SSNV150 - Triaxial traction with the law of behavior BETON_DOUBLE_DP

Summary

This case of validation is intended to check the model of behavior 3D BETON_DOUBLE_DP formulated within the framework of thermoplasticity, for the description of the nonlinear behavior of the concrete, in traction, and compression, with the taking into account of the irreversible variations of the thermal and mechanical characteristics of the concrete, particularly sensitive at high temperature.

The description of cracking is treated within the framework of plasticity, using an energy equivalence, by identifying the density of energy of cracking in mode I, with the plastic work of a homogeneous medium are equivalent, where the plastic deformation is uniformly distributed, in an “elementary” zone. This approach preserves the continuity of the formulation of the model, on the whole of its behavior, and contributes to avoid the possible digital difficulties during the change of state of material.
The pathological sensitivity of the digital solution to the space discretization (grid), generated by the introduction of a softening behavior of the concrete in traction and compression, is partially solved by introducing an energy of cracking or rupture, dependent a characteristic length l_c, dependent in keeping with elements. The resolution of the equations constitutive of the model is carried out by an implicit scheme.

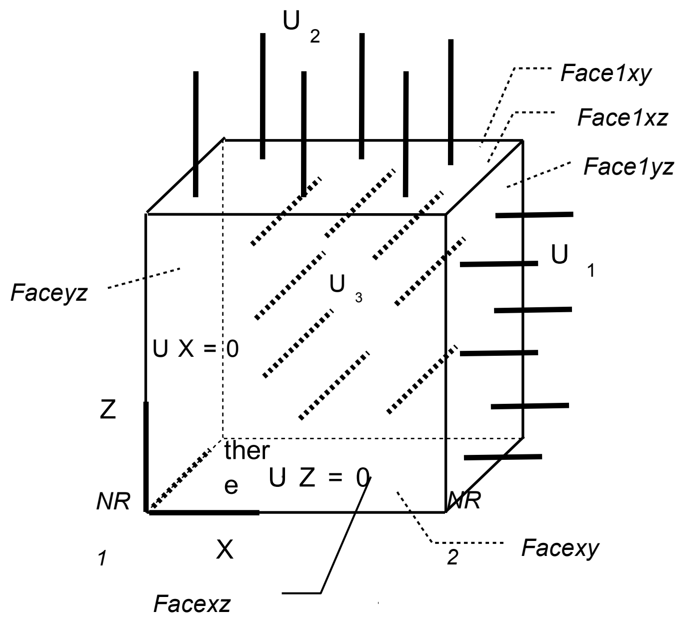
It is about a cube with 8 nodes subjected to a triaxial traction, in imposed displacement. This loading led to the typical case of a hydrostatic state of stress, solved by projection at the top of the cone of traction, when one places oneself in a hydrostatic diagram forced equivalent/forced. It is about a case test with analytical solution.

1 Problem of reference

1.1 Geometry

It is about a cube with 8 nodes, whose three faces have a normal displacement no one and the three opposite faces an imposed and identical normal displacement.
The made cube $1\,mm$ on side. In modeling A, the cube is directed according to the reference mark $Oxyz$.

Modeling A



$U_2 = U_1 = U_3$

1.2 Material properties

To test the irreversible evolution of the mechanical characteristics with the temperature, one applies a field of temperature decreasing. Certain variables depend on the temperature, others of drying. Lastly, one applies a coefficient of withdrawal of desiccation not no one, equal to the thermal dilation coefficient, to test “data-processing” operation. The thermal deformations thus equal and will be opposed to the deformations of withdrawal of desiccation. These dependences intervene only for purely data-processing checks, the mechanical characteristics can be regarded as constants.

For the usual linear mechanical characteristics:

Young modulus:	$E=32\,000\,MPa$	of	$0°C$ with $20°C$
	$E=15\,000\,MPa$	with	$400°C$ (linear decrease)
	$E=5\,000\,MPa$	with	$800°C$ (linear decrease)
Poisson's ratio:	$\nu=0.18$		
Thermal dilation coefficient:	$a=10^{-5}/°C$		
Coefficient of withdrawal of desiccation:	$k=10^{-5}$		

For the nonlinear mechanical characteristics of the model `BETON_DOUBLE_DP` :

Resistance in uniaxial pressing:	$f'c=40\,N/mm^2$	of	$0°C$ with $400°C$
	$f'c=15\,N/mm^2$	with	$800°C$ (linear decrease)
Resistance in uniaxial traction:	$f't=4\,N/mm^2$	of	$0°C$ with $400°C$
	$f't=1.5\,N/mm^2$	with	$800°C$ (linear decrease)
Report of resistances in compression biaxial/uniaxial pressing:	$b=1.16$		
Energy of rupture in compression:	$G_c=10\,Nmm/mm^2$		
Energy of rupture in traction:	$G_t=0.1\,Nmm/mm$		
Report of the limit elastic to resistance in uniaxial pressing:	30%		

1.3 Boundary conditions and loadings mechanical

Field of temperature decreasing of $20°C$ with $0°C$.

