SSNV112 - Hollow roll into incompressible (great deformations)

Summary:

This test makes it possible to validate the quasi-incompressible elements in great deformations, in statics for a three-dimensional, axisymmetric or two-dimensional problem (plane deformations). One considers a hollow roll subjected to an internal radial displacement. The material has a Poisson's ratio equal to 0.4999 and one uses the quasi-incompressible elements INCO_UPG with the deformations of SIMO_MIEHE and GDEF_LOG and elements INCO_UP with the deformations GDEF_LOG.

Fifteen modelings are carried out for this problem:

Modeling a: modeling 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') with HEXA20,

Modeling b: modeling 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') with TETRA10,

Modeling C: modeling D_PLAN_INCO_UPG (DEFORMATION=' SIMO_MIEHE') with QUAD8 and TRIA6,

Modeling D: modeling AXIS_INCO_UPG (DEFORMATION=' SIMO_MIEHE') with QUAD8 and TRIA6,

Modeling E: modeling 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') with PENTA15,

Modeling F: modeling 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') with HEXA20,

Modeling G: modeling 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') with TETRA10,

Modeling H: modeling D_PLAN_INCO_UPG (DEFORMATION=' GDEF_LOG') with QUAD8 and TRIA6,

Modeling I: modeling AXIS_INCO_UPG (DEFORMATION=' GDEF_LOG') with QUAD8 and TRIA6,

Modeling J: modeling 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') with PENTA15,

Modeling K: modeling 3D_INCO_UP with HEXA20,

Modeling L: modeling 3D_INCO_UP with TETRA10,

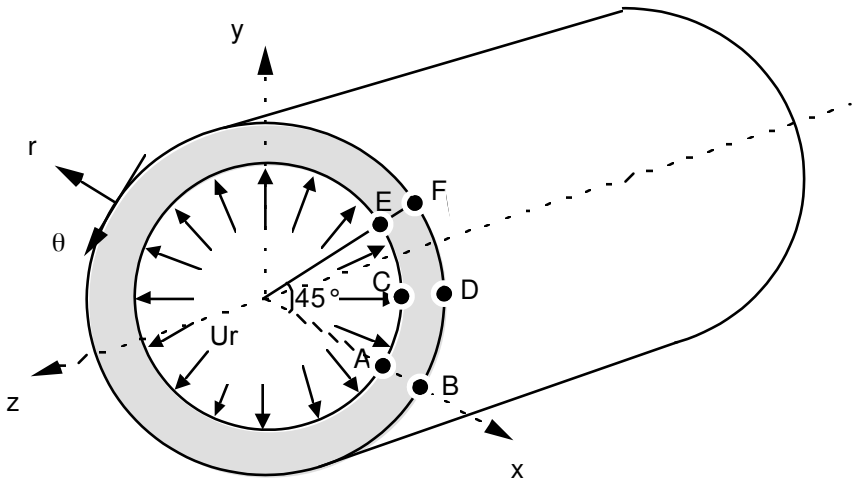
Modeling M: modeling D_PLAN_INCO_UP with QUAD8 and TRIA6,

Modeling NR: modeling AXIS_INCO_UP with QUAD8 and TRIA6,

Modeling O: modeling `3D_INCO_UP` with `PENTA15`,

1 Problem of reference

1.1 Geometry



Rayon interne a = 0.1 m
Rayon externe b = 0.2 m

Coordinates of the points:

	A	B	E	F	C	D
x	0.1	0.2	$0.1\times\cos(45)$	$0.2\times\cos(45)$	$0.1\times\cos(22.5)$	$0.2\times\cos(22.5)$
y	0	0	$0.1\times\sin(45)$	$0.1\times\sin(45)$	$0.1\times\sin(22.5)$	$0.1\times\sin(22.5)$
z	0	0	0	0	0	0

1.2 Properties of material

$E=2.10^5\ MPa$
$\nu=0.4999$

1.3 Boundary conditions and loadings

Radial displacement $U_0=6.10^{-5}m$ (expansion)

2 Reference solution

2.1 Method of calculating

For the studied problem, displacement $\boldsymbol{u}$ is radial and thus of the form $\boldsymbol{u}=[u,0,0]$.

One from of deduced the general form from the tensor of the deformations in great deformations:

$$\boldsymbol{b}=\boldsymbol{F}\,\boldsymbol{F}^{T}=\begin{bmatrix}(1+u')^2 & 0 & 0\\ 0 & \left(1+\frac{u}{r}\right)^2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

as well as the form of the tensor of the constraints, which is written simply if one takes into account the fact that $J=\det \boldsymbol{F}=1$ for an incompressible problem:

$\boldsymbol{\sigma}=-p\,\boldsymbol{I}_d+\mu\,\boldsymbol{b}^d$, that is to say:

$$\begin{cases}\sigma_{rr}=-p+\mu\left(\frac{2}{3}(1+u')^2-\frac{1}{3}\left(1+\frac{u}{r}\right)^2-\frac{1}{3}\right)\\ \sigma_{\theta}\theta=-p+\mu\left(-\frac{1}{3}(1+u')^2+\frac{2}{3}\left(1+\frac{u}{r}\right)^2-\frac{1}{3}\right)\\ \sigma_{zz}=-p+\mu\left(-\frac{1}{3}(1+u')^2-\frac{1}{3}\left(1+\frac{u}{r}\right)^2+\frac{2}{3}\right)\\ \sigma_{r\theta}=\sigma_{rz}=\sigma_{\theta z}=0\end{cases}$$

The writing of the equilibrium equations leads to the checking of only one equation:

$$\sigma'_{rr}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0$$

who allows to determine the pressure p knowing the field of radial displacement u :

$$p'=\mu\left(\frac{4}{3}(1+u')u''-\frac{2}{3}\left(1+\frac{u}{r}\right)\left(\frac{u'}{r}-\frac{u}{r^2}\right)+\frac{(1+u')^2}{r}-\frac{\left(1+\frac{u}{r}\right)^2}{r}\right)$$

2.2 Particularization of the solution

The condition of incompressibility is written $\det F=1$ with $\boldsymbol{F}=\begin{bmatrix}1+u' & 0 & 0\\ 0 & 1+\frac{u}{r} & 0\\ 0 & 0 & 1\end{bmatrix}$. Displacement u thus check the following differential equation:

$$ru'+u+u'u=0 \qquad \text{éq 2.2-1}$$

The imposed loading is the following $u=U_0$ in $r=a$.

The solution in displacement is thus:

$$\begin{cases} u_r = -r + r\sqrt{r^2 + U_0(U_0+2a)} \\ u_\theta = u_z = 0 \end{cases}$$

The tensor of the deformations thus has as an expression:

$$\begin{cases} b_{rr} = \dfrac{r^2}{r^2 + U_0(U_0+2a)} \\ b_{\theta\theta} = \dfrac{r^2 + U_0(U_0+2a)}{r^2} \\ b_{zz} = 1 \\ b_{r\theta} = b_{z\theta} = b_{\theta z} = 0 \end{cases}$$

