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## SSNV137 - Cable of prestressed in a beam concrete right-hand side

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### Summary

One considers a right concrete beam, of square section, crossed over his length by a cable of prestressed out of steel. With at-rest state, the cable is parallel to the average of beam and excentré fibre compared to the two principal plans. The beam and the cable are embed-free. The cable is put in traction at its loose lead, in order to prestress the beam in inflection-compression. The losses of tension along the cable are neglected.

The goal of this CAS-test is to validate the method of calculating of the state of balance of a prestressed concrete structure, when this structure is modelled by elements 3D, associated with the basic elements representing the cable of prestressing.

The features particular to test are the following ones:

- operator `DEFI_CABLE_BP` : determination of the relations kinematics between the degrees of freedom of the nodes of a cable and the degrees of freedom of the nodes "close" to a concrete structure modelled by elements 3D;
- operator `STAT_NON_LINE`, option `BEHAVIOR` : calculation of the state of balance.

The got results are validated by comparison with an analytical solution of reference.

## 1 Problem of reference

### 1.1 Geometry

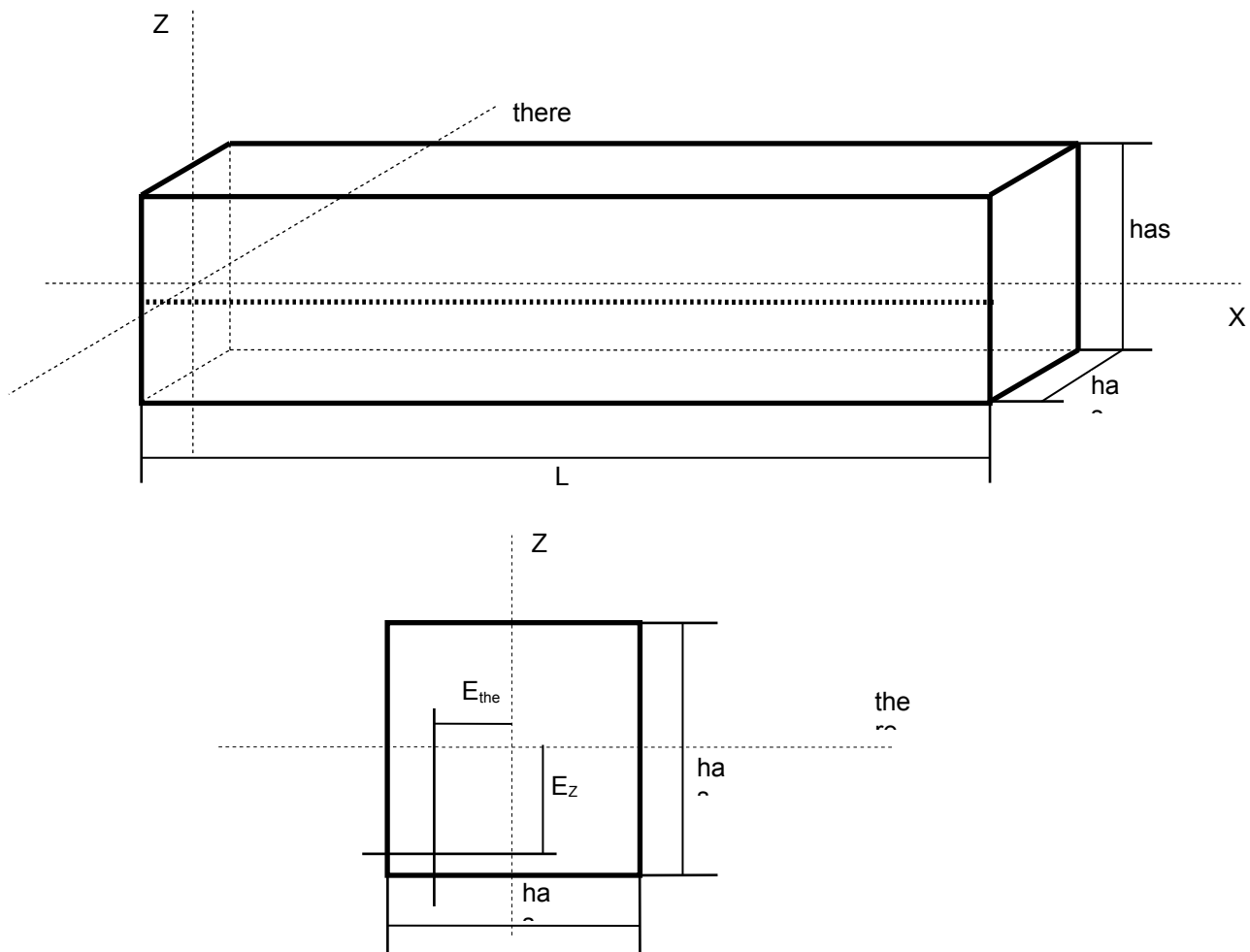
The concrete beam is right, of square section.

