SSNP150 – Method of the solutions manufactured in contact 2D and great deformations

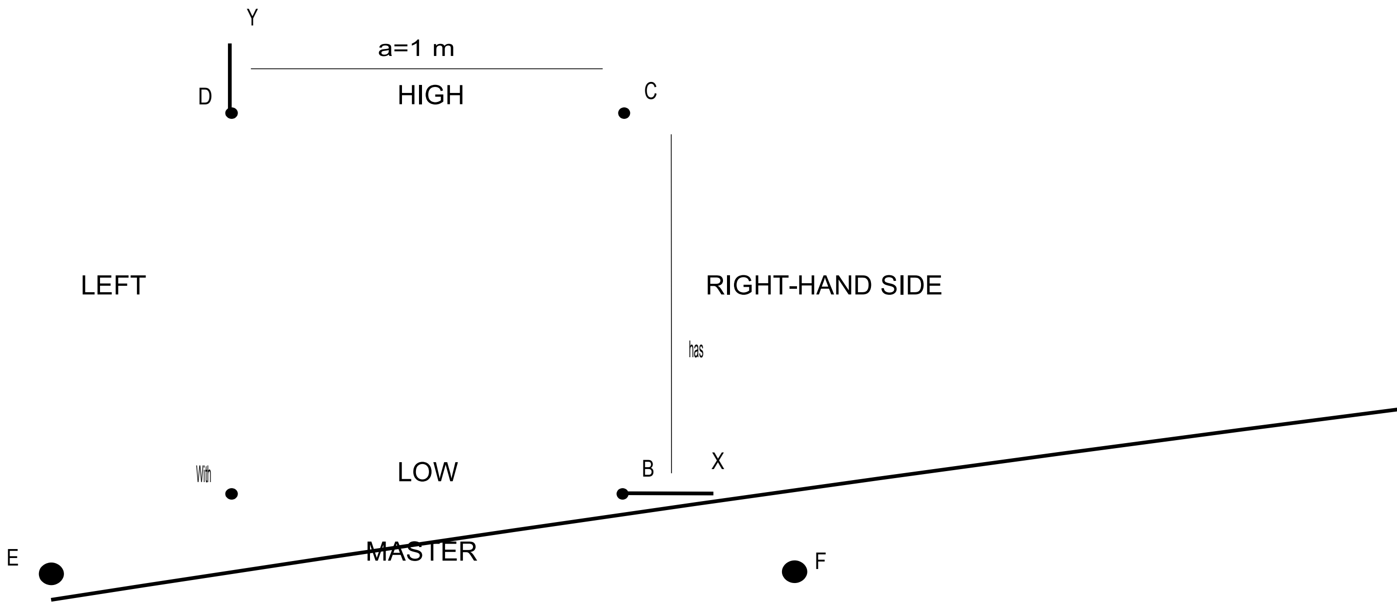
Summary:

The objective of this test is to check the modeling of the contact 2D in great deformations thanks to the method of the manufactured solutions [bib1].

1 Problem of reference

1.1 Geometry

One considers a square of with dimensions $1\,m$.



1.2 Properties of material

$E=1\text{MPa}$	Young modulus
$\nu=0.3$	Poisson's ratio

1.3 Boundary conditions and loadings

On the edge HAUT , one forces a displacement (see paragraph 2).
On the edges GAUCHE , BAS and DROITE , one forces a pressure (see paragraph 2).
In all the field, one forces a force of volume (see paragraph 2).
Surface MAITRE of natural paraboloid is described by the equation:

$$Y=-0.05\times(X-0.5)^2 \quad (1)$$

1.4 Initial conditions

Nothing

2 Reference solution

2.1 Method of calculating

The analytical reference solution is given by:

$$\begin{aligned} Ux &= -0.2\times Y\times Y\times Y\times(X-0.5) \\ Uy &= -0.05\times(X-0.5)\times(X-0.5)\times(1+Y)-0.01\times Y \end{aligned} \quad (2)$$

The conditions of Dirichlet, Neumann and the source term are obtained by the method of the manufactured solutions [bib1].
One starts by determining the gradient of the transformation $\underline{\underline{F}}$:

$$\underline{\underline{F}}=\nabla\underline{U}+\underline{\underline{Id}} \quad (3)$$

Knowing the normal $\underline{N}=[0,-1]^T$ on the surface slave in the configuration not-deformation, one obtains his expression in the configuration deformed by the formula of Nanson:

$$\underline{n}=\frac{\underline{\underline{F}}^{-T}\underline{N}}{\|\underline{\underline{F}}^{-T}\underline{N}\|} \quad (4)$$

