SSNV102 - Tensile test shearing with the model of TAHERI

Summary:

The problem is quasi-static nonlinear in mechanics of the structures.

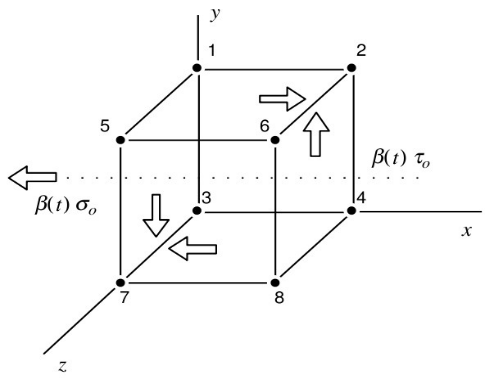
One analyzes the response of an element of volume to a loading in traction-shearing, carried out in such way that imposes a state of uniform stress-strain in the element.

There are 2 modelings: one in $3D$ voluminal and another in plane constraints $2D$.

One validates by this test the digital integration of the elastoplastic model of behavior of Saïd Taheri.

1 Problem of reference

1.1 Geometry



Face YZ: $(1,3,5,7)$
Face XZ: $(3,4,7,8)$
Face $1YZ$: $(2,4,6,8)$
Face $1XZ$: $(1,2,5,6)$

$\beta(t)\tau_0$ imposed shearing $\begin{cases}\text{Face } 1XZ \\ \text{Face } 1YZ\end{cases}$

$\beta(t)\sigma_0$ imposed pressure

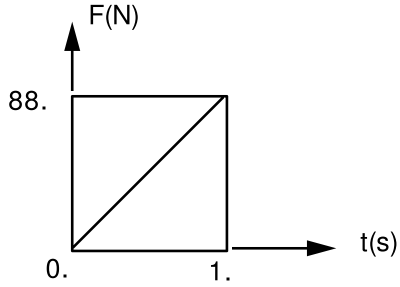
$\beta(t)$ function of effort

1.2 Material properties

isotropic elasticity	$E=200\,000\ MPa$	$\nu=0,3$		
Taheri plasticity	$C_{inf}=0.065\ MPa$	$C_1=-0.012\ Mpa$	$s=450$	$b=30$
	$m=0.1$	$a=312$	$\alpha=0.3$	$R_o=72$

1.3 Boundary conditions and loadings

$N04$	$dx=dy=0$	Face YZ:	$FX=FY=-F(t)$
$N08$	$dx=dy=dz=0$	Face XZ:	$FX=-F(t)$
$N02, N06$	$dx=0$	Face $1YZ$:	$FY=F(t)$
		Face $1XZ$:	$FX=F(t)$



1.4 Initial conditions

Worthless constraints and deformations with $t=0$.

2 Reference solution

2.1 Method of calculating used for the reference solution

One integrates the following system numerically enters $t=0$ and $t=1$.

(Nonlinear ordinary Differential connection of 6 equations to 6 unknown factors solved using library NAG by a 'Backward difference method')

$$\dot{\varepsilon} - \dot{\varepsilon} p = \dot{\beta} \frac{\sigma_o}{E}$$ **éq 2.1- 1**

$$\dot{\gamma} - \dot{\gamma} p = \dot{\beta} \frac{\tau_o}{2\mu}$$ **éq 2.1- 2**

$$\dot{\varepsilon} p - \dot{p} \frac{\partial F}{\partial \sigma} = 0$$ **éq 2.1- 3**

$$\dot{\gamma} p - \dot{p} \frac{\partial F}{\partial \tau} = 0$$ **éq 2.1- 4**

$$\left[-\frac{3}{2}\frac{\partial F}{\partial \sigma}\left(Kx + Cs\frac{\partial F}{\partial \sigma}\right) - 2\frac{\partial F}{\partial \tau}\left(Ky + Cs\frac{\partial F}{\partial \tau}\right) - HR - a\, D\, \alpha\, Z^{(\alpha-2)} \left[\left(\varepsilon_p - \varepsilon_p^n\right)\frac{\partial F}{\partial \sigma} + \frac{4}{3}\left(\gamma_p - \gamma_p^n\right)\frac{\partial F}{\partial \tau} \right] \right] \dot{p} + \left[\frac{3}{2}\frac{\partial F}{\partial \sigma}\left(Qx + C\varepsilon_p^n\right) + 2\frac{\partial F}{\partial \tau}\left(Qy + C\gamma_p^n\right) + JR \right] \dot{\sigma}_p = -\frac{\partial F}{\partial \sigma}\dot{\beta}\,\sigma_o - 2\frac{\partial F}{\partial \tau}\dot{\beta}\,\tau_o$$ **éq 2.1- 5**