Its dimensions are  $L \times a \times a = 3\text{ m} \times 0,4\text{ m} \times 0,4\text{ m}$ .

The cable crosses the beam parallel with average fibre and it is excentré compared to the two principal plans. Eccentricities according to the directions  $y$  and  $z$  are worth respectively

$e_y = -0,12\text{ m}$  and  $e_z = -0,16\text{ m}$ .

The surface of the cross-section of the cable is worth  $S_a = 2,5 \cdot 10^{-3}\text{ m}^2$ .



### 1.2 Properties of materials

Material concrete constituting the beam: Young modulus  $E_b = 4,5 \cdot 10^{10}\text{ Pa}$

Material steel constituting the cable: Young modulus  $E_a = 1,85 \cdot 10^{11}\text{ Pa}$

The Poisson's ratio is taken equal to 0 for two materials. One thus cancels the effects Fish in the directions  $y$  and  $z$ .

The losses of tension in the cable being neglected, the various parameters being used for their estimate are fixed at 0.

## 1.3 Boundary conditions and loadings

Nodes of the beam located on the face  $x=0$  are blocked in translation according to the three directions. Among these nodes are the "neighbors" of the left node end of the cable, which is thus blocked in translation by the relations kinematics. One thus should not impose boundary conditions additional in this node, which would be redundant with the relations kinematics and would make impossible the resolution in displacements (singular matrix).

One applies to the node right end of the cable a normal effort of traction  $(F_0; 0; 0)$ , with  $F_0=10^6 N$ .

## 2 Reference solution

The analytical solution of reference is determined by the theory of the beams. A embed-free beam is considered. The geometrical characteristics are those defined in paragraph [§2.1]. One applies at the loose lead a normal effort of compression  $(-F ; 0 ; 0)$  and a bending moment  $(0 ; e_z \cdot F ; -e_y \cdot F)$ .

The solution of this problem is the following one:

**Tensor of the constraints:**

$$\Rightarrow \begin{matrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \text{ with } \sigma_{xx} = - \frac{F}{a^2} \left[ 1 + \frac{12e_y}{a^2} y + \frac{12e_z}{a^2} z \right] \quad \text{éq 2-1}$$

**Displacements :** by neglecting the effects Fish one obtains

$$\begin{matrix} u(x, y, z) = - \frac{F}{E_b a^2} \left[ 1 + \frac{12e_y}{a^2} y + \frac{12e_z}{a^2} z \right] x \\ v(x, y, z) = \frac{6Fe_y}{E_b a^4} x^2 \\ w(x, y, z) = \frac{6Fe_z}{E_b a^4} x^2 \end{matrix} \quad \text{éq 2-2}$$

with the boundary conditions  $\begin{matrix} u = v = w = 0 \\ \frac{\partial v}{\partial x} = \frac{\partial w}{\partial x} = 0 \end{matrix}$  en  $x = 0$

In the expressions above,  $F$  indicate the residual normal effort in the cable after shortening elastic of the beam, which can be clarified according to the initial tension  $F_0$ .

