$

Since $\nu=0$, one has $\boldsymbol{\varepsilon}=\frac{1}{E}\boldsymbol{\sigma}$. From where:

$$\boldsymbol{\sigma}:\boldsymbol{\varepsilon}=\frac{p^2}{E}\,.$$

One thus has, for p expressed in MPa:

$$E^e=\frac{1}{4\pi}\frac{p^2}{E}\int_0^{2\pi}\left(\int_0^{LZ}\left(\int_0^{LY} r\,dr\right)dz\right)d\theta=\frac{p^2}{E}\frac{LZ\,LY^2}{4}\approx 3{,}6585365837\times 10^3\ \mathrm{MJ}\times\mathrm{rad}^{-1}\,.$$

One tests the value of E^e product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
E E	'ANALYTICAL'	$3.6585365837\ 10^3$	0.1%

24.3.3 L normalizes[2] displacement

The field of analytical solution displacement is:

$$\boldsymbol{u} = \frac{-p}{E}\left(z - \frac{LZ}{2}\right)\boldsymbol{e}_z .$$

The standard L^2 displacement is defined by:

$$\|\boldsymbol{u}\|^2_{L^2} = \frac{1}{2\pi}\int_{\Omega}\|\boldsymbol{u}\|^2\, dV .$$

One a:

$$\|\boldsymbol{u}\|^2 = \frac{p^2}{E^2}\left(z - \frac{LZ}{2}\right)^2 ,$$

for any point of Ω . One thus has:

$$\|\boldsymbol{u}\|^2_{L^2} = \frac{1}{2\pi}\frac{p^2}{E^2}\int_0^{2\pi}\left(\int_0^{LZ}\left(\int_0^{LY}\left(z - \frac{LZ}{2}\right)^2 r\, dr\right)dz\right)d\theta = \frac{1}{24}\frac{p^2}{E^2}LZ^3\,LY^2 .$$

That is to say:

$$\|\boldsymbol{u}\|_{L^2} = \frac{1}{2}\frac{p}{E}LY\sqrt{\frac{LZ^3}{6}} \approx 1{,}6361473006 \times 10^{-3}\ \mathrm{m}^{\frac{5}{2}} \times \mathrm{rad}^{\frac{-1}{2}} .$$

One tests the value of $\|\boldsymbol{u}\|_{L^2}$ product by the operator POST_ERREUR.

Identification	Type of reference	Value of reference	Tolerance
L normalizes 2	'ANALYTICAL'	1.6361473006 10^{-3}	0.1%

25 Summary of the results

The goals of this test are achieved:

- to validate on a simple case the taking into account of singular enrichment in bottom of crack with the method X-FEM ;
- to validate the calculation of the stress intensity factors (here only mode I) for the elements X-FEM , whatever the load (fixed or function);
- to validate the contact on a case of compression in mode 1 of closing;
- to validate the calculation of K_I on a case of multi-cracking.
- to test the case of the imposed voluminal efforts.

It will be retained that the use of the method X-FEM allows to appreciably improve the precision of calculation of K_I, and that this one increases when the zone of enrichment is not restricted to only one elements in bottom of crack sleep.