Lower face of the cube ($facexy$) :	blocked according to oz .
Higher face of the cube ($face1xy$) :	displacement $U_z=0,15\,mm$
Left face of the cube ($faceyz$) :	blocked according to ox .
Right face of the cube ($face1yz$) :	displacement $U_x=0,15\,mm$
Face before cube ($facexz$) :	blocked according to oy .
Face postpones cube ($face1xz$) :	displacement $U_y=0,15\,mm$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is calculated in an analytical way, knowing that in traction, only the criterion of traction is activated, and that in the case of a hydrostatic loading, one is projected at the top of the cone of traction. It is thus necessary to solve a linear system of an equation to an unknown factor, which makes it possible to obtain the plastic deformation cumulated in traction. This one makes it possible to calculate strains and stresses then.

2.2 Calculation of the reference solution of reference

For more detail on the notations and the setting in equation, one will refer to the reference document [R7.01.03]. Only, the principal equations are pointed out here.
One notes " a ", imposed displacement following the directions x, y and z. The tensor of deformation is form $(a, a, a, 0., 0., 0.)$ by taking the usual notations of *Code_Aster* (three principal components, three components of shearing).
The tensor of constraint is form $(\sigma, \sigma, \sigma, 0., 0., 0.)$, in modeling A.

General equations of the model:

The equations constitutive of the model are written by distinguishing the isotropic part of the deviatoric part of the tensors of constraints and deformations.

$$\sigma_H = \frac{1}{3} tr(\sigma) \quad s = \sigma - \frac{1}{3} tr(\sigma) I \quad \varepsilon_H = \frac{1}{3} tr(\varepsilon) \quad \tilde{\varepsilon} = \varepsilon - \frac{1}{3} tr(\varepsilon) I$$

$$\sigma = s + \sigma_H I \quad \text{and} \quad \varepsilon = \tilde{\varepsilon} + \varepsilon_H I$$

The equivalent constraint is written then: $\sigma^{eq} = \sqrt{\frac{3}{2} tr(s^2)}$

In the case of an incremental formulation, and of a variable law of behavior, while noting with an exhibitor *"E"* the elastic components of the constraint and the deformation, one obtains:

$$s^e = \frac{\mu^+}{\mu^-} s^- + 2\mu^+ \Delta\tilde{\varepsilon} \quad \text{and} \quad \sigma_H{}^e = \frac{K^+}{K^-} \sigma_H{}^- + 3K^+ \Delta\varepsilon_H$$

Criteria in compression (f_{comp}) and in traction (f_{trac}) express themselves in the following way:

$$f_{comp} = \frac{\tau_{oct} + a.\sigma_{oct}}{b} - f_c(\lambda_c) = \frac{\sqrt{2}}{3b} \sigma^{eq} + \frac{a}{b} \sigma_H - f_c(\lambda_c)$$

$$f_{trac} = \frac{\tau_{oct} + c.\sigma_{oct}}{d} - f_t(\lambda_t) = \frac{\sqrt{2}}{3d} \sigma^{eq} + \frac{c}{d} \sigma_H - f_t(\lambda_t)$$

avec $\tau_{oct}=\sqrt{\frac{tr(s^2)}{3}}$

$\sigma_{oct}=\sqrt{\frac{tr(\sigma)}{3}}$

λ_c : plastic multiplier in compression

λ_T : plastic multiplier in traction

and a,b,c,d coefficients of the model

The plastic deformations in traction and compression are expressed:

$$\Delta\tilde{\varepsilon}^p{}_c=\frac{\Delta\lambda_c}{\sqrt{2}b}\frac{s}{\sigma^{eq}} \qquad \Delta\varepsilon^p{}_{H_c}=\Delta\lambda_c\frac{a}{3b}$$

$$\Delta\tilde{\varepsilon}^p{}_t=\frac{\Delta\lambda_t}{\sqrt{2}d}\frac{s}{\sigma^{eq}} \qquad \Delta\varepsilon^p{}_{H_t}=\Delta\lambda_t\frac{c}{3d}$$