And the constraints are worth:

$$\begin{cases} \sigma_{rr} = -p + \mu\left(\dfrac{2}{3}\dfrac{r^2}{r^2 + U_0(U_0+2a)} - \dfrac{1}{3}\dfrac{r^2 + U_0(U_0+2a)}{r^2} - \dfrac{1}{3}\right) \\ \sigma_\theta\theta = -p + \mu\left(-\dfrac{1}{3}\dfrac{r^2}{r^2 + U_0(U_0+2a)} + \dfrac{2}{3}\dfrac{r^2 + U_0(U_0+2a)}{r^2} - \dfrac{1}{3}\right) \\ \sigma_{zz} = -p + \mu\left(-\dfrac{1}{3}\dfrac{r^2}{r^2 + U_0(U_0+2a)} - \dfrac{1}{3}\dfrac{r^2 + U_0(U_0+2a)}{r^2} + \dfrac{2}{3}\right) \\ \sigma_{r\theta} = \sigma_{z\theta} = \sigma_{\theta z} = 0 \end{cases}$$

with p obtained by integration of [éq 2.2-1] which is worth:

$$p = \mu\left(\frac{U_0(U_0+2a)}{6r^2} - \frac{2U_0(U_0+2a)}{3\left(U_0(U_0+2a)+r^2\right)} - \log(r) + \frac{1}{2}\log\left(U_0(U_0+2a)+r^2\right)\right) + C$$

where C is a constant

One obtains finally the following digital values:

in $r=0.1$:	in $r=0.2$:
$u_r = 6.10^{-5}$	$u_r = 3.006710^{-5}$
$\sigma_{rr} = -59.9955$	$\sigma_{rr} = 0.$
$\sigma_{\theta\theta} = 99.9566$	$\sigma_{\theta\theta} = 40.006$
$\sigma_{zz} = 19.9326$	$\sigma_{zz} = 20.$

The passage in the Cartesian system is done using the following relations:

$$\sigma_{xx}=\sigma_{rr}\cos^2\theta+\sigma_{\theta\theta}\sin^2\theta-2\sigma_{r\theta}\sin\theta\cos\theta$$
$$\sigma_{\theta\theta}=\sigma_{rr}\sin^2\theta+\sigma_{\theta\theta}\cos^2\theta+2\sigma_{r\theta}\sin\theta\cos\theta$$
$$\sigma_{zz}=\sigma_{rr}\sin\theta\cos\theta-\sigma_{\theta\theta}\sin\theta\cos\theta-2\sigma_{r\theta}(\cos^2\theta-\sin^2\theta)$$

2.3 Sizes and results of reference

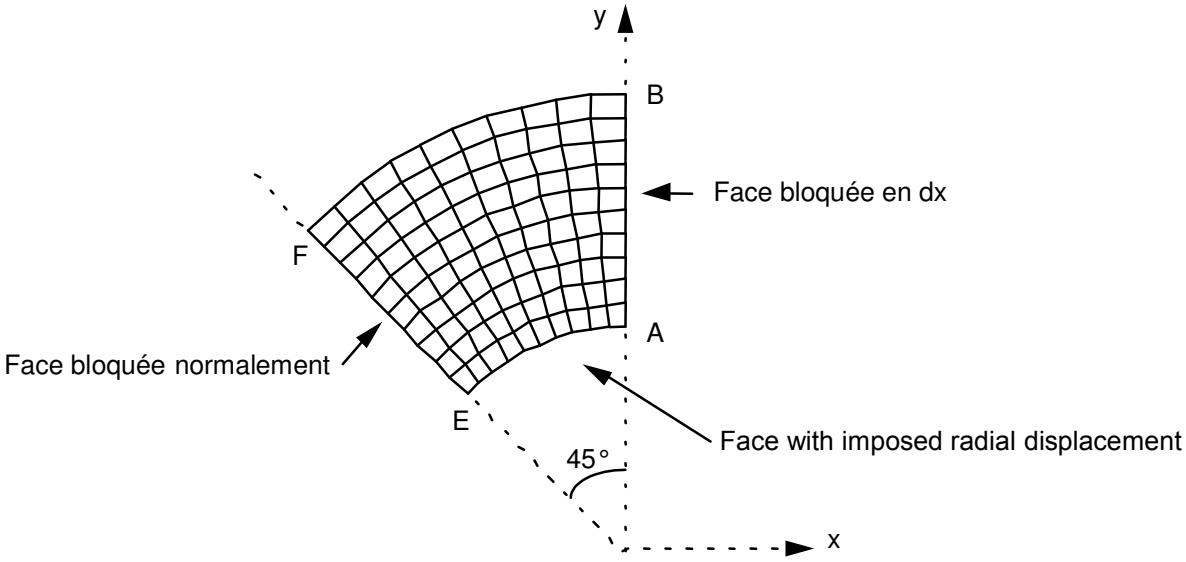
One compared to values of reference:

- displacements (u,v) at the points A and F,
- constraints $(\sigma_{xx},\sigma_{yy},\sigma_{zz},\sigma_{xy})$ at the points A and F,
- constraints of Von Mises and Tresca as well as the eigenvalues of the tensor of the constraints at the point A.

3 Modeling A

3.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') incompressible of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of elements

Limiting conditions:

```
DDL_IMPO   =  GROUP_NO = ' FACSUP'    DZ    = 0.
              GROUP_NO = ' FACINF'    DZ    = 0.
FACE_IMPO =   GROUP_MA = ' FACEEF'    DNOR = 0.
              GROUP_MA = ' FACEAE'    DNOR = -6.10 -5
               GROUP_NO = ' FACEAB'   DX    = 0.
```

faces $AEFD$ ($z=0$ and $z=0.01$)

face AB

face EF

face AE

3.2 Characteristics of the grid

Many nodes: 1501 nodes
Many meshs: 240 HEXA20

3.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-5}
	v	ANALYTICAL	6. 10-5	1. 10^{-3}
	σ_{xx}	ANALYTICAL	99.9566	0.01
	σ_{yy}	ANALYTICAL	-59.9955	0.02
	σ_{zz}	ANALYTICAL	19.9326	0,035
	σ_{xy}	ANALYTICAL	0.	0,012
	VMIS	ANALYTICAL	138.5226	0.02
	TRESCA	ANALYTICAL	159.9521	0.02
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0,035
	PRIN_3	ANALYTICAL	99.9566	0.01
	VMIS_SG	ANALYTICAL	138.5226	0.02

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	1. 10^{-3}
	v	ANALYTICAL	2.1217 10^{-5}	1. 10^{-3}
	σ_{xx}	ANALYTICAL	20,003	0,005
	σ_{yy}	ANALYTICAL	20,003	0,005
	σ_{zz}	ANALYTICAL	20,003	0,005
	σ_{xy}	ANALYTICAL	20,003	0,005

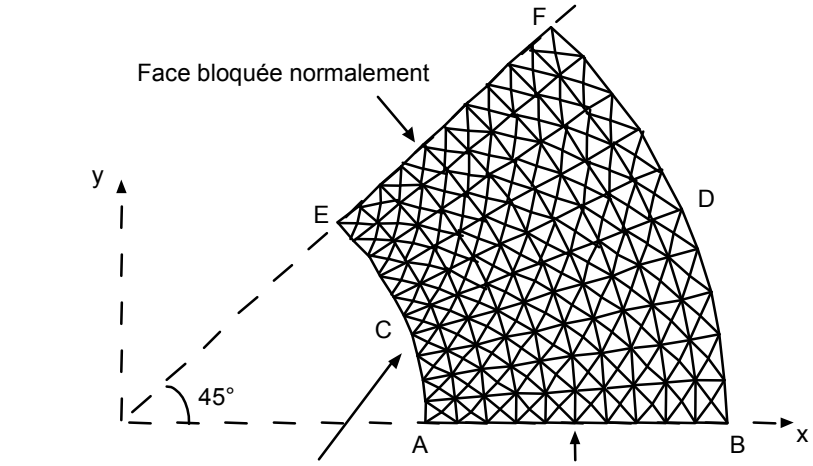
3.4 Remarks

One obtains very good performances since for all the examined sizes, the difference between the solution obtained with the code and the analytical solution is lower than 0.1% for displacements and lower than $3.5\ \%$ for the constraints.

4 Modeling B

4.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') incompressible of type TETRA10 only



AB is on the axis OX (contrary to modeling A).
The grid was obtained with GMSH for a density of $0,01$.

