



## SSNP131 - Identification of the energy parameter $G_p$ in 2D and 3D

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### Summary

This test of nonlinear quasi-static mechanics makes it possible to present the calculation of the parameter  $G_p$  resulting from the energy approach from the elastoplastic rupture and the identification from the breaking values corresponding to values of experimental tenacity given. It requires to represent the crack by a notch and to finely net the vicinity of the bottom of notch. It also requires to calculate elastic energy on the zone of virtual propagation of the notch, cut out in "chips".

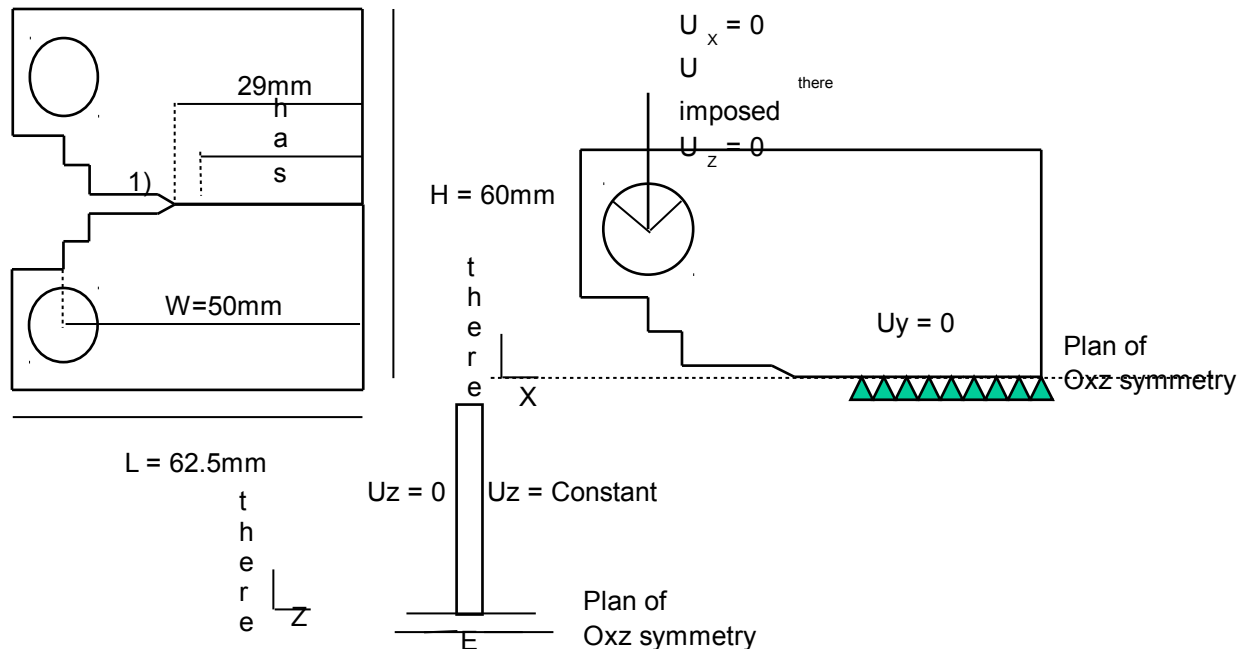
Modeling A is carried out with elements 2D quadratic, in plane deformation. The grid represents the zones known as chips; the calculation of the parameter is carried out by using the properties of the grid (POST\_GP and CALC\_GP) or by automatic creation of these zones (CALC\_GP).

Modeling B is identical to modeling A, but uses an unspecified grid; only the automatic definition of the chips is used (CALC\_GP).

Modeling C is carried out with elements 3D quadratic and the phase of prediction (one tests checks that the value of  $G_p$  is quite identical in 2D and 3D).