The axial rate of deformation of the concrete on the level of the cable is written

$$\varepsilon_{xx}^{béton} = \frac{\sigma_{xx}}{E_b} = - \frac{F}{E_b a^2} \left[ 1 + \frac{12e_y^2}{a^2} + \frac{12e_z^2}{a^2} \right]$$

The residual normal effort in the cable results from the initial tension  $F_0$  by the relation

$$\varepsilon_{xx}^{béton} = \varepsilon_{xx}^{acier} \text{ and } \varepsilon_{xx}^{acier} = \frac{F - F_0}{E_a S_a}; \text{ from where:}$$

$$F = F_0 + E_a S_a \varepsilon_{xx} \text{ that is to say } F = \frac{F_0}{1 + \frac{E_a S_a}{E_b a^2} \left[ 1 + \frac{12e_y^2}{a^2} + \frac{12e_z^2}{a^2} \right]}$$

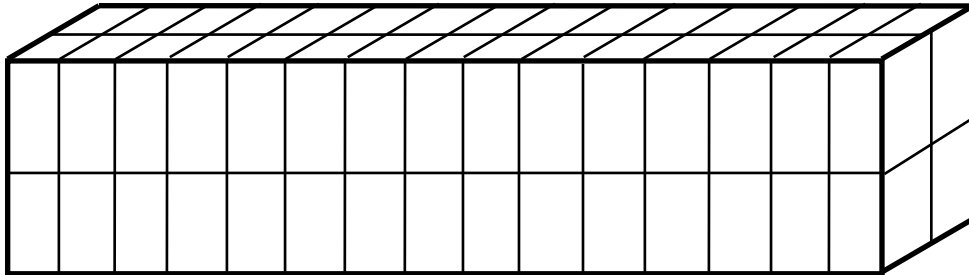
éq 2-3

The digital values of reference are calculated using the formulas [éq 2-1], [éq 2-2] and [éq 2-3].

## 3 Modeling A

### 3.1 Characteristics of modeling

The concrete beam is represented by 60 elements `MECA_HEXA20`, supported per as many meshes hexahedrons to 20 nodes. The figure below gives a simplified representation of the grid of the beam.



A material concrete is affected with the elements, for which the behaviors are defined `ELAS` (Young modulus  $E_b = 4,5 \cdot 10^{10} Pa$ ) and `BPEL_BETON`: the parameters characteristic of this relation are fixed at 0 because one neglects the losses of tension along the cable of prestressing.

Degrees of freedom  $DX$ ,  $DY$ , and  $DZ$  nodes of the face  $x=0$  are blocked.

The cable is represented by 30 elements `MECA_BARRE`, supported per as many meshes segments to 2 nodes. The ends left and right-hand side are respectively the nodes `NC000001` and `NC000031`.

A surface of cross-section  $S_a = 2,5 \cdot 10^{-3} m^2$  is assigned to the elements, as well as a material steel for which the behaviors are defined `ELAS` (Young modulus  $E_a = 1,85 \cdot 10^{11} Pa$ ) and `BPEL_ACIER`: the parameters characteristic of this relation are fixed at 0 (neglected losses of tension), except for the elastic ultimate stress for which a zero value is illicit ( $f_{prg} = 1,77 \cdot 10^9 Pa$ ).

To avoid any redundancy with the relations kinematics, no blocking is forced on the node `NC000001` (cf notices paragraph [§2.3]).

The tension  $F_0 = 10^6 N$  is applied to the node `NC000031`. This value of tension is coherent with the values of section and yield stress, for a cable of prestressed of standard strand.

The calculation of the state of balance of the beam unit and cable is carried out in only one step, the behavior being elastic. One carries out then a complementary calculation allowing to determine the constraints with the nodes of the elements of the beam.