Limiting conditions:

```
DDL_IMPO  =  GROUP_NO = ' FACSUP'   DZ   = 0.
             GROUP_NO = ' FACINF'   DZ   = 0.          faces AEFD ( z=0 and z=0.01 )
              GROUP_NO = ' FACEAB'  DY   = 0.          face AB
FACE_IMPO =  GROUP_MA = ' FACEEF'   DNOR = 0.          face EF
             GROUP_MA = ' FACEAE'   DNOR = -6.10 -5    face AE
```

4.2 Characteristics of the grid

Many nodes: 2064
Many meshs: 1121 TETRA10

4.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	6. 10^{-5}	2. 10^{-3}
	v	ANALYTICAL	0.	1. 10^{-5}
	σ_{xx}	ANALYTICAL	-59.9955	0,025
	σ_{yy}	ANALYTICAL	99.9566	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.03
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0.01
	TRESCA	ANALYTICAL	159.9521	0.01
	PRIN_1	ANALYTICAL	-59.9955	0,025
	PRIN_2	ANALYTICAL	19.9326	0.03
	PRIN_3	ANALYTICAL	99.9566	0,015
	VMIS_SG	ANALYTICAL	138.5226	0.01

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	2.1217 10^{-5}	1. 10^{-3}
	v	ANALYTICAL	2.1217 10^{-5}	1. 10^{-3}
	σ_{xx}	ANALYTICAL	20,003	0,005
	σ_{yy}	ANALYTICAL	20,003	0,005
	σ_{zz}	ANALYTICAL	20,003	0,005
	σ_{xy}	ANALYTICAL	-20,003	0.01

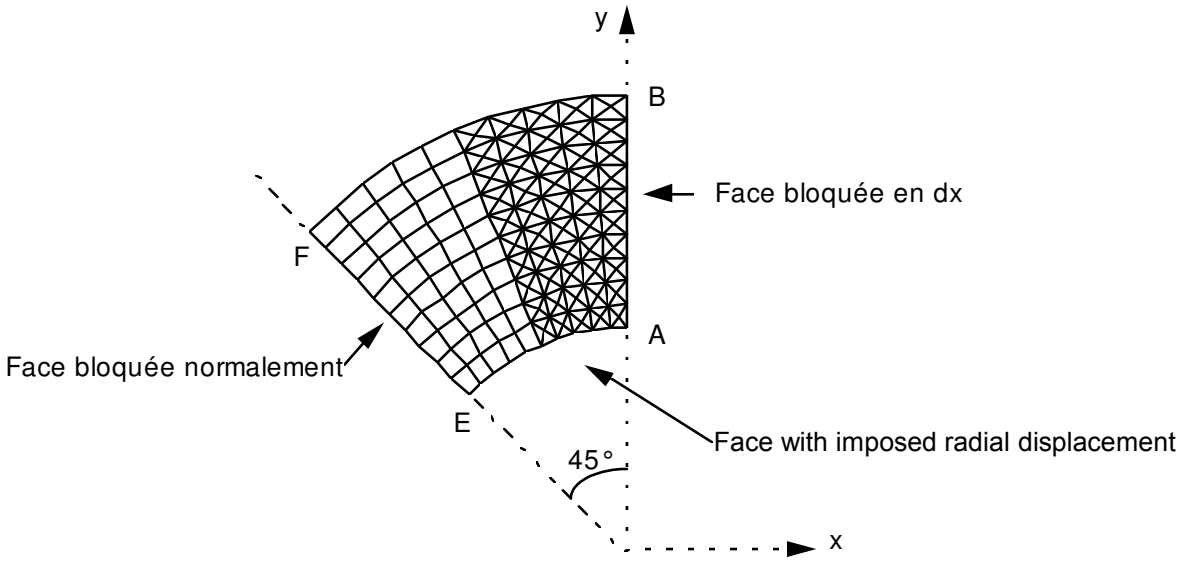
4.4 Remarks

The got results are completely correct since the constraints are obtained with a precision lower than $3\,\%$ even $1\,\%$ at the point F. the variation is a little more important here than for HEXA20, but can be explained by the fact that the loading is imposed here in a way a little less precise since displacement U at point A, is defined only with one precision of 0.158% against 0.077% (evening the factor 2, qu ' one finds on the constraints).

5 Modeling C

5.1 Characteristics of modeling

Grid with elements D_PLAN_INCO_UPG (DEFORMATION=' SIMO_MIEHE') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' GRNM11'	DX = 0.	side AB
FACE_IMPO =	GROUP_MA = ' GRMA12'	DNOR = 0.	dimensioned EF
=	GROUP_MA = ' GRMA13'	DNOR = -6. 10^{-5}	face AE

Name of the nodes:

$A=N2$, $B=N361$, $C=N121$, $D=N584$, $E=N155$, $F=N503$

5.2 Characteristics of the grid

Many nodes: 591
Many meshs: 200 TRIA6, 50 QUAD8.

5.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-5}
	v	ANALYTICAL	6. 10-5	5. 10^{-3}
	σ_{xx}	ANALYTICAL	99.9566	5. 10^{-3}
	σ_{yy}	ANALYTICAL	-59.9955	0.03
	σ_{zz}	ANALYTICAL	19.9326	0.03
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0.02
	TRESCA	ANALYTICAL	159.9521	0.02
	PRIN_1	ANALYTICAL	-59.9955	0.03
	PRIN_2	ANALYTICAL	19.9326	0.03
	PRIN_3	ANALYTICAL	99.9566	0.02
	VMIS_SG	ANALYTICAL	138.5226	0.02

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	5. 10^{-3}
	v	ANALYTICAL	2.1217 10^{-5}	5. 10^{-3}
	σ_{xx}	ANALYTICAL	20,003	5. 10^{-3}
	σ_{yy}	ANALYTICAL	20,003	5. 10^{-3}
	σ_{zz}	ANALYTICAL	20,003	5. 10^{-3}
	σ_{xy}	ANALYTICAL	20,003	5. 10^{-3}

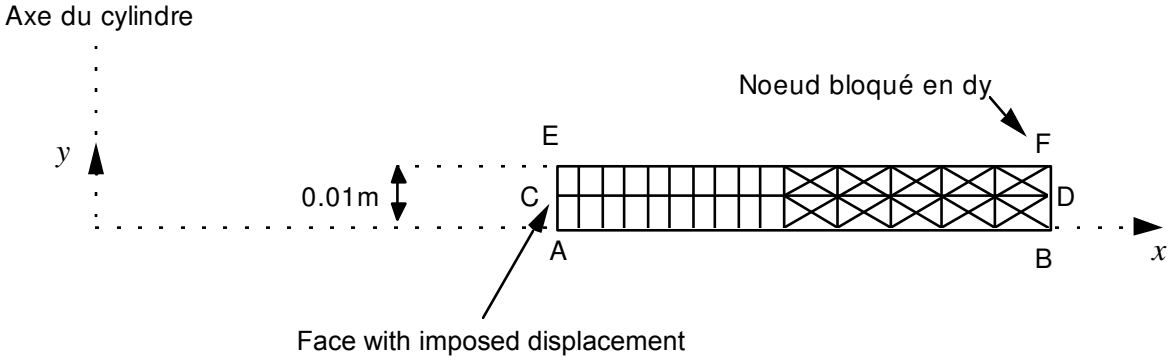
5.4 Remarks

As for the modelisation 3D, the got results are completely satisfactory.

6 Modeling D

6.1 Characteristics of modeling

Grid with elements AXIS_INCO_UPG (DEFORMATION=' SIMO_MIEHE') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DY = 0.	side EF
	GROUP_NO = ' FACINF'	DY = 0.	side AB
FACE_IMPO =	GROUP_MA = ' FACEAE'	DX = 6. 10^{-5}	face AE

6.2 Characteristics of the grid

Many nodes: 175.
Many meshs and types: 20 QUAD8, 40 TRIA6.