## 1 Problem of reference

### 1.1 Geometry



A test-tube is considered *CT25* with a length of ligament:  $a = 27.5\text{mm}$  ( $a/W = 0.55$ ). Along the axis  $z$ , the thickness is  $e = 1\text{mm}$ . The test-tube *CT25* is modelled first of all in plane deformations, then in 3D. By reason of symmetry, half of this one is represented in 2D and a quarter in 3D.

### 1.2 Material properties

Young modulus:  $214100\text{ Mpa}$

Poisson's ratio:  $\nu = 0.3$ . The traction diagram used is presented in the following table:

| $\epsilon$  | $\sigma$ (MPa) |
|-------------|----------------|
| 0.003439678 | 740.6632663    |
| 0.004628373 | 842.148772     |
| 0.00607988  | 876.3117064    |
| 0.007654628 | 895.2063119    |
| 0.010417548 | 911.0718694    |
| 0.014178015 | 925.022448     |
| 0.017543214 | 935.2135771    |
| 0.021942493 | 945.6948965    |
| 0.027416704 | 960.732311     |
| 0.033866984 | 975.8041996    |
| 0.040205805 | 988.2450325    |
| 0.046616375 | 1000.143035    |
| 0.052903597 | 1010.004051    |
| 0.058235889 | 1017.5664      |

Table 1.1

## 1.3 Boundary conditions and loadings

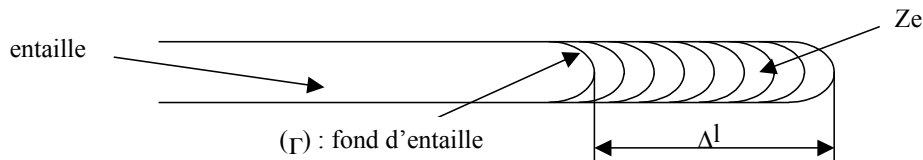
In 2D , the loading is of standard displacement imposed in a point located at the center of the pin which is modelled by four indeformable angular sectors. Half of the test-tube being modelled, a condition of symmetry is applied to the ligament (  $y=0$  ).

In 3D , the loading is of standard displacement imposed on the segments located on the axis of the pin which is modelled by four indeformable angular sectors. The quarter of the test-tube being modelled, a condition of symmetry is applied to the face corresponding to the ligament (  $y=0$  ), and another on the face  $z=0$  .

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution in 2D

One uses the energy method of the elastoplastic rupture based on the parameter  $G_p$  [1], [2]. The bottom of notch is made of a half-circle of ray  $R$ . The zone  $Z_e$  of length  $\Delta l$  corresponds to the virtual propagation of the notch and is cut out in "chips".



The evolution of the quantity at every moment there is determined  $G_p(\Delta l)$  defined by:

$$G_p(\Delta l) = 2[W_{elas}(\Delta l)]/\Delta l$$

where  $W_{elas}(\Delta l)$  is the elastic energy calculated on the zone  $Z_e$ . One must then calculate the maximum of this quantity compared to  $\Delta l$ , that one calls " $G_p$ ".

$$G_p = \underset{\Delta l}{Max}\{G_p(\Delta l)\}$$

The moment criticizes where the propagation of the defect will start is then that where tenacity  $K_j = K_{j_{crit}}$ . It is said whereas  $G_p$  reached the value criticizes " $G_{pc}$ ".

### 2.2 Method of calculating used for the reference solution in 3D

The bottom of notch is rectilinear. The face corresponding is described by slices. Each slice consists of chips. This zone corresponds to the virtual propagation of the notch.