$$\left[1 + JR + \frac{3C}{2U}\left[\frac{3}{2}C\left(\varepsilon_p\, s - \sigma_p\, \varepsilon_p^n\right)\varepsilon_p^n + 2C\left(s\,\gamma_p - \sigma_p\,\gamma_p^n\right)\gamma_p^n \right] \right] \dot{\sigma}_p - \left[HR + a\, D\, \alpha\, Z^{(\alpha-2)} \left[\left(\varepsilon_p - \varepsilon_p^n\right)\frac{\partial F}{\partial \sigma} + \frac{4}{3}\left(\gamma_p - \gamma_p^n\right)\frac{\partial F}{\partial \tau} \right] + KX + 3\frac{CS}{2U}\left[\frac{3}{2}C\left(s\,\varepsilon_p - \sigma_p\,\varepsilon_p^n\right)\frac{\partial F}{\partial \sigma} + 2C\left(s\,\gamma_p - \sigma_p\,\gamma_p^n\right)\frac{\partial F}{\partial \tau} \right] \right] \dot{p} = 0$$ **éq 2.1- 6**

avec

$$D = 1 - m e^{-up} \quad \begin{cases} u = b\left(1 - \dfrac{\sigma_p}{S}\right) \\ v = \dfrac{C\infty - C}{C} \\ w = \dfrac{1 - D}{D} \\ C = C\infty - C_1 e^{-up} \\ K = v\,u \qquad Q = v\dfrac{bp}{s} \\ H = w\,u \qquad J = w\dfrac{bp}{s} \end{cases}$$

et

$$\begin{cases} X = C\left(s\,\varepsilon_p - \sigma_p\,\varepsilon_p^n\right) \\ Y = C\left(s\,\gamma_p - \sigma_p\,\gamma_p^n\right) \\ U = \left[\dfrac{9}{4}X^2 + 3\,Y^2\right]^{1/2} \\ R = D\left(a\,Z^{\alpha} + r0\right) \\ Z = \left[\left(\varepsilon_p - \varepsilon_p^n\right)^2 + \dfrac{4}{3}\left(\gamma_p - \gamma_p^n\right)^2\right]^{1/2} \end{cases}$$

with the initial conditions:

$$\begin{cases} \beta(0) = \dfrac{R(0)}{\left(\sigma_o^2 + 3\,\tau_o^2\right)^{1/2}} \\ \varepsilon(0) = \beta(0)\,\dfrac{\sigma_o}{E} \\ \gamma(0) = \beta(0)\,\dfrac{\tau_o}{2\mu} \\ p(0) = \varepsilon_p(0) = \varepsilon_p^n = 0 \\ R(0) = (1-m)\,r_o = \sigma_p(o) \end{cases}$$

$$\text{d'où } \sigma(t=1) = \begin{bmatrix} 88. & 88. & 0 \\ 88. & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2.2 Results of reference

Values of $\varepsilon, \gamma, \varepsilon_p, \gamma_p, p$ and σ_p with the nodes with $t=1\ s$.

2.3 Uncertainty on the solution

Uncertainty of library NAG.

2.4 Bibliographical references

1) User's manual library NAG on CRAY.

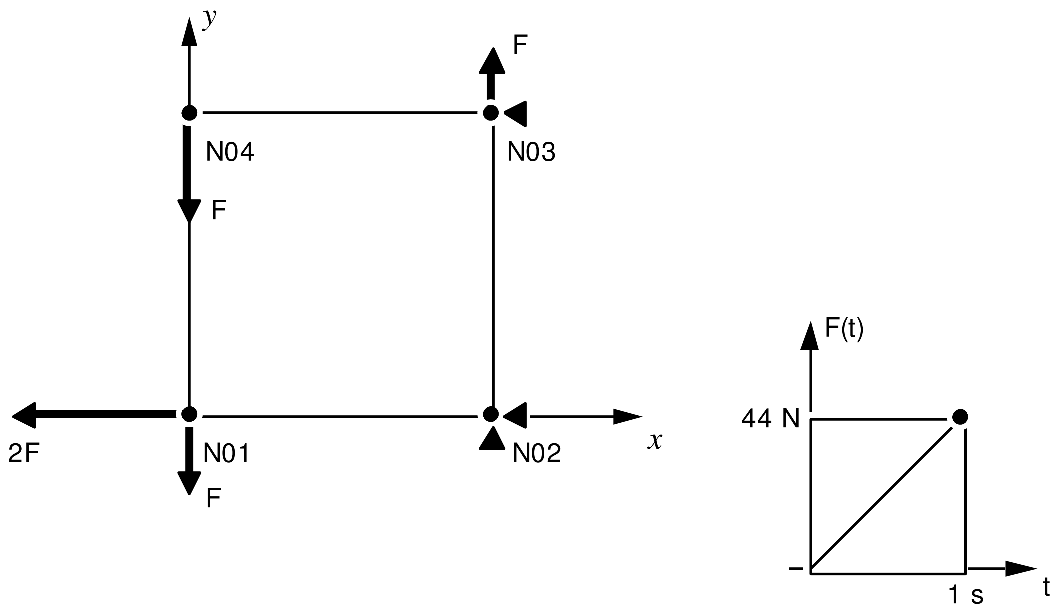
2) S. ANDRIEUX - P. SCHOENBERGER - S. TAHERI: With three dimensional cyclic constitutive law for metals with has semi-discrete memory variable - HI - 71/8147 (1992)

3) P. GEYER - J.M. PROIX - P. SCHOENBERGER - S. TAHERI: Modeling of the phenomena of progressive deformation - Collection of the internal notes of DER 93NB00153

3 Modeling A

3.1 Characteristics of modeling

Modeling in plane constraints $2\mathrm{D}$ **, C_PLAN**



3.2 Characteristics of the grid

Square quadrangle with 4 nodes in plane constraints with:

- $largeur = 1\,mm$,
- $épaisseur = 1\,mm$.