Knowing the tensor of Hooke $\underline{\underline{\underline{A}}}$ and the tensor of Green-Lagrange $\underline{\underline{E}}$, one calculates the second tensor of Piola-Kirchhoff $\underline{\underline{S}}$:

$$\underline{\underline{E}}=\frac{1}{2}(\underline{\underline{F}}^T\cdot\underline{\underline{F}}-\underline{\underline{Id}}) \quad (5)$$

$$\underline{\underline{S}}=\underline{\underline{\underline{A}}}:\underline{\underline{E}} \quad (6)$$

It is pointed out that the second tensor of Piola-Kirchhoff $\underline{\underline{S}}$ allows to obtain efforts in configuration not deformed per not deformed unit of area:

$$\frac{d\,f_0}{dA}=\underline{\underline{S}}\cdot\underline{N} \quad (7)$$

As we seek to determine efforts in deformed configuration, we will determine the first tensor of Piola-Kirchhoff $\underline{\underline{\Pi}}$

$$\underline{\underline{\Pi}}=\underline{\underline{F}}\cdot\underline{\underline{S}} \quad (8)$$

One can thus determine the forces of volume $\underline{f}_{vol}$:

$$\underline{f}_{vol}=\text{-}div\,\underline{\underline{\Pi}} \quad (9)$$

Knowing the normal in initial configuration on the various faces and the first tensor of Piola-Kirchhoff $\underline{\underline{\Pi}}$, one can calculate the efforts of surface in deformed configuration:

$$\underline{f}_{surf} = \underline{\underline{\Pi}} \cdot \underline{N} \qquad (10)$$

On surface BAS who is in contact, one needs a particular treatment. Indeed, the normal efforts are taken there into account by the contact:

$$\begin{aligned} \underline{f}^{BAS}_{surf} &= \underline{f}^{BAS}_{surf_n} + \underline{f}^{BAS}_{surf_t} \\ &= \underline{f}_{contact} + \underline{f}^{BAS}_{surf_t} \\ &= p * \underline{n} + \underline{f}^{BAS}_{surf_t} \end{aligned} \qquad (11)$$

Where p indicate the contact pressure. It can be given by the expression:

$$p = (\underline{\underline{\Pi}} \cdot \underline{N}) \cdot \underline{n} \qquad (12)$$

One thus should apply only the tangential stresses to it. One calculates them by the expression:

$$\begin{aligned} \underline{f}^{BAS}_{surf_t} &= \underline{f}^{BAS}_{surf} - \underline{f}^{BAS}_{surf_n} \\ &= \underline{f}^{BAS}_{surf} - (\underline{f}^{BAS}_{surf_n} \cdot \underline{n}) \underline{n} \end{aligned} \qquad (13)$$

Concerning the efforts of contact, *it is absolutely essential to build the manufactured solution so that they check the equations of the contact* [bib2], namely:

$$\begin{aligned} gap(\underline{U}) &\geqslant 0 \\ p &\leqslant 0 \\ p \cdot gap(\underline{U}) &= 0 \end{aligned} \qquad (14)$$