6.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	6. 10^{-5}	1. 10^{-3}
	v	ANALYTICAL	0.	1. 10^{-5}
	σ_{xx}	ANALYTICAL	-59.9955	5. 10^{-3}
	σ_{yy}	ANALYTICAL	19.9326	5. 10^{-3}
	σ_{zz}	ANALYTICAL	99.9566	5. 10^{-3}
	σ_{xy}	ANALYTICAL	0.	1. 10^{-5}
	VMIS	ANALYTICAL	138.5226	5. 10^{-3}
	TRESCA	ANALYTICAL	159.9521	5. 10^{-3}
	PRIN_1	ANALYTICAL	-59.9955	5. 10^{-3}
	PRIN_2	ANALYTICAL	19.9326	5. 10^{-3}
	PRIN_3	ANALYTICAL	99.9566	5. 10^{-3}
	VMIS_SG	ANALYTICAL	138.5226	5. 10^{-3}

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	3. 10^{-5}	1. 10^{-3}
	v	ANALYTICAL	0.	1. 10^{-5}
	σ_{xx}	ANALYTICAL	0.	0.03
	σ_{yy}	ANALYTICAL	20.0	5. 10^{-3}
	σ_{zz}	ANALYTICAL	40,006	5. 10^{-3}
	σ_{xy}	ANALYTICAL	0.	5. 10^{-3}

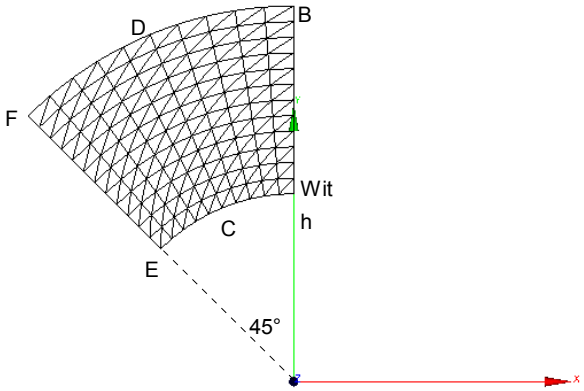
6.4 Remarks

The precision obtained is very good since all the constraints are obtained with a precision lower than 0.5%.

7 Modeling E

7.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' SIMO_MIEHE') incompressible of type PENTA15 only



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DZ = 0.		
	GROUP_NO = ' FACINF'	DZ = 0.	faces $AEFD$ ($z=0$ and $z=0.01$)	
	GROUP_NO = ' FACEAB'	DX = 0.	face AB	
FACE_IMPO =	GROUP_MA = ' FACEEF'	DNOR = 0.	face EF	
	GROUP_MA = ' FACEAE'	DNOR = -6.10 -5	face AE	

7.2 Characteristics of the grid

Many nodes: 1861
Many meshs: 480 PENTA15

7.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

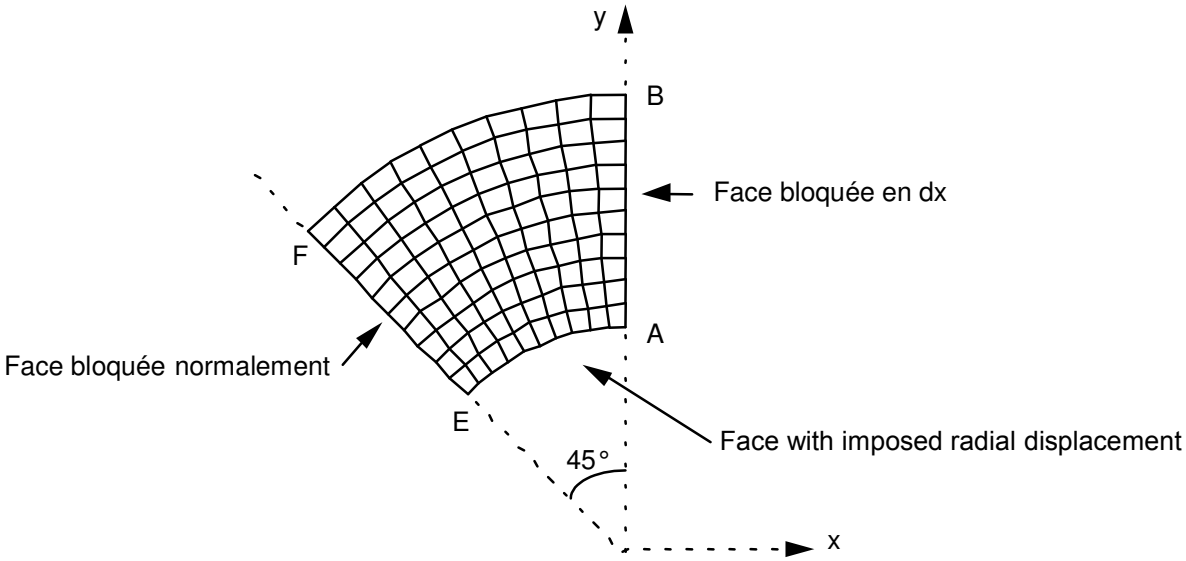
Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-5}
	v	ANALYTICAL	6. 10-5	1. 10^{-3}
	σ_{xx}	ANALYTICAL	99.9566	0.03
	σ_{yy}	ANALYTICAL	-59.9955	0.09
	σ_{zz}	ANALYTICAL	19.9326	0.08
	σ_{xy}	ANALYTICAL	0.	0,005
	VMIS	ANALYTICAL	138.5226	0.05
	TRESCA	ANALYTICAL	159.9521	0.05
	PRIN_1	ANALYTICAL	-59.9955	0.09
	PRIN_2	ANALYTICAL	19.9326	0.08
	PRIN_3	ANALYTICAL	99.9566	0.03
	VMIS_SG	ANALYTICAL	138.5226	0.05

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	3. 10^{-3}
	v	ANALYTICAL	2.1217 10^{-5}	3. 10^{-3}
	σ_{xx}	ANALYTICAL	20,003	0.04
	σ_{yy}	ANALYTICAL	20,003	0.17
	σ_{zz}	ANALYTICAL	20,003	0.07
	σ_{xy}	ANALYTICAL	20,003	0,008

8 Modeling F

8.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of elements

Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DZ = 0.		
	GROUP_NO = ' FACINF'	DZ = 0.	faces $AEFD$ ($z=0$ and $z=0.01$)	
	GROUP_NO = ' FACEAB'	DX = 0.	face AB	
FACE_IMPO =	GROUP_MA = ' FACEEF'	DNOR = 0.	face EF	
	GROUP_MA = ' FACEAE'	DNOR = -6.10 -5	face AE	

8.2 Characteristics of the grid

Many nodes: 1501 nodes
Many meshs: 240 HEXA20

8.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-3}
	v	ANALYTICAL	6. 10-5	1. 10^{-3}
	σ_{xx}	ANALYTICAL	99.9566	0.01
	σ_{yy}	ANALYTICAL	-59.9955	0.03
	σ_{zz}	ANALYTICAL	19.9326	0.05
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0,001
	TRESCA	ANALYTICAL	159.9521	0,001
	PRIN_1	ANALYTICAL	-59.9955	0.0025
	PRIN_2	ANALYTICAL	19.9326	0,005
	PRIN_3	ANALYTICAL	99.9566	0.0005
	VMIS_SG	ANALYTICAL	138.5226	0,001

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	0,005
	v	ANALYTICAL	2.1217 10^{-5}	0,005
	σ_{xx}	ANALYTICAL	20,003	0,002
	σ_{yy}	ANALYTICAL	20,003	0,002
	σ_{zz}	ANALYTICAL	20,003	0.0025
	σ_{xy}	ANALYTICAL	20,003	0.0015