One at every moment determines in each chip the evolution of the quantity  $G_p(\Delta S)$  defined by:

$$G_p(\Delta S) = 2[W_{elas}(\Delta S)]/\Delta S$$

where  $W_{elas}(\Delta S)$  is the elastic energy calculated on the zone cumulated of chip according to a slice and  $\Delta S$  is the surface of the chips cumulated of the bottom of notch to the chip concerned. Then one calculates on each slice the maximum of this quantity called " $G_p$ " compared to  $\Delta S$ .

$$(G_p)_{tranchei} = \underset{\Delta S}{Max}\{G_p(\Delta S)\}$$

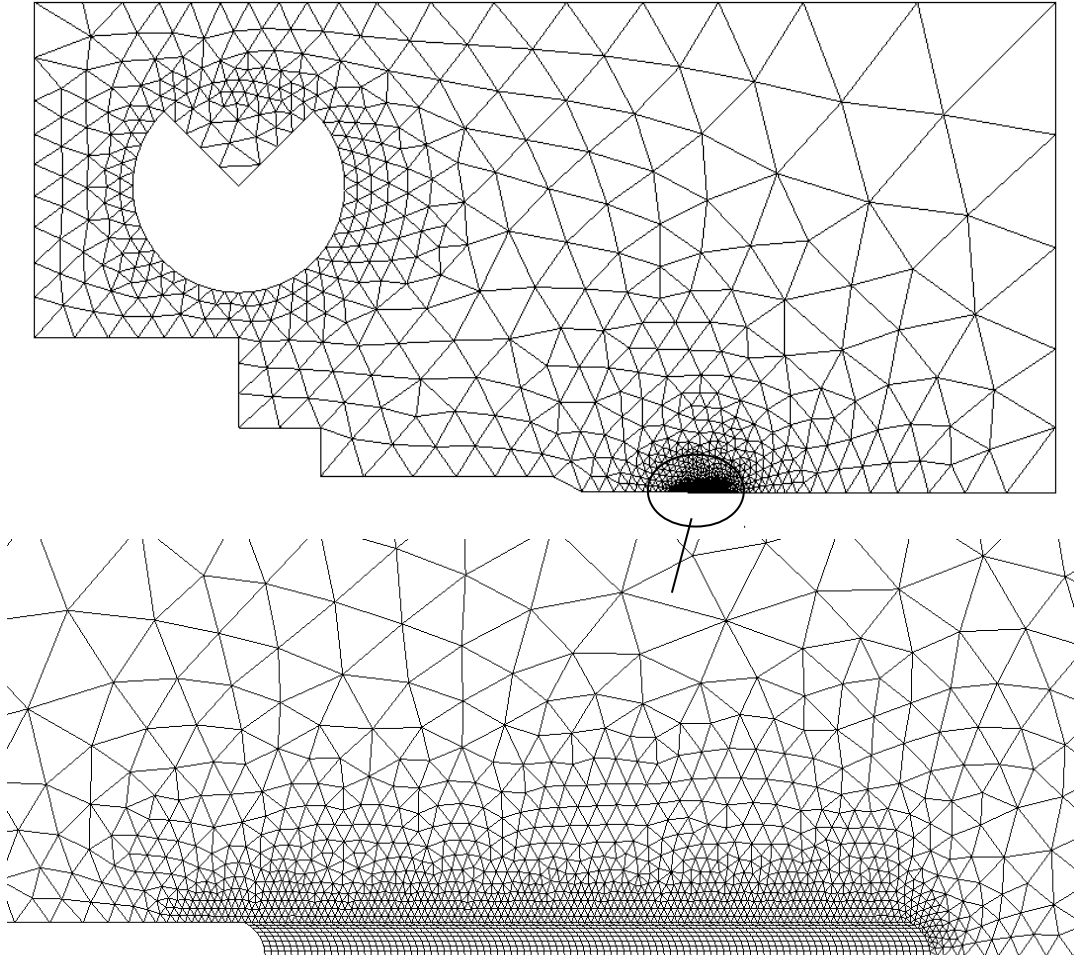
With each chip, a distance is associated  $\Delta L$  who corresponds to the interior distance from the chip at the bottom of notch.

