SSNV122 - Rotation and following traction very-rubber band of a bar

Summary:

This test of quasi-static mechanics consists in making turn of 90° a parallelepipedic bar and subjecting it to an important traction by means of following forces. One thus validates the kinematics of the great deformations very-rubber bands (order `STAT_NON_LINE`, keyword `BEHAVIOR`), and thus in particular great rotations, for a relation of elastic behavior linear, as well as the taking into account of following forces (order `STAT_NON_LINE` keyword `TYPE_CHARGE=' SUIV'`).

The bar is modelled by a voluminal element (`HEXA8`, modeling A).
Modeling B validates as for it the following loads of functions type depending on the initial geometry.

Results got by *Code_Aster* do not differ from the theoretical solution.

1 Problem of reference

1.1 Geometry

1.2 Material properties

Behavior very-rubber band of COMING SAINT - KIRCHHOFF:

$$\mathbf{S} = \frac{\nu E}{(1+\nu)(1-2\nu)} \operatorname{tr}(\mathbf{E})\, \mathbf{1} + \frac{E}{1+\nu} \mathbf{E}$$

$E = 200\,000.$ MPa

$\nu = 0.3$

1.3 Boundary conditions and loadings

The loading is applied in two times: first of all, an overall rotation of the structure, followed by a traction exerted by following forces.

For modeling B, the value of p depends on the coordinate Y .

2 Reference solution

2.1 Method of calculating used for the reference solution of modeling A

It is about a problem plan. One can seek the solution in the form of a rigid rotation followed by a dilation of a factor a in a direction and b in the other:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \xrightarrow{\text{rotation}} \begin{pmatrix} -Y \\ X \\ Z \end{pmatrix} \xrightarrow{\text{traction}} \begin{pmatrix} b(-Y) \\ a\ X \\ Z \end{pmatrix} \quad \text{soit} \quad u = \begin{pmatrix} -X & -bY \\ AX & -Y \\ 0 & \end{pmatrix}$$

The gradient of the transformation and the deformation of Green-Lagrange are then:

$$\mathbf{F} = \begin{pmatrix} 0 & -b & 0 \\ a & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{E} = \begin{pmatrix} e_x & 0 & 0 \\ 0 & e_y & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{où} \quad e_x = \frac{a^2-1}{2} \quad e_y = \frac{b^2-1}{2}$$

The relation of behavior leads to a tensor of Lagrangian constraints diagonal (with λ and μ coefficients of Lamé):

$$S_{xx} = (\lambda + 2\mu)\ e_x + \lambda\, e_y$$

$$S_{yy} = \lambda\, e_x + (\lambda + 2\mu)\ e_y \qquad \text{où} \quad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \quad \mu = \frac{E}{2(1+\nu)}$$

$$S_{zz} = \lambda\, e_x + \lambda\, e_y$$

One from of deduced the tensor from the constraints of Cauchy, so diagonal:

$$\sigma_x = \frac{b}{a}\ S_y \qquad \sigma_y = \frac{a}{b}\ S_x \qquad \sigma_z = \frac{1}{a\,b}\ S_z$$

Finally the boundary conditions are written:

$$\sigma_x = 0 \quad (\text{bord libre}) \qquad \sigma_y = -p \quad (\text{traction})$$

One can moreover calculate the efforts exerted on the faces:

$[1, 3]$ $\quad \mathsf{F}_y = -\sigma_y\, b\, S_{o[1,3]}$

$[3, 4]$ $\quad \mathsf{F}_x = 0$

$[1, 2, 3, 4]$ $\quad \mathsf{F}_z = \begin{cases} -\sigma_z\, a\, b\, S_{o[1,2,3,4]} & \text{sur le côté inférieur de la face} \\ \sigma_z\, a\, b\, S_{o[1,2,3,4]} & \text{sur le côté supérieur de la face} \end{cases}$

where $S_o[\ ]$ initial surfaces of the faces represent.

2.2 Reference of modeling B

The results of references for this modeling are provided by the same calculation to which the stage of rotation was not applied.

2.3 Results of reference

One adopts like results of reference displacements, the strains of Grenn - Lagrange, the stresses of Cauchy and the forces exerted on the faces $[1,3]$, $[3,4]$ and $[1,2,3,4]$ at the end of the loading ($t=2s$).

One seeks p such as dilation a =1,1

that is to say $p=-26610.3\,MPa$.

Dilation b and displacements are then:

$$b=0.9539 \quad e_x=0.105 \quad e_y=-0.045$$

The constraints of Cauchy are worth:

$$\sigma_x=0 \quad \sigma_y=26610.3 \text{ MPa} \quad \sigma_z=6597.6 \text{ MPa}$$

Lastly, the exerted forces are:

$$\begin{aligned} F_x &= 0 \\ F_y &= -25384\ S_{o[1,3]}\ N \\ F_z &= -6.9228\ 10^9\ N \quad (\text{côté inférieur}) \end{aligned}$$

2.4 Uncertainty on the solution

Analytical solution.