### 3.2 Stages of calculation and features tested

The principal stages of calculation correspond to the features which one wishes to validate:

- operator `DEFI_MATERIAU`: definition of the relations of behavior `BPEL_BETON` and `BPEL_ACIER`, in the typical case where the losses of tension along the cable of prestressing are neglected (values by default of the parameters);
- operator `DEFI_CABLE_BP`: determination of a constant profile of tension along the cable of prestressing, losses being neglected; calculation of the coefficients of the relations kinematics

- between the degrees of freedom of the nodes of the cable and the degrees of freedom of the nodes "close" to the beam out of concrete, in the case of a beam modelled by elements 3D;
- operator `AFFE_CHAR_MECA` : definition of a loading of the type `RELA_CINE_BP` ;
  - operator `STAT_NON_LINE`, option `BEHAVIOR` : calculation of the state of balance by taking account of the loading of the type `RELA_CINE_BP`, in the case of a beam modelled by elements 3D.

## 3.3 Results of modeling A

### 3.3.1 Displacements of the nodes of the beam

One compares the values extracted the field `DEPL` resulting from `STAT_NON_LINE` with the theoretical values of reference. The tolerance of relative variation compared to the reference is worth:

- 3% for the node `NB010527` ;
- 1% for the nodes `NB030127`, `NB050127` and `NB050527` ;
- 0,1% for the other nodes.

Node	Component	Value of reference	Tolerance (%)
NB010105	DX	- 2,298342.10 <sup>-4</sup> m	1,0
NB010305	DX	- 1,237569.10 <sup>-4</sup> m	1,0
NB010505	DX	- 1,767956.10 <sup>-5</sup> m	1,0
NB030105	DX	- 1,502762.10 <sup>-4</sup> m	1,0
NB030305	DX	- 4,419890.10 <sup>-5</sup> m	1,0
NB030305	DY	- 7,955801.10 <sup>-5</sup> m	1,0
NB030305	DZ	- 1,060773.10 <sup>-4</sup> m	1,0
NB030505	DX	+6,187845.10 <sup>-5</sup> m	1,0
NB050105	DX	- 7,071823.10 <sup>-5</sup> m	1,0
NB050305	DX	+3,535912.10 <sup>-5</sup> m	1,0
NB050505	DX	+1,414365.10 <sup>-4</sup> m	1,0
NB010116	DX	- 8,618785.10 <sup>-4</sup> m	1,0
NB010316	DX	- 4,640884.10 <sup>-4</sup> m	1,0
NB010516	DX	- 6,629834.10 <sup>-5</sup> m	1,0
NB030116	DX	- 5,635359.10 <sup>-4</sup> m	1,0
NB030316	DX	- 1,657459.10 <sup>-4</sup> m	1,0
NB030316	DY	- 1,118785.10 <sup>-3</sup> m	1,0
NB030316	DZ	- 1,491713.10 <sup>-3</sup> m	1,0
NB030516	DX	+2,320442.10 <sup>-4</sup> m	1,0
NB050116	DX	- 2,651934.10 <sup>-4</sup> m	1,0
NB050316	DX	+1,325967.10 <sup>-4</sup> m	1,0
NB050516	DX	+5,303867.10 <sup>-4</sup> m	1,0
NB010127	DX	- 1,493923.10 <sup>-3</sup> m	1,0
NB010327	DX	- 8,044199.10 <sup>-4</sup> m	1,0
NB010527	DX	- 1,149171.10 <sup>-4</sup> m	3,0
NB030127	DX	- 9,767956.10 <sup>-4</sup> m	1,0
NB030327	DX	- 2,872928.10 <sup>-4</sup> m	1,0
NB030327	DY	- 3,361326.10 <sup>-3</sup> m	1,0
NB030327	DZ	- 4,481768.10 <sup>-3</sup> m	1,0
NB030527	DX	+4,022099.10 <sup>-4</sup> m	1,0

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NB050127	DX	- 4,596685.10 <sup>-4</sup> m	1,0
NB050327	DX	+2,298343.10 <sup>-4</sup> m	1,0
NB050527	DX	+9,193370.10 <sup>-4</sup> m	1,0

## 3.3.2 Displacements of the nodes of the cable of prestressing

One compares the values extracted the field `DEPL` resulting from `STAT_NON_LINE` with the theoretical values of reference. The tolerance of relative variation compared to the reference is worth:

- 1% for the node `NC000031`, component `DZ` ;
- 0,1% for the other nodes.