### 2.3 Bibliographical references

- 1) WADIER Y.: "Brief presentation of the energy approach of the elastoplastic rupture applied to the rupture by cleavage", Notes EDF R & D HT-64/03/001/A, January 2003.
- 2) WADIER Y., LORENTZ E.: "Breaking process in the presence of plasticity: modeling of the crack by a notch". C.R.A.S.T. 332, 11b series, 2004.

## 3 Modeling A

### 3.1 Characteristics of modeling



The crack is modelled by a notch of ray 100 microns. The zone  $Z_e$  of 2 mm of length is divided into layers of 20 microns thickness elements (also called "chips").

### 3.2 Characteristics of the grid

Many nodes: 9368

Many meshes and types: 3350 TRIA6, 800 QUAD8

### 3.3 Sizes tested and results

#### 3.3.1 Values tested

With the operator `CALC_GP` and definition of the chips by the grid:

| Identification                 | Reference Aster | Tolerance (%) |
|--------------------------------|-----------------|---------------|
| $G_p$ at moment 4 with chip 8  | 0.023662413     | 0,010         |
| $G_p$ at moment 40 with chip 3 | 0.727738207     | 0,010         |

With the operator `CALC_GP` and automatic definition of the chips

| Identification                 | Reference Aster | Tolerance (%) |
|--------------------------------|-----------------|---------------|
| $G_p$ at moment 4 with chip 8  | 0.0234148559    | 0,010         |
| $G_p$ at moment 40 with chip 3 | 0.674949376     | 0,010         |

With the operator `POST_GP`

| Identification                      | Reference Aster | Tolerance (%) |
|-------------------------------------|-----------------|---------------|
| $G_{PC}$ such as $K_j(t) = K_j c_1$ | 0.678875        | 0,010         |
| $G_{PC}$ such as $K_j(t) = K_j c_2$ | 0.812359        | 0,010         |
| $G_{PC}$ such as $K_j(t) = K_j c_3$ | 0.929797        | 0,010         |

### 3.3.2 Notice

The results observed to make sure of the not-regression of the code are the breaking values of the energy parameter corresponding to the values of following critical tenacities:  $K_j c_1 = 27,2 \text{ MPa} \sqrt{m}$  ;  $K_j c_2 = 34 \text{ MPa} \sqrt{m}$  ;  $K_j c_3 = 40 \text{ MPa} \sqrt{m}$  .

With these values correspond of the critical loadings identified by calculating the quantity  $G$  by the method Theta which is connected to tenacity via the formula of Irwin:  $G = \frac{1-\nu^2}{E} K^2$  . The crowns chosen for the field Theta are:  $[0.25 \text{ mm}; 0.5 \text{ mm}]$  ,  $[0.5 \text{ mm}; 1.0 \text{ mm}]$  ,  $[1.0 \text{ mm}; 2.0 \text{ mm}]$  ,  $[2.0 \text{ mm}; 5.0 \text{ mm}]$  ,  $[5.0 \text{ mm}; 10.0 \text{ mm}]$  . With these critical loadings the breaking values of the parameter correspond  $G_p$  whose values are tested with the §3.3.1.

The macro-order `POST_GP` product two types of table:

- 1) A table under the produced concept of the macro-order containing for each moment of calculation:
  2. Young and Poisson's ratio modulus at the temperature of calculation,
  3.  $G$  and  $K_j$  deduced by Irwin for each crown  $i$  of calculation ( $G_i, K_i$ )
  4.  $G$  and  $K_j$  means ( $GMOY, KMOY$ )
  5. the index of chip where  $G_p$  is maximum (`ICOPMAX`)
  6. the distance compared to the bottom of notch where  $G_p$  is maximum (`DELTALMAX`)
  7. the value of  $G_p$  maximum (`GPMAX`)
- 2) A table containing the result of the identification (keyword `TABL_GPMAX`) for each critical tenacity:
  1. Critical tenacity  $K_j c_i$  ,
  2. The moment of calculation interpolated in the preceding table where  $K_j = K_j c_i$  ,
  3.  $G_p$  critical,  $G_{pc}$  , at the moment of interpolated calculation (`GP_CRIT`),
  4. Quantity  $K_{Gp_{max}}$  deduced by the relation from Irwin (`KGP_CRIT`),
  5. The distance to the bottom of notch where  $G_p = G_{pc}$  (`DELTA_CRIT`)