2.5 Bibliographical references

[1] Eric LORENTZ “a nonlinear relation of behavior hyperelastic” Notes intern EDF/DER HI - 74/95/011/0

3 Modeling A

3.1 Characteristics of modeling

Voluminal modeling: 1 mesh HEXA 8
1 mesh QUAD4

- rigid phase of rotation $0 \le t \le 1\,s$

$[3,7]$	$DX=0$	$DY=0$	$DZ=0$
$[1,5]$	$DX=-1000\,\sin\left(\frac{\pi t}{2}\right)$	$DY=-1000\left(1-\cos\frac{\pi t}{2}\right)$	$DZ=0$
$[2,6]$			$DZ=0$
$[4,8]$			$DZ=0$

- phase of traction: $1\mathrm{s} \le t \le 2\mathrm{s}$
  - boundary conditions (TYPE_CHARGE: 'DIDI')

$[3,7]$	$\Delta DX=0$	$\Delta DY=0$	$DZ=0$
$[1,5]$		$\Delta DY=0$	$DZ=0$
$[2,6]$			$DZ=0$
$[4,8]$			$DZ=0$

  - loading: pressure (negative) on the face $[2,4,8,6]$ (PRES_REP) : mesh $[2,4,8,6]$ (QUAD4) : $PRES=-26610.3\,(t-1)$.

3.2 Characteristics of the grid

Many nodes: 8 Many meshs: 2
1 HEXA8
1 QUAD4

3.3 Sizes tested and results

The values are tested at the end of the loading ($t=2\mathrm{s}$)

Identification	Reference	*Aster*	% difference
Displacement DX (NO2)	– 1953.94	– 1953.92	0
Displacement DY (NO2)	100.	100.	0
Constraints SIXX (PG1)	0	$8.\ 10^{-10}$	
Constraints SIYY (PG1)	26610.3	26610.3	0
Constraints SIZZ (PG1)	6597.6	6597.6	0
Constraints SIXY (PG1)	0	□ 10^{-26}	
Constraints SIXZ (PG1)	0	□ 10^{-11}	
Constraints SIYZ (PG1)	0	□ 10^{-10}	
Deformation EPXX (PG1)	0,105	0,105	0
Deformation EPYY (PG1)	– 0,045	– 0,045	0
Deformation EPZZ (PG1)	0	□ 10^{-16}	
Deformation EPXY (PG1)	0	□ 10^{-14}	

Deformation EPXZ (PG1)	0	□ 10^{-14}	
Deformation EPYZ (PG1)	0	□ 10^{-16}	
Nodal reaction DX (NO3)	0	□ 10^{-3}	
Nodal reaction DY (NO3)	$-6.3462\ 10^{9}$	$-6.3461\ 10^{9}$	– 0,001
Nodal reaction DZ (NO3)	$-1.7307\ 10^{9}$	$-1.7307\ 10^{9}$	0,004

4 Modeling B

4.1 Characteristics of modeling

The characteristics of modeling are the same ones as for modeling A. One carries out two calculations. The first is used as reference, only the phase of traction is made. In the second one applies the followed rotation by the phase of traction.

Following rotation, the values of Y on the face on which one applies the pressure changed. However that should not modify the values of the pressure on the face, because the function of pressure depends on the initial geometry and not on the reactualized geometry. With the change of reference mark near, one must thus find the same values of constraints and reactions of supports that those of the case of reference without rotation.

For the phase of traction, pressure east defines as follows:

$$p = -26610.3(t-1)$$

Coordinate Y	Pressure
0	0
1000	p

4.2 Characteristics of the grid

Idem modeling A

4.3 Sizes tested and results

The values are tested at the end of the loading ($t=2\text{s}$ for complete calculation)

Identification	Reference	% tolerance
Constraints SIYY (MA1, PG 1)	6499.1355823353	1,00E-004
Constraints SIXX (MA1, PG 1)	-2742.7772229229	1,00E-004
Constraints SIZZ (MA1, PG 1)	953.28609907388	1,00E-004
Constraints SIXY (MA1, PG 1)	999.17919974535	1,00E-004
Nodal reaction DX (NO3)	4.7691648248739E+08	1,00E-004
Nodal reaction DZ (NO3)	-1.3348410985705E+09	1,00E-004

5 Summary of the results

It appears at the conclusion of this test (modeling A) which the digital solution coincides remarkably with the analytical solution. One will notice however that the strong not linearity due to great rotations requires a relatively fine discretization in time, without being penalizing on the precision since, contrary to an incremental relation of behavior, the errors do not cumulate a step of time on the other.

Modeling B validates the dependence of the pressure to the initial geometry.