Node	Component	Value of reference	Tolerance (%)
NC000006	DY	$-1,243094 \cdot 10^{-4}$ m	1,0
NC000006	DZ	$-1,657459 \cdot 10^{-4}$ m	1,0
NC000011	DY	$-4,972376 \cdot 10^{-4}$ m	1,0
NC000011	DZ	$-6,629834 \cdot 10^{-4}$ m	1,0
NC000016	DY	$-1,118785 \cdot 10^{-3}$ m	1,0
NC000016	DZ	$-1,491713 \cdot 10^{-3}$ m	1,0
NC000021	DY	$-1,988950 \cdot 10^{-3}$ m	1,0
NC000021	DZ	$-2,651934 \cdot 10^{-3}$ m	1,0
NC000026	DY	$-3,107735 \cdot 10^{-3}$ m	1,0
NC000026	DZ	$-4,143646 \cdot 10^{-3}$ m	1,0
NC000031	DY	$-4,475138 \cdot 10^{-3}$ m	1,0
NC000031	DZ	$-5,966851 \cdot 10^{-3}$ m	10,0

## 3.3.3 Normal effort in the cable of prestressing

One compares the value extracted the field `SIEF_ELNO` resulting from `STAT_NON_LINE` with the theoretical value of reference.

The component to which the test relates is `N`.

Node	Mesh	Value of reference	Tolerance (%)
NC000016	SG000015	$+7,955801 \cdot 10^5$ N	1,0



## 4 Modeling B

### 4.1 Characteristics of modeling

The characteristics of modeling B are the same ones as modeling A, except the cable is modelled by 30 elements `CABLE_GAINE` with the adherent case of the law `CABLE_GAINE_FROT` ( `TYPE_ ' ADHERENT'` ).

The meshes of the cable are `SEG3` .

### 4.2 Stages of calculation and features tested

The stages are the same ones as for modeling A. All the features are tested with elements `CABLE_GAINE`.

### 4.3 Results of modeling B

#### 4.3.1 Displacements of the nodes of the beam

One compares the values extracted the field `DEPL` resulting from `STAT_NON_LINE` with the theoretical values of reference. The tolerance of relative variation compared to the reference is worth:

- 3% for the node `NB010527` ;
- 1% for the nodes `NB030127` , `NB050127` and `NB050527` ;
- 0,1% for the other nodes.

Node	Component	Value of reference	Tolerance (%)
NB010105	DX	- 2,298342.10 <sup>-4</sup> m	1,0
NB010305	DX	- 1,237569.10 <sup>-4</sup> m	1,0
NB010505	DX	- 1,767956.10 <sup>-5</sup> m	1,0
NB030105	DX	- 1,502762.10 <sup>-4</sup> m	1,0
NB030305	DX	- 4,419890.10 <sup>-5</sup> m	1,0
NB030305	DY	- 7,955801.10 <sup>-5</sup> m	1,0
NB030305	DZ	- 1,060773.10 <sup>-4</sup> m	1,0
NB030505	DX	+6,187845.10 <sup>-5</sup> m	1,0
NB050105	DX	- 7,071823.10 <sup>-5</sup> m	1,0
NB050305	DX	+3,535912.10 <sup>-5</sup> m	1,0
NB050505	DX	+1,414365.10 <sup>-4</sup> m	1,0
NB010116	DX	- 8,618785.10 <sup>-4</sup> m	1,0
NB010316	DX	- 4,640884.10 <sup>-4</sup> m	1,0
NB010516	DX	- 6,629834.10 <sup>-5</sup> m	1,0
NB030116	DX	- 5,635359.10 <sup>-4</sup> m	1,0
NB030316	DX	- 1,657459.10 <sup>-4</sup> m	1,0
NB030316	DY	- 1,118785.10 <sup>-3</sup> m	1,0
NB030316	DZ	- 1,491713.10 <sup>-3</sup> m	1,0
NB030516	DX	+2,320442.10 <sup>-4</sup> m	1,0
NB050116	DX	- 2,651934.10 <sup>-4</sup> m	1,0
NB050316	DX	+1,325967.10 <sup>-4</sup> m	1,0
NB050516	DX	+5,303867.10 <sup>-4</sup> m	1,0