## 4 Modeling B

### 4.1 Characteristics of modeling

This modeling is two-dimensional, in plane deformations. One uses an initial grid of CT. The crack is modelled there by a notch of ray 100 microns, with a fairly fine grid. The initial grid comprises 2937 nodes and 1377 elements. It is visible on the Figure 4.1 and 4.2.

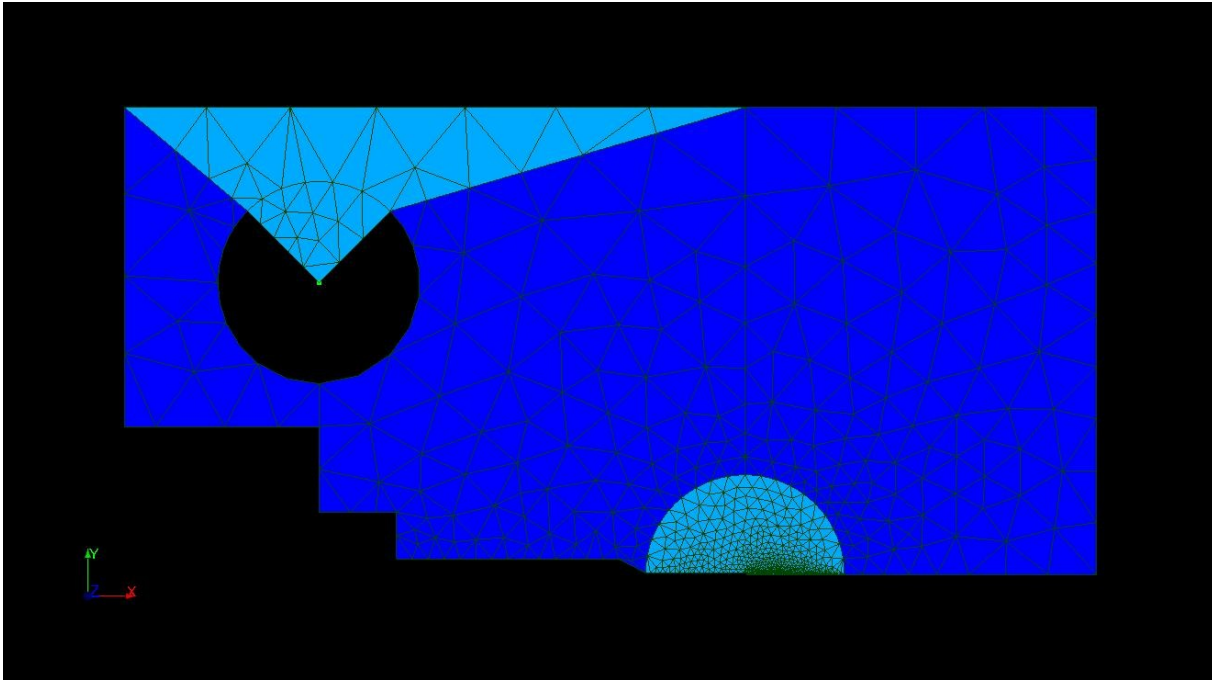


Figure 4.1 : Overall picture of the initial grid.