One obtains for the constraint:

$$s=s^e-2\mu^+\left(\Delta\tilde{\varepsilon}^p{}_c+\Delta\tilde{\varepsilon}^p{}_t\right) \qquad \sigma_H=\sigma_H{}^e-3K^+\left(\Delta\varepsilon_H{}^p{}_c+\Delta\varepsilon_H{}^p{}_t\right)$$

$$s=\left(1-2\mu^+\left(\frac{\Delta\lambda_c}{\sqrt{2}b}+\frac{\Delta\lambda_t}{\sqrt{2}d}\right)\frac{1}{\sigma^{e\,eq}}\right)s^e \qquad \sigma_H=\sigma_H{}^e-3K^+\left(\Delta\lambda_c\frac{a}{3b}+\Delta\lambda_t\frac{c}{3d}\right)$$

for the equivalent constraint: $\sigma^{eq}=\sigma^{e\,eq}-2\mu^+\left(\frac{\Delta\lambda_c}{\sqrt{2}b}+\frac{\Delta\lambda_t}{\sqrt{2}d}\right)$

The two criteria lead then to a system of two equations to two unknown factors $\Delta\lambda_c$ and $\Delta\lambda_t$ to solve:

$$\begin{cases}\frac{\sqrt{2}}{3b}\sigma^{e\,eq}+\frac{a}{b}\sigma_H{}^e-\Delta\lambda_c\left(\frac{2\mu^+}{3b^2}+\frac{K^+a^2}{b^2}\right)-\Delta\lambda_t\left(\frac{2\mu^+}{3bd}+\frac{K^+ac}{bd}\right)-f_c\left(\lambda_c^-+\Delta\lambda_c\right)=0\\ \frac{\sqrt{2}}{3d}\sigma^{e\,eq}+\frac{c}{d}\sigma_H{}^e-\Delta\lambda_c\left(\frac{2\mu^+}{3bd}+\frac{K^+ac}{bd}\right)-\Delta\lambda_t\left(\frac{2\mu^+}{3d^2}+\frac{K^+c^2}{d^2}\right)-f_t\left(\lambda_t^-+\Delta\lambda_t\right)=0\end{cases}$$

In a similar way, in the case of the only criterion of traction activated, **configuration of the case test**, one obtains a system of an equation to an unknown factor $\Delta\lambda_t$ to solve:

$$\left\{\frac{\sqrt{2}}{3d}\sigma^{e\,eq}+\frac{c}{d}\sigma_H{}^e-\Delta\lambda_t\left(\frac{2\mu^+}{3d^2}+\frac{K^+c^2}{d^2}\right)-f_t\left(\lambda_t^-+\Delta\lambda_t\right)=0\right.$$

Resolution with projection at the top of the cone of traction:
One thus seeks to solve this system, by using the particular shape of the tensors of constraints and deformations, uniforms on the structure.
On the basis of $\varepsilon=(a,a,a,0.,0.,0.)$ and of $\sigma=(\sigma,\sigma,\sigma,0.,0.,0.)$. one obtains:

The elastic tensor of constraint $\begin{cases}\sigma_x = a(3\lambda+2\mu)\\ \sigma_y = a(3\lambda+2\mu)\\ \sigma_z = a(3\lambda+2\mu)\end{cases}$

The elastic diverter of constraint $\begin{cases}s_x = 0\\ s_y = 0\\ s_z = 0\end{cases}$

The elastic hydrostatic constraint $\sigma^e{}_H = (3\lambda+2\mu)(a) = 3aK$

The elastic equivalent constraint $\sigma^e{}_{eq} = 0$

In the case of a curve of linear work hardening post-peak in traction, the expression of the parameter of work hardening is the following one:

$$f_t\left(\theta,\left\|\varepsilon_t^p\right\|\right) = \tau(\theta,\kappa) = f_t'(\theta)\left(1-\frac{\left\|\varepsilon_t^p\right\|}{\kappa_u(\theta)}\right) \text{ with } \kappa_u(\theta) = \frac{2.G_f(\theta)}{l_c . f_t'(\theta)}$$

where θ indicate the maximum of temperature during the history of loading, $f_t'(\theta)$ resistance in traction.

$$\Delta\tilde{\varepsilon}^p{}_t = 0 \qquad \Delta\varepsilon^p{}_{H_t} = \Delta\lambda_t \frac{c}{3d} \qquad \sigma_H = \sigma_H{}^e - 3K^+\left(\Delta\lambda_t \frac{c}{3d}\right)$$

The equation characterizing projection at the top of the cone of traction is the following one:

$$\left(\frac{c}{d}\sigma_H{}^e - \Delta\lambda_t \frac{K^+c^2}{d^2} - f_t'\left(1-\Delta\lambda_t \frac{l_c . f_t'}{2.G_t}\right)\right) = 0 \qquad \text{éq 2.2-1}$$

G_t being the energy of rupture in traction (characteristic of material).