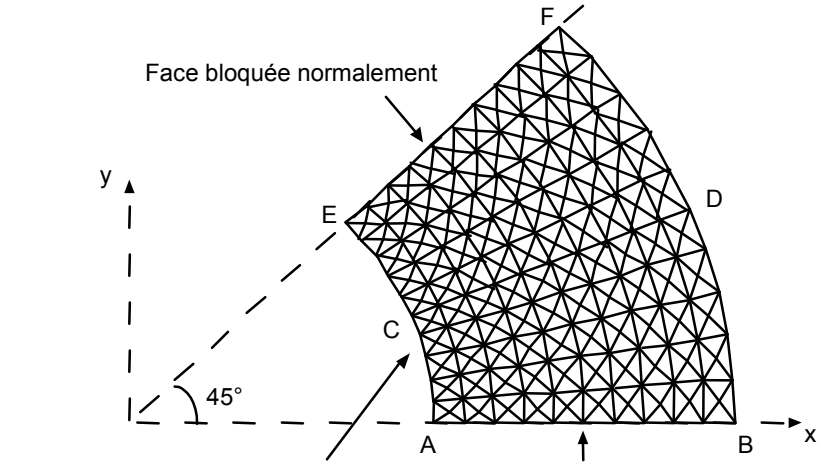
8.4 Remarks

One obtains very good performances since for all the examined sizes, the difference between the solution obtained with the code and the analytical solution is lower than 0.5% for displacements and lower than $5\ \%$ for the constraints.

9 Modeling G

9.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type TETRA10 only



AB is on the axis OX (contrary to modeling A).
The grid was obtained with GMSH for a density of $0,01$.

Limiting conditions:

```
DDL_IMPO   =  GROUP_NO = ' FACSUP'    DZ    = 0.
              GROUP_NO = ' FACINF'    DZ    = 0.        faces AEFD ( z=0 and z=0.01 )
               GROUP_NO = ' FACEAB'   DY    = 0.        face AB
FACE_IMPO  =  GROUP_MA = ' FACEEF'    DNOR  = 0.        face EF
              GROUP_MA = ' FACEAE'    DNOR  = -6.10 -5     face AE
```

9.2 Characteristics of the grid

Many nodes: 2064
Many meshs: 1121 TETRA10

9.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	6. 10^{-5}	2. 10^{-3}
	v	ANALYTICAL	0.	1. 10^{-3}
	σ_{xx}	ANALYTICAL	-59.9955	0.02
	σ_{yy}	ANALYTICAL	99.9566	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.03
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0,002
	TRESCA	ANALYTICAL	159.9521	0,002
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0.03
	PRIN_3	ANALYTICAL	99.9566	0,015
	VMIS_SG	ANALYTICAL	138.5226	0,002

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	2.1217 10^{-5}	1. 10^{-4}
	v	ANALYTICAL	2.1217 10^{-5}	1. 10^{-4}
	σ_{xx}	ANALYTICAL	20,003	0,003
	σ_{yy}	ANALYTICAL	20,003	0,005
	σ_{zz}	ANALYTICAL	20,003	0,002
	σ_{xy}	ANALYTICAL	-20,003	0.01

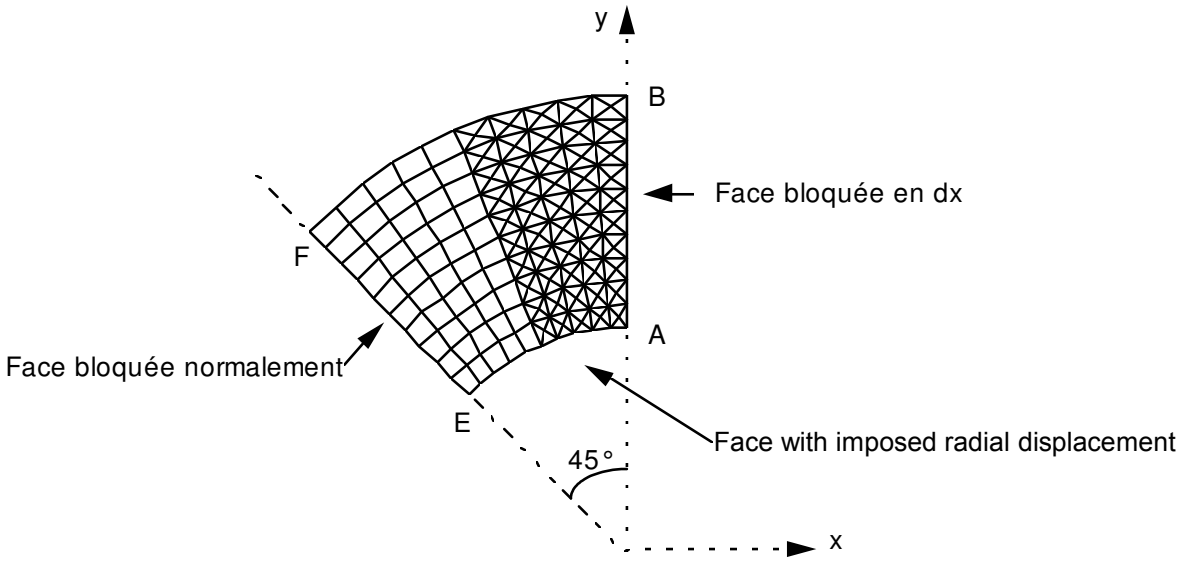
9.4 Remarks

The got results are completely correct since the constraints are obtained with a precision lower than $3\,\%$ even $1\,\%$ at the point F. the variation is a little more important here than for HEXA20, but can be explained by the fact that the loading is imposed here in a way a little less precise since displacement U at point A, is defined only with one precision of 0.158% against 0.077% (evening the factor 2, qu ' one finds on the constraints).

10 Modeling H

10.1 Characteristics of modeling

Grid with elements D_PLAN_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' GRNM11'	DX = 0.	side AB
FACE_IMPO =	GROUP_MA = ' GRMA12'	DNOR = 0.	dimensioned EF
=	GROUP_MA = ' GRMA13'	DNOR = -6. 10^{-5}	face AE

Name of the nodes:

$A=N2$, $B=N361$, $C=N121$, $D=N584$, $E=N155$, $F=N503$

10.2 Characteristics of the grid

Many nodes: 591
Many meshs: 200 TRIA6, 50 QUAD8.

10.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-5}
	v	ANALYTICAL	6. 10-5	1. 10^{-4}
	σ_{xx}	ANALYTICAL	99.9566	5. 10^{-3}
	σ_{yy}	ANALYTICAL	-59.9955	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.02
	σ_{xy}	ANALYTICAL	0.	0.02
	VMIS	ANALYTICAL	138.5226	0,006
	TRESCA	ANALYTICAL	159.9521	0,006
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0.02
	PRIN_3	ANALYTICAL	99.9566	0,003
	VMIS_SG	ANALYTICAL	138.5226	0,006

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	3. 10^{-4}
	v	ANALYTICAL	2.1217 10^{-5}	3. 10^{-4}
	σ_{xx}	ANALYTICAL	20,003	5. 10^{-3}
	σ_{yy}	ANALYTICAL	20,003	2.5 10^{-3}
	σ_{zz}	ANALYTICAL	20,003	5. 10^{-4}
	σ_{xy}	ANALYTICAL	20,003	2. 10^{-3}

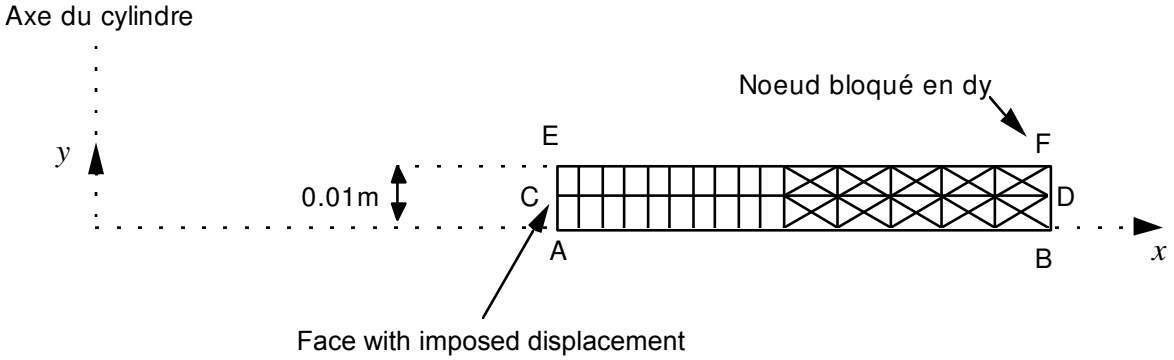
10.4 Remarks

As for modeling 3D, the got results are completely satisfactory.