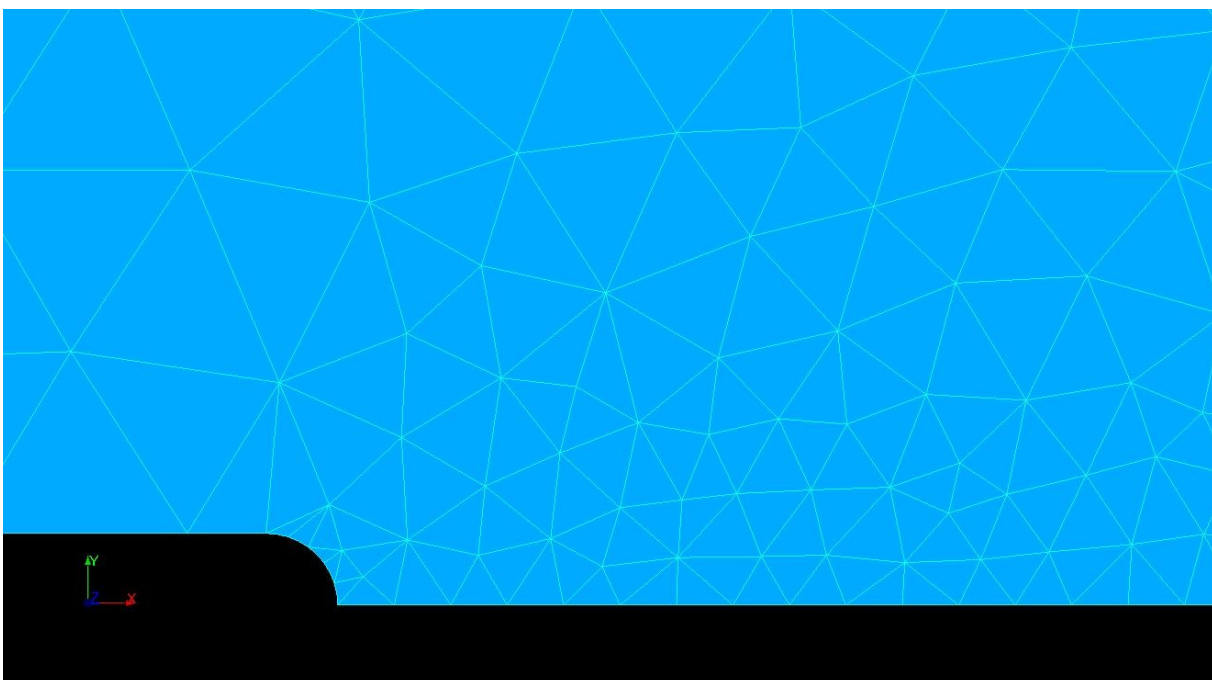


Figure 4.2 : Zoom on the notch of the initial grid.

The grid is then refined with order RAFF\_GP on a zone of 10 chips of 20 microns (either on 0,2mm). The Figure presents the grid finally obtained, comprising 4920 nodes for 1377 elements.

