

## SSNP130 - Detection of the singularities in a fissured plate

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### Summary:

The goal of this test is to test the detection of the singularities in a plate fissured as well as the calculation of map of size (for an error targets given).

## 1 Problem of reference

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### 1.1 Geometry

The geometry is a fissured plate of which one models only the upper part.

### 1.2 Properties of material

The solid mass consists of a linear isotropic elastic material:

- Isotropic elasticity Young Modulus:  $E = 200000 \text{ Pa}$
- Poisson's ratio:  $\nu = 0,3$

### 1.3 Boundary conditions and loadings

One imposes a unit pressure on the top of the plate.

For reasons of symmetry, the ligament of the crack is blocked according to the axis  $Y$  and the node at the end of ligament is blocked according to the axis  $X$ .

## 2 Reference solution

### 2.1 Method of calculating

When the exact solution of the studied problem present of the singularities, the order of convergence of the solution finite elements is modified. Let us consider, for example, a problem of plane elasticity discretized with triangular elements of degree  $p$ .

If the exact solution  $U_{ex}$  is regular, one knows that ([bib1]):

$$\|u - u_h\|_{\Omega} = \|e\|_{\Omega} \leq C h^p \quad \text{éq 2.1-1}$$

With  $C$  a constant,  $h$  size of the elements.

Where  $\|e\|_{\Omega} \leq C h^p$  is the contribution to the error in energy, that is to say:

$$\|e\|_{\Omega} \leq \frac{1}{2} \int_{\Omega} \varepsilon(e_h) K \varepsilon(e_h) d\Omega \quad \text{éq 2.1-2}$$

On the other hand, if the exact solution presents a singularity, for example if, locally in the vicinity of a point  $M_0$ , the field of displacement is form (with  $r$  and  $\theta$  polar coordinates in the vicinity of the point  $M_0$ ):

$$U_{ex} = r^{\alpha} V(\theta) + W \quad \text{with } 0 < \alpha < 1 \quad \text{éq 2.1-3}$$

With  $V$  a function of  $\theta$  and  $W$  a constant.

Then, it is shown that [bib1]:

$$\|e_h\|_{\Omega} \leq C h^{\alpha} \quad \text{éq 2.1-4}$$

It results from it that the rate of convergence of the total error in energy becomes independent of the degree  $p$  finite elements used and it is the same of that of the measurement of the error (for example, if  $p=1$  or  $p=2$  then  $\alpha=1/2$  for a crack).

Thus, at a peak of crack the order of the singularity will be worth 0.5, and far from the singularity (where the solution finite element is regular the order of the singularity is worth  $p$  (1 for the linear elements, 2 for the quadratic elements)

### 2.2 Sizes and results of reference

One will test the value of the singularity at a peak of crack (analytical solution), in his vicinity (not-regression) and far from the crack (analytical solution).

One also tests the report of size to be applied to the grid for a target error (not-regression) and the new size of the elements (not-regression).

It is noted that for modeling A, the target error is the error (in quantity of interest) on average displacement on the structure. For other modelings, the error used is the error in standard of energy.

### 2.3 Uncertainty on the solution

Analytical solution and not