11 Modeling I

11.1 Characteristics of modeling

Grid with elements AXIS_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DY = 0.	side EF
	GROUP_NO = ' FACINF'	DY = 0.	side AB
FACE_IMPO =	GROUP_MA = ' FACEAE'	DX = 6. 10^{-5}	face AE

11.2 Characteristics of the grid

Many nodes: 175.
Many meshs and types: 20 QUAD8, 40 TRIA6.

11.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	6. 10^{-5}	1. 10^{-3}
	v	ANALYTICAL	0.	1. 10^{-5}
	σ_{xx}	ANALYTICAL	-59.9955	5. 10^{-3}
	σ_{yy}	ANALYTICAL	19.9326	5. 10^{-3}
	σ_{zz}	ANALYTICAL	99.9566	5. 10^{-3}
	σ_{xy}	ANALYTICAL	0.	1. 10^{-5}
	VMIS	ANALYTICAL	138.5226	2. 10^{-3}
	TRESCA	ANALYTICAL	159.9521	2. 10^{-3}
	PRIN_1	ANALYTICAL	-59.9955	3. 10^{-3}
	PRIN_2	ANALYTICAL	19.9326	5. 10^{-3}
	PRIN_3	ANALYTICAL	99.9566	5. 10^{-4}
	VMIS_SG	ANALYTICAL	138.5226	2. 10^{-3}

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	3. 10^{-5}	5. 10^{-4}
	v	ANALYTICAL	0.	1. 10^{-5}
	σ_{xx}	ANALYTICAL	0.	0.03
	σ_{yy}	ANALYTICAL	20.0	3. 10^{-3}
	σ_{zz}	ANALYTICAL	40,006	3. 10^{-3}
	σ_{xy}	ANALYTICAL	0.	5. 10^{-3}

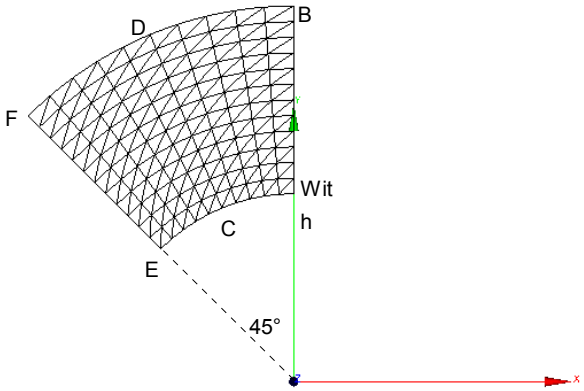
11.4 Remarks

The precision obtained is very good since all the constraints are obtained with a precision lower than 0.5%.

12 Modeling J

12.1 Characteristics of modeling

Grid with elements 3D_INCO_UPG (DEFORMATION=' GDEF_LOG') incompressible of type PENTA15 only



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DZ = 0.		
	GROUP_NO = ' FACINF'	DZ = 0.	faces $AEFD$ ($z=0$ and $z=0.01$)	
	GROUP_NO = ' FACEAB'	DX = 0.	face AB	
FACE_IMPO =	GROUP_MA = ' FACEEF'	DNOR = 0.	face EF	
	GROUP_MA = ' FACEAE'	DNOR = -6.10 -5	face AE	

12.2 Characteristics of the grid

Many nodes: 1861
Many meshs: 480 PENTA15

12.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

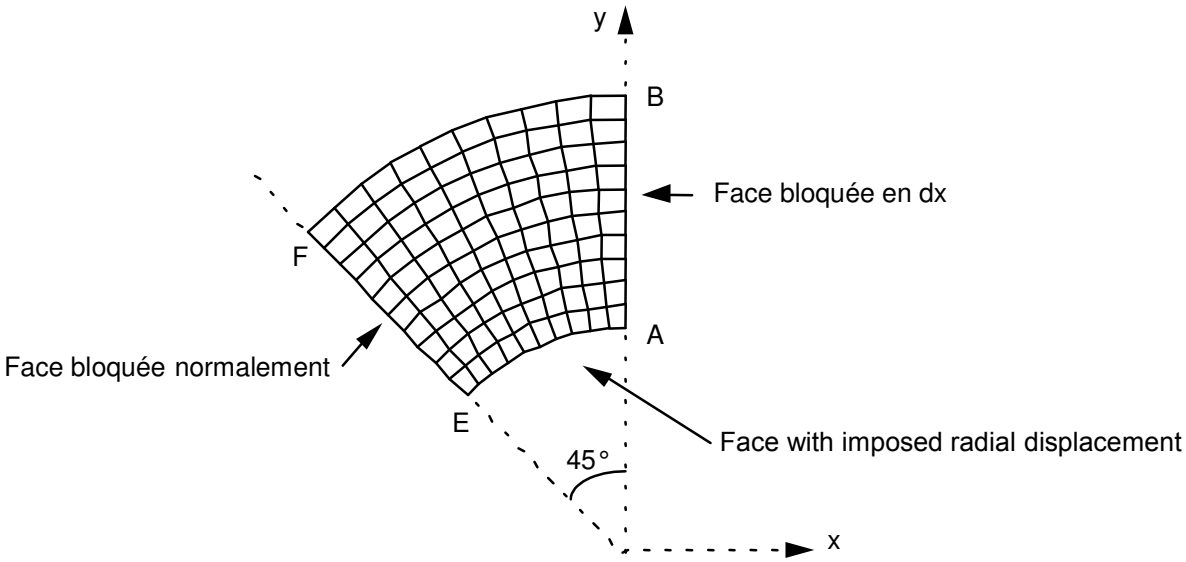
Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-5}
	v	ANALYTICAL	6. 10-5	1. 10^{-3}
	σ_{xx}	ANALYTICAL	99.9566	0.0002
	σ_{yy}	ANALYTICAL	-59.9955	0,003
	σ_{zz}	ANALYTICAL	19.9326	0,007
	σ_{xy}	ANALYTICAL	0.	0.0008
	VMIS	ANALYTICAL	138.5226	0.05
	TRESCA	ANALYTICAL	159.9521	0.05
	PRIN_1	ANALYTICAL	-59.9955	0.09
	PRIN_2	ANALYTICAL	19.9326	0.08
	PRIN_3	ANALYTICAL	99.9566	0.03
	VMIS_SG	ANALYTICAL	138.5226	0.05

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	4. 10^{-3}
	v	ANALYTICAL	2.1217 10^{-5}	4. 10^{-3}
	σ_{xx}	ANALYTICAL	20,003	0,007
	σ_{yy}	ANALYTICAL	20,003	0.0006
	σ_{zz}	ANALYTICAL	20,003	3. 10^{-5}
	σ_{xy}	ANALYTICAL	20,003	0.0004

13 Modeling K

13.1 Characteristics of modeling

Grid with elements 3D_INCO_UP (DEFORMATION=' GDEF_LOG') incompressible of type HEXA20 only



Along the axis z :