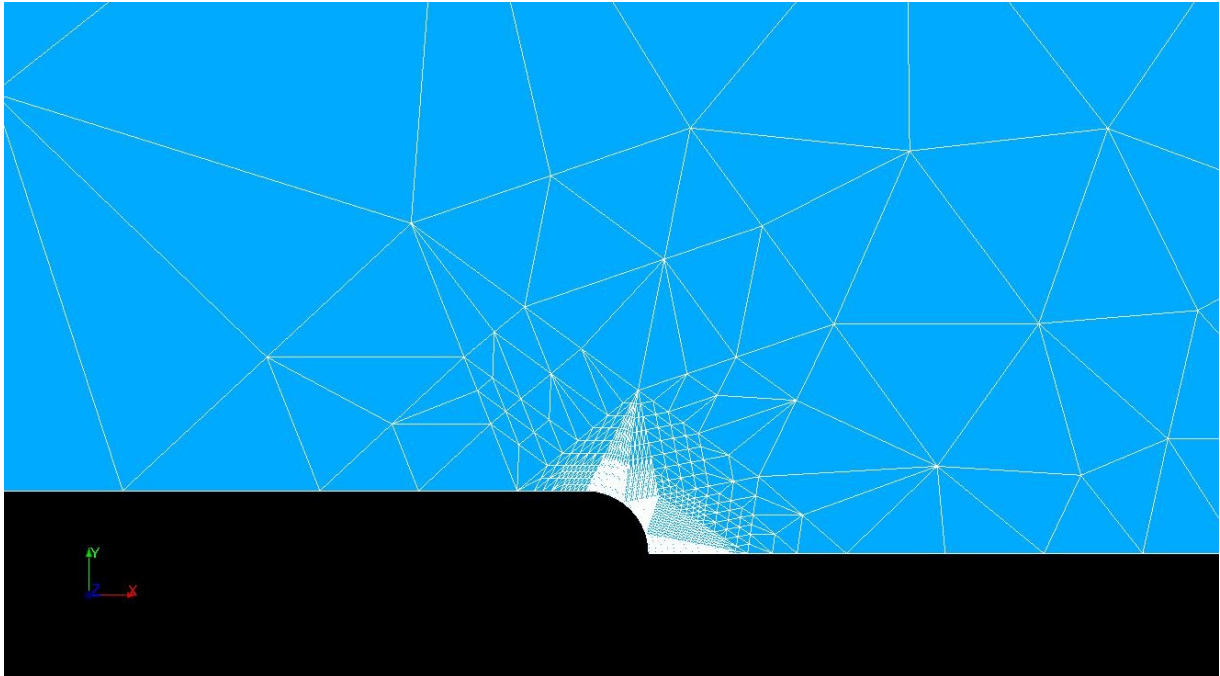


Figure 4.3 : Zoom on the notch of the refined grid.

## 4.2 Sizes tested and results

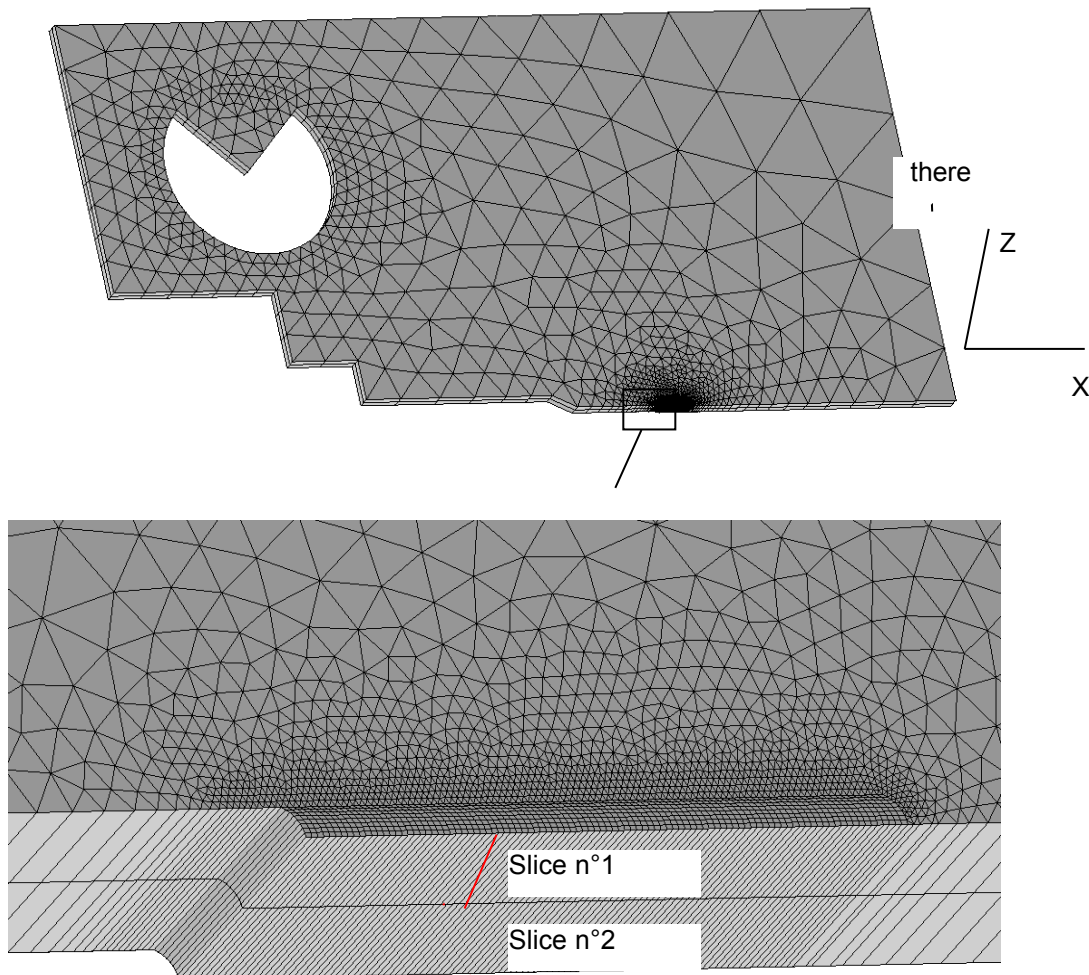
With the operator `CALC_GP` and automatic definition of the chips by the grid:

| Identification                 | Reference Aster | Tolerance (%) |
|--------------------------------|-----------------|---------------|
| $G_p$ at moment 40 with chip 3 | 0.7171792604019 | 0,010         |

The differences noticed between the various possibilities of calculation, although rather weak, are due with the geometry different from the zones of calculation and with the singularity created by using the geometry of the grid of modeling A.

## 5 Modeling C

### 5.1 Characteristics of modeling



The crack is modelled by a notch of ray 100 microns. The zone  $Z_e$  of 2 mm of length and 1 mm of width is divided into layers of 20 microns thickness elements, and of 0,5 mm of width. These layers are called "chips". In addition, the zone  $Z_e$  is divided into "parts" along the axis  $z$ .

### 5.2 Characteristics of the grid

Many nodes: 33322

Many meshes and types: 3544 SEG2, 10050 TRIA6, 4760 QUAD8, 1600 HEXA20 and 6700 PENTA15

### 5.3 Sizes tested and results

#### 5.3.1 Values tested

Here, one will not test the keyword "IDENTIFICATION" but the keyword "PREDICTION". One makes use of the value of  $G_{PC}$  such as  $K_j(t) = K_j c_1$  determined in 2D for the prediction. Calculation is carried out only until the moment 8s not to increase the duration of calculation

unnecessarily. This level of loading, the critical sizes are not reached, the rupture should not be announced, therefore the prediction must be to 0.

One also tests the value of  $G_p$  at this moment by reference to the value determined in 2D .

| Size tested       | Reference Aster 2D           | Tolerance (%) |
|-------------------|------------------------------|---------------|
| $G_p$ for INST=8s | 2,36E-001                    | 0,010         |
| PREDICTION        | 0 (the lower than $G_{PC}$ ) | 1,00E-008     |

## 5.3.2 Notice

One notes that in this CAS-test the law of behavior in CALC\_G (ELAS\_VMIS\_TRAC) differ law of behavior of STAT\_NON\_LINE (VMIS\_ISOT\_TRAC). This is due to the fact that one wants to calculate  $G$  by supposing that the loading is monotonous proportional. The use of the law VMIS\_ISOT\_TRAC in CALC\_G would have resulted in calculating the parameter  $GTP$  (see U2.82.03 documentation).

Results got in 3D are very close to modeling 2D in plane deformations. That makes it possible to validate the operation of the macro-order POST\_GP in 3D .

## 6 Summary of the results

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In both cases ( 2D and 3D ), the tests are validated with a lower deviation than 2nd-04%.  
One notes moreover one great coherence of the results 2D and 3D .