What makes it possible to obtain the plastic multiplier:

$$\Delta\lambda_t = \frac{\frac{c}{d}\sigma_H{}^e - f_t'}{\frac{K^+c^2}{d^2} - \frac{l_c.(f_t')^2}{2.G_t}} = \frac{\frac{c}{d}(3aK) - f_t'}{\frac{K^+c^2}{d^2} - \frac{l_c.(f_t')^2}{2.G_t}}$$

Then the constraint: $\sigma_H = \sigma_H{}^e - 3K^+ \left(\Delta\lambda_t \frac{c}{3d} \right) = K \left(3.a - \Delta\lambda_t \frac{c}{d} \right)$

Knowing has, imposed displacement, one obtains all the unknown factors of the problem.

2.3 Uncertainty on the solution

The solution being analytical, uncertainty is negligible, about the precision of the machine.

2.4 Bibliographical references

The model was defined starting from the following theses and is described in the report of specification:

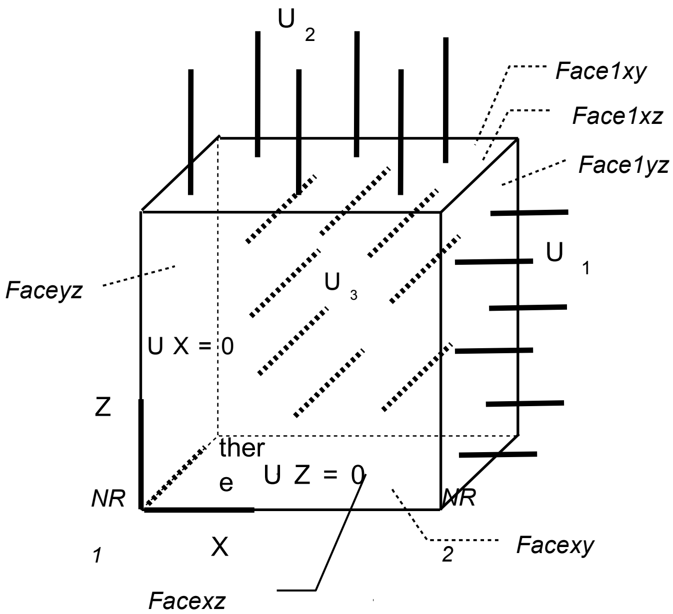
- G. Heinfling, at the time of its thesis “Contribution to the digital modeling of the behavior of the concrete and the reinforced concrete structures under thermomechanical requests at high temperature”,
- J.F. Georgin, at the time of its thesis “Contribution to the modeling of the concrete under fast request of dynamics. The taking into account of the effect speed by viscoplasticity”.
- SCSA/128IQ1/RAP/00.034 Version 1.2, Development of a model of behavior 3D concrete with double criterion of plasticity in *Code_Aster* - Specifications “.

3 Modeling A

3.1 Characteristics of modeling

3D (HEXA8)

1 element, stress field and uniform deformation.



3.2 Characteristics of the grid

Many nodes: 8
Number of meshs and type: 1 HEXA8

3.3 Values tested

The components were tested xx and zz stress field `SIGM_ELNO`, and plastic deformation cumulated in traction (second variable internal, second component of the field `VARI_ELNO`). Displacement being imposed, the field `EPSI_ELNO` is not tested.

The three moments correspond to a displacement of 0.005, 0.01 and $0.015\,mm$.

Field `SIGM_ELNO` component `SIXX`

Identification	Reference
For a displacement imposed in load $U_1 = U_2 = U_3 = 0.005$	1.9182065
For a displacement imposed in load $U_1 = U_2 = U_3 = 0.010$	1,161 6770
For a displacement imposed in load $U_1 = U_2 = U_3 = 0.015$	0.4051470

Field `SIGM_ELNO` component `SIZZ`

Identification	Reference
For a displacement imposed in load $U_1=U_2=U_3=0.005$	1.9182065
For a displacement imposed in load $U_1=U_2=U_3=0.010$	1.1616770
For a displacement imposed in load $U_1=U_2=U_3=0.015$	0.4051470

Field VARI_ELNO component V2 (plastic deformation cumulated in traction)

Identification	Reference
For a displacement imposed in load $U_1=U_2=U_3=0.005$	0.0099232717
For a displacement imposed in load $U_1=U_2=U_3=0.010$	0.0199535329
For a displacement imposed in load $U_1=U_2=U_3=0.015$	0.0299837941

4 Summary of the results

This case test offers very satisfactory results compared to the analytical solution, lower than 7.10^{-5}% with a low iteration count (1 or 2 iterations). The solution is obtained starting from a linear equation in the case of a linear curve of work hardening in traction, but the resolution uses an algorithm of Newton within a framework plus general.

One can note the work hardening of the criterion of traction which takes place during the loading, involving a reduction in the constraint (component xx, yy and zz) in addition equalizes with the hydrostatic constraint.