3.3 Sizes tested and results

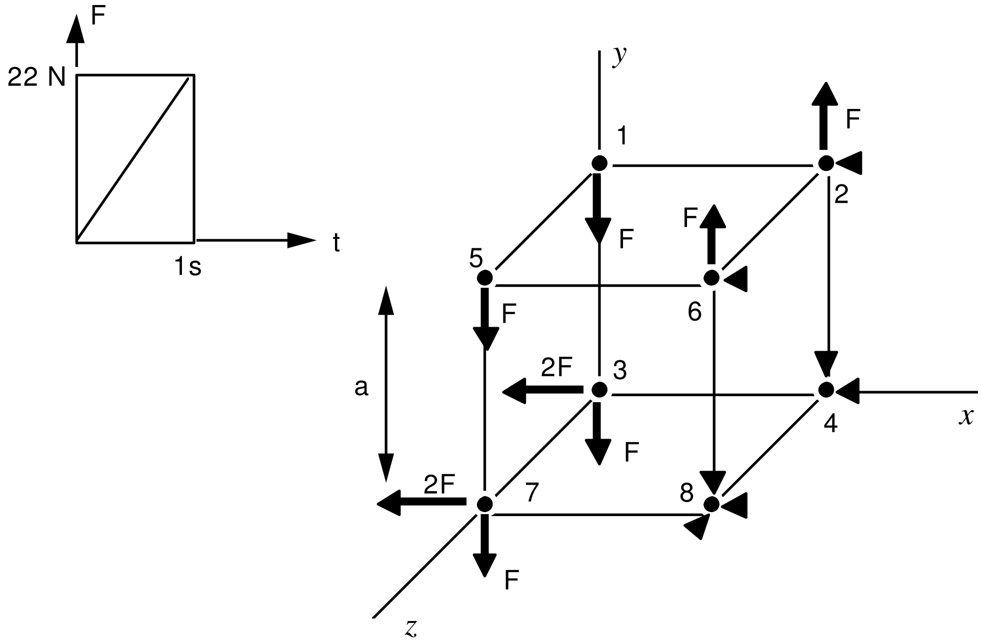
Identification	Reference	Test	Tolerance
ε on $NO4$ with $t=1\mathrm{s}$	0.01721	ANALYTICAL	2,5%
γ on $NO4$ with $t=1\mathrm{s}$	0.02573	ANALYTICAL	2,5%
ε_p on $MA1$, not of Gauss 4 with $t=1\mathrm{s}$	0.01678	ANALYTICAL	2,5%
γ_p on $MA1$, not of Gauss 4 with $t=1\mathrm{s}$	0.02515	ANALYTICAL	2,5%
ε_p on $NO4$ with $t=1\mathrm{s}$	0.01678	ANALYTICAL	2,5%
γ_p on $NO4$ with $t=1\mathrm{s}$	0.02515	ANALYTICAL	2,5%
σ_p on $NO1$ with $t=1\mathrm{s}$	176.0	ANALYTICAL	0,10%

4 Modeling B

4.1 Characteristics of modeling

Modeling 3D :

Cubic elementary with a grid using a hexahedron with 8 nodes.



4.2 Characteristics of the grid

1 mesh HEXA8, width side $a=1\,mm$.

4.3 Sizes tested and results

4.3.1 Values tested

Identification	Reference	Test	Tolerance
ε on $NO4$ with $t=1s$	0.01721	ANALYTICAL	2,5%
γ on $NO4$ with $t=1s$	0.02573	ANALYTICAL	2,5%
ε_p on $MA1$, not of Gauss 1 with $t=1s$	0.01678	ANALYTICAL	2,5%
γ_p on $MA1$, not of Gauss 1 with $t=1s$	0.02515	ANALYTICAL	2,5%
ε_p on $NO4$ with $t=1s$	0.01678	ANALYTICAL	2,5%
γ_p on $NO4$ with $t=1s$	0.02515	ANALYTICAL	2,5%
σ_p on $NO1$ with $t=1s$	176.0	ANALYTICAL	0,10%
p on $NO1$ with $t=1s$	0,03	ANALYTICAL	0,10%

One also tests the structural parameters of data results:

Identification	Reference	Test	Tolerance
INST for NUME_ORDRE= 6	1	ANALYTICAL	0%
ITER_GLOB for NUME_ORDRE= 6	12	NON_REGRESSION	8

4.3.2 Remarks

The reduction in the tolerance on total convergence in displacement does not bring significant profit in precision.

The number of increments of load (6) led to a satisfactory precision of the result.

5 Summary of the results

Good precision at the time of the comparison with NAG in spite of some difficulties of convergence with this mathematical library.