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NB010127	DX	- 1,493923.10 <sup>-3</sup> m	1,0
NB010327	DX	- 8,044199.10 <sup>-4</sup> m	1,0
NB010527	DX	- 1,149171.10 <sup>-4</sup> m	3,0
NB030127	DX	- 9,767956.10 <sup>-4</sup> m	1,0
NB030327	DX	- 2,872928.10 <sup>-4</sup> m	1,0
NB030327	DY	- 3,361326.10 <sup>-3</sup> m	1,0
NB030327	DZ	- 4,481768.10 <sup>-3</sup> m	1,0
NB030527	DX	+4,022099.10 <sup>-4</sup> m	1,0
NB050127	DX	- 4,596685.10 <sup>-4</sup> m	1,0
NB050327	DX	+2,298343.10 <sup>-4</sup> m	1,0
NB050527	DX	+9,193370.10 <sup>-4</sup> m	1,0

## 4.3.2 Displacements of the nodes of the cable of prestressing

One compares the values extracted the field `DEPL` resulting from `STAT_NON_LINE` with the theoretical values of reference. The tolerance of relative variation compared to the reference is worth:

- 1% for the node `NC000031` , component `DZ` ;
- 0,1% for the other nodes.

Node	Component	Value of reference	Tolerance (%)
NC000006	DY	- 1,243094.10 <sup>-4</sup> m	1,0
NC000006	DZ	- 1,657459.10 <sup>-4</sup> m	1,0
NC000011	DY	- 4,972376.10 <sup>-4</sup> m	1,0
NC000011	DZ	- 6,629834.10 <sup>-4</sup> m	1,0
NC000016	DY	- 1,118785.10 <sup>-3</sup> m	1,0
NC000016	DZ	- 1,491713.10 <sup>-3</sup> m	1,0
NC000021	DY	- 1,988950.10 <sup>-3</sup> m	1,0
NC000021	DZ	- 2,651934.10 <sup>-3</sup> m	1,0
NC000026	DY	- 3,107735.10 <sup>-3</sup> m	1,0
NC000026	DZ	- 4,143646.10 <sup>-3</sup> m	1,0
NC000031	DY	- 4,475138.10 <sup>-3</sup> m	1,0
NC000031	DZ	- 5,966851.10 <sup>-3</sup> m	10,0

## 4.3.3 Normal effort in the cable of prestressing

One compares the value extracted the field `SIEF_ELNO` resulting from `STAT_NON_LINE` with the theoretical value of reference.

The component to which the test relates is `N`.

Node	Mesh	Value of reference	Tolerance (%)
NC000016	SG000015	+7,955801.10 <sup>5</sup> N	1,0

## 5 Summary of the results

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The computed values correspond indeed to those theoretically expected. One obtains well a state of inflection-compression for the concrete beam.

The more important variations observed in certain nodes closer to the loose lead can be explained by the more or less good adequacy of a modeling 3D for a structure of type beam. Thus the grid remains enough coarse not to increase the cost of calculation. It is pointed out finally that the reference solution is established under the assumptions of the theory of the beams.