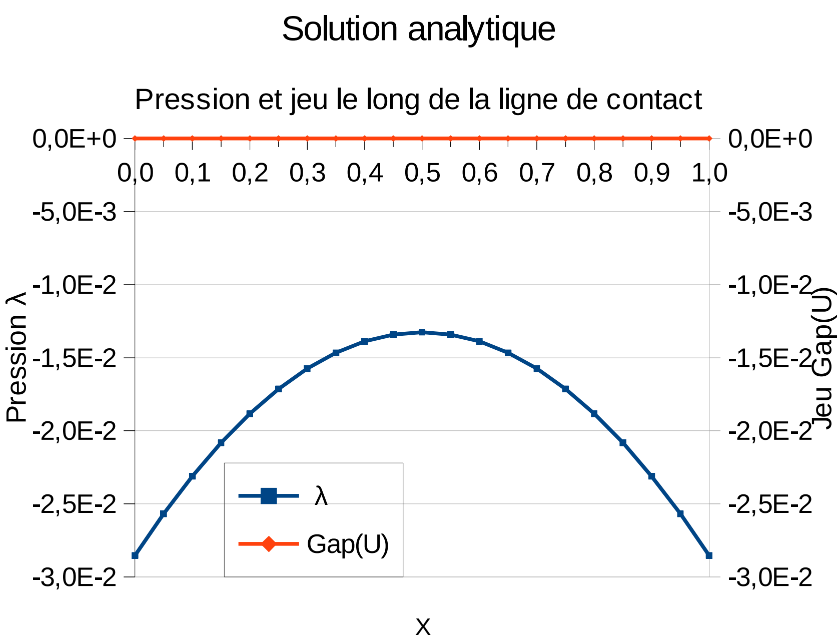


Figure 2.1-1: checking has postérori validity of the solution

This checking is done after having calculated in an analytical way the pressure and the jump of displacement associated with the manufactured solution, in general with a formal computational tool (in fact, it acts of the module Python *sympy*). One must then visualize them, in order to check *retrospectively* that the solution which one has contruite checks well (14). In the case of this test, we represented pressure and jump of displacement analytical in fig.2.1-1. It is noticed that it check $p<0$ and $\mathrm{gap}(\underline{U})=0$, which is characteristic of a surface entirely contacting, and conforms to (14).

2.2 Sizes and results of reference

The value of the difference between solutions analytical and calculated on the grid: $\sum^{\mathrm{noeuds}\ n} |\underline{U}_{\mathrm{n}}^{\mathrm{calc}}-\underline{U}_{\mathrm{n}}^{\mathrm{ref}}|$.

In the case of modelings which carry out an analysis of convergence with the smoothness of the grid, the speed of convergence with the smoothness of the grid of the solution calculated towards the analytical solution in standard L_2 :

– greatest reality $\alpha_U>0$ such as $\|\underline{U}^{\mathrm{calc}}-\underline{U}^{\mathrm{ref}}\|_{0,\Omega}<C_U\times h^{\alpha_U}$ where C_U is independent of h for displacement;

– greatest reality $\alpha_p>0$ such as $\|p^{\mathrm{calc}}-p^{\mathrm{ref}}\|_{0,\Gamma_c}<C_p\times h^{\alpha_p}$ where C_p is independent of h for the contact pressure.

2.3 Uncertainties on the solution

None

2.4 Bibliographical references

1 U2.08.08 document, Use of the Method of the Solutions Manufactured for the software validation, Documentation U2 de Code_Aster

2 R5.03.50 document, discrete Formulation of contact-friction, Documentation R of Code_Aster

3 Modeling A

3.1 Characteristics of modeling

A modeling is used D_PLAN.

3.2 Characteristics of the grid

The grid contains 65 elements of the type SEG3 and 256 elements of the type QUAD8.
Curved surface Master is represented by single SEG3 .

3.3 Sizes tested and results

One tests the sum of the absolute values of the difference between the calculated solution and the analytical solution.

Identification	Type of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	'NON_REGRESSION'	4.03888411513E-05

4 Modeling B

4.1 Characteristics of modeling

A modeling is used D_PLAN.

4.2 Characteristics of the grid

One carries out a study of convergence with the smoothness of the grid of the solution calculated towards the analytical solution. A succession of grids obtained by uniform refinement using the order MACR_ADAP_MAIL is used:

- grid 0: 5 SEG3, 1 QUAD8
- grid 1: 9 SEG3, 4 QUAD8
- grid 2: 17 SEG3, 16 QUAD8
- grid 3: 32 SEG3, 64 QUAD8
- grid 4: 64 SEG3, 256 QUAD8

Curved surface Master is represented by single SEG3 .

4.3 Sizes tested and results

One tests the speed of convergence with the smoothness of the grid of the solution calculated towards the analytical solution in standard L_2 :

– greatest reality $\alpha_U > 0$ such as $\|\underline{U}^{\text{calc}} - \underline{U}^{\text{ref}}\|_{0,\Omega} < C_U \times h^{\alpha_U}$ where C_U is independent of h for displacement;

– greatest reality $\alpha_p > 0$ such as $\|p^{\text{calc}} - p^{\text{ref}}\|_{0,\Gamma_c} < C_p \times h^{\alpha_p}$ where C_p is independent of h for the contact pressure.

One tests also the sum of the absolute values of the difference between the calculated solution and the analytical solution for displacement.

Identification	Type of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	'NON_REGRESSION'	6.62799356621E-07
α_U	'ANALYTICAL'	3.0
α_p	'ANALYTICAL'	2.5
α_p	'NON_REGRESSION'	2.8035

5 Summary of the results

The results are in very good agreement with the theory.