- total thickness $e=0.01$
- 2 layers of elements

Limiting conditions:

```
DDL_IMPO  =  GROUP_NO = ' FACSUP'   DZ   = 0.
             GROUP_NO = ' FACINF'   DZ   = 0.          faces AEFD ( z=0 and z=0.01 )
              GROUP_NO = ' FACEAB'  DX   = 0.          face AB
FACE_IMPO =  GROUP_MA = ' FACEEF'   DNOR = 0.         face EF
             GROUP_MA = ' FACEAE'   DNOR = -6.10 -5       face AE
```

13.2 Characteristics of the grid

Many nodes: 1501 nodes
Many meshs: 240 HEXA20

13.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-3}
	v	ANALYTICAL	6. 10-5	1. 10^{-4}
	σ_{xx}	ANALYTICAL	99.9566	0.01
	σ_{yy}	ANALYTICAL	-59.9955	0.03
	σ_{zz}	ANALYTICAL	19.9326	0.05
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0,001
	TRESCA	ANALYTICAL	159.9521	0,001
	PRIN_1	ANALYTICAL	-59.9955	0.0025
	PRIN_2	ANALYTICAL	19.9326	0,005
	PRIN_3	ANALYTICAL	99.9566	0.0005
	VMIS_SG	ANALYTICAL	138.5226	0,001

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	0,005
	v	ANALYTICAL	2.1217 10^{-5}	0,005
	σ_{xx}	ANALYTICAL	20,003	0,002
	σ_{yy}	ANALYTICAL	20,003	0,002
	σ_{zz}	ANALYTICAL	20,003	0.0025
	σ_{xy}	ANALYTICAL	20,003	0.0015

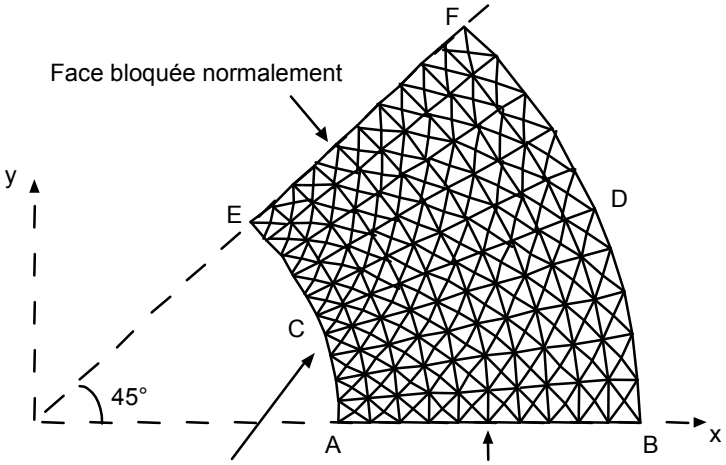
13.4 Remarks

One obtains very good performances since for all the examined sizes, the difference between the solution obtained with the code and the analytical solution is lower than 0.5% for displacements and lower than $5\ \%$ for the constraints.

14 Modeling L

14.1 Characteristics of modeling

Grid with elements 3D_INCO_UP (DEFORMATION=' GDEF_LOG') incompressible of type TETRA10 only



AB is on the axis OX (contrary to modeling A).
The grid was obtained with GMSH for a density of $0,01$.

Limiting conditions:

```
DDL_IMPO   =  GROUP_NO = ' FACSUP'    DZ   = 0.
              GROUP_NO = ' FACINF'    DZ   = 0.        faces AEFD ( z=0 and
z=0.01 )
               GROUP_NO = ' FACEAB'   DY   = 0.        face AB
FACE_IMPO  =  GROUP_MA = ' FACEEF'    DNOR = 0.        face EF
              GROUP_MA = ' FACEAE'    DNOR = -6.10 -5     face AE
```

14.2 Characteristics of the grid

Many nodes: 2064
Many meshs: 1121 TETRA10

14.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	6. 10^{-5}	2. 10^{-4}
	v	ANALYTICAL	0.	1. 10^{-3}
	σ_{xx}	ANALYTICAL	-59.9955	0.02
	σ_{yy}	ANALYTICAL	99.9566	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.03
	σ_{xy}	ANALYTICAL	0.	0.03
	VMIS	ANALYTICAL	138.5226	0,002
	TRESCA	ANALYTICAL	159.9521	0,002
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0.03
	PRIN_3	ANALYTICAL	99.9566	0,015
	VMIS_SG	ANALYTICAL	138.5226	0,002

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	2.1217 10^{-5}	1. 10^{-4}
	v	ANALYTICAL	2.1217 10^{-5}	1. 10^{-4}
	σ_{xx}	ANALYTICAL	20,003	0,003
	σ_{yy}	ANALYTICAL	20,003	0,005
	σ_{zz}	ANALYTICAL	20,003	0,002
	σ_{xy}	ANALYTICAL	-20,003	0.01

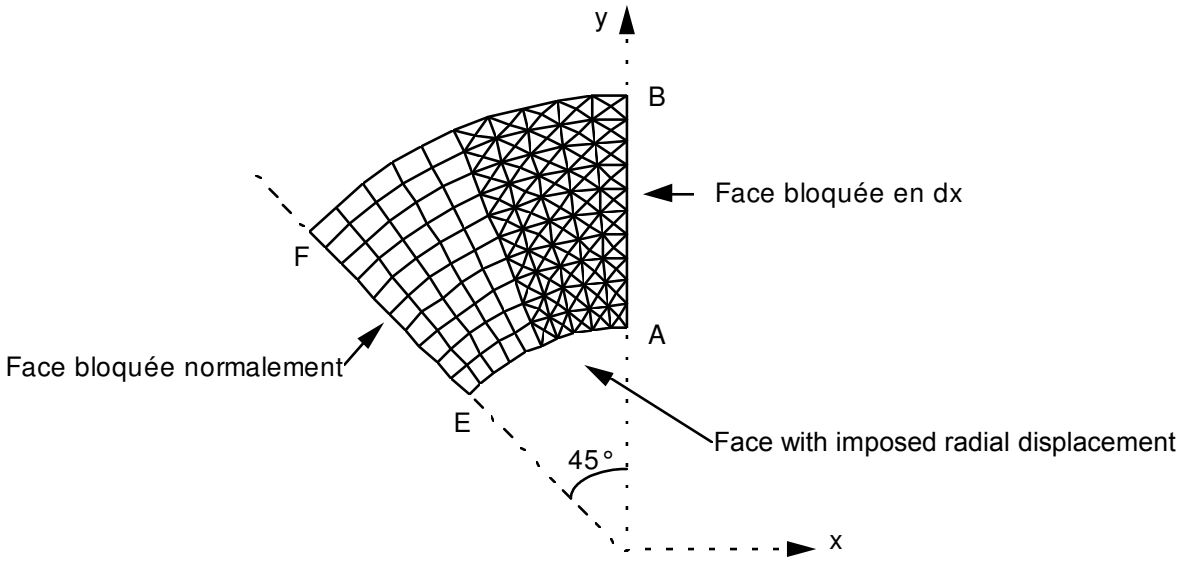
14.4 Remarks

The got results are completely correct since the constraints are obtained with a precision lower than $3\,\%$ even $1\,\%$ at the point F. the variation is a little more important here than for HEXA20, but can be explained by the fact that the loading is imposed here in a way a little less precise since displacement U at point A, is defined only with one precision of 0.158% against 0.077% (evening the factor 2, qu ' one finds on the constraints).

15 Modeling M

15.1 Characteristics of modeling

Grid with elements D_PLAN_INCO_UP (DEFORMATION=' GDEF_LOG') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' GRNM11'	DX = 0.	side AB
FACE_IMPO =	GROUP_MA = ' GRMA12'	DNOR = 0.	dimensioned EF
=	GROUP_MA = ' GRMA13'	DNOR = -6. 10^{-5}	face AE

Name of the nodes:

$A=N2$, $B=N361$, $C=N121$, $D=N584$, $E=N155$, $F=N503$

15.2 Characteristics of the grid

Many nodes: 591
Many meshs: 200 TRIA6, 50 QUAD8.

15.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-5}
	v	ANALYTICAL	6. 10-5	1. 10^{-4}
	σ_{xx}	ANALYTICAL	99.9566	5. 10^{-3}
	σ_{yy}	ANALYTICAL	-59.9955	0.02
	σ_{zz}	ANALYTICAL	19.9326	0.02
	σ_{xy}	ANALYTICAL	0.	0.02
	VMIS	ANALYTICAL	138.5226	0,006
	TRESCA	ANALYTICAL	159.9521	0,006
	PRIN_1	ANALYTICAL	-59.9955	0.02
	PRIN_2	ANALYTICAL	19.9326	0.02
	PRIN_3	ANALYTICAL	99.9566	0,003
	VMIS_SG	ANALYTICAL	138.5226	0,006

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	3. 10^{-4}
	v	ANALYTICAL	2.1217 10^{-5}	3. 10^{-4}
	σ_{xx}	ANALYTICAL	20,003	5. 10^{-3}
	σ_{yy}	ANALYTICAL	20,003	2.5 10^{-3}
	σ_{zz}	ANALYTICAL	20,003	5. 10^{-4}
	σ_{xy}	ANALYTICAL	20,003	2. 10^{-3}

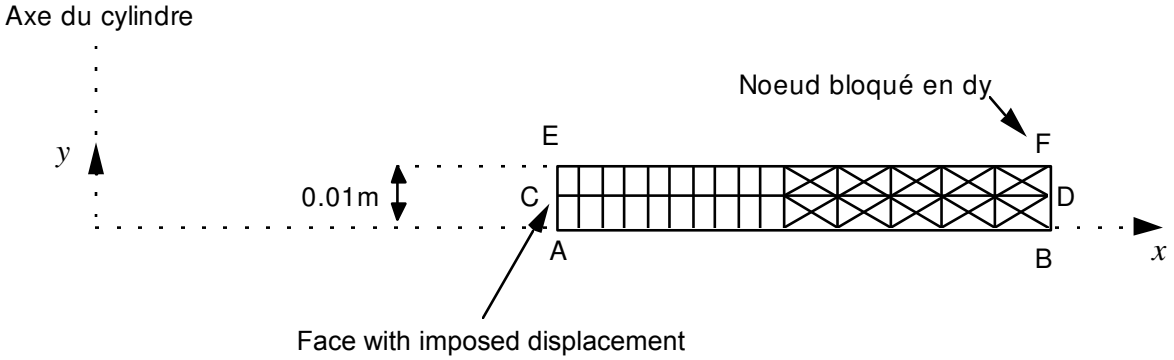
15.4 Remarks

As for modeling 3D, the got results are completely satisfactory.

16 Modeling NR

16.1 Characteristics of modeling

Grid with elements AXIS_INCO_UP (DEFORMATION=' GDEF_LOG') incompressible of type TRIA6 and QUAD8



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DY = 0.	side EF
	GROUP_NO = ' FACINF'	DY = 0.	side AB
FACE_IMPO =	GROUP_MA = ' FACEAE'	DX = 6. 10^{-5}	face AE

16.2 Characteristics of the grid

Many nodes: 175.
Many meshs and types: 20 QUAD8, 40 TRIA6.

16.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	6. 10^{-5}	1. 10^{-3}
	v	ANALYTICAL	0.	1. 10^{-5}
	σ_{xx}	ANALYTICAL	-59.9955	5. 10^{-3}
	σ_{yy}	ANALYTICAL	19.9326	5. 10^{-3}
	σ_{zz}	ANALYTICAL	99.9566	5. 10^{-3}
	σ_{xy}	ANALYTICAL	0.	1. 10^{-5}
	VMIS	ANALYTICAL	138.5226	2. 10^{-3}
	TRESCA	ANALYTICAL	159.9521	2. 10^{-3}
	PRIN_1	ANALYTICAL	-59.9955	3. 10^{-3}
	PRIN_2	ANALYTICAL	19.9326	5. 10^{-3}
	PRIN_3	ANALYTICAL	99.9566	5. 10^{-4}
	VMIS_SG	ANALYTICAL	138.5226	2. 10^{-3}

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	3. 10^{-5}	5. 10^{-4}
	v	ANALYTICAL	0.	1. 10^{-5}
	σ_{xx}	ANALYTICAL	0.	0.03
	σ_{yy}	ANALYTICAL	20.0	3. 10^{-3}
	σ_{zz}	ANALYTICAL	40,006	3. 10^{-3}
	σ_{xy}	ANALYTICAL	0.	5. 10^{-3}

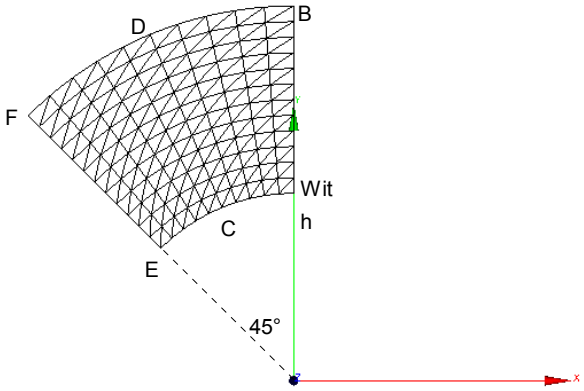
16.4 Remarks

The precision obtained is very good since all the constraints are obtained with a precision lower than 0.5%.

17 Modeling O

17.1 Characteristics of modeling

Grid with elements 3D_INCO_UP (DEFORMATION=' GDEF_LOG') incompressible of type PENTA15 only



Limiting conditions:

DDL_IMPO =	GROUP_NO = ' FACSUP'	DZ = 0.		
	GROUP_NO = ' FACINF'	DZ = 0.	faces $AEFD$ ($z=0$ and $z=0.01$)	
	GROUP_NO = ' FACEAB'	DX = 0.	face AB	
FACE_IMPO =	GROUP_MA = ' FACEEF'	DNOR = 0.	face EF	
	GROUP_MA = ' FACEAE'	DNOR = -6.10 -5	face AE	

17.2 Characteristics of the grid

Many nodes: 1861
Many meshs: 480 PENTA15

17.3 Sizes tested and results

Displacements and the constraints are evaluated at the points A and F. Components of the field SIEQ_NOEU are tested at the point A only.

Identification		Type of reference	Reference	Tolerance
A	u	ANALYTICAL	0.	1. 10^{-5}
	v	ANALYTICAL	6. 10-5	1. 10^{-3}
	σ_{xx}	ANALYTICAL	99.9566	0.0002
	σ_{yy}	ANALYTICAL	-59.9955	0,003
	σ_{zz}	ANALYTICAL	19.9326	0,007
	σ_{xy}	ANALYTICAL	0.	0.0008
	VMIS	ANALYTICAL	138.5226	0,001
	TRESCA	ANALYTICAL	159.9521	0,001
	PRIN_1	ANALYTICAL	-59.9955	0,003
	PRIN_2	ANALYTICAL	19.9326	0,008
	PRIN_3	ANALYTICAL	99.9566	0.0002
	VMIS_SG	ANALYTICAL	138.5226	0,001

Identification		Type of reference	Reference	Tolerance
F	u	ANALYTICAL	-2.1217 10^{-5}	4. 10^{-3}
	v	ANALYTICAL	2.1217 10^{-5}	4. 10^{-3}
	σ_{xx}	ANALYTICAL	20,003	0,007
	σ_{yy}	ANALYTICAL	20,003	0.0006
	σ_{zz}	ANALYTICAL	20,003	3. 10^{-5}
	σ_{xy}	ANALYTICAL	20,003	0.0004

18 Summary of the results

With a Poisson's ratio ν very near to 0.5, one finds the results of the incompressible analytical solution in great deformations, with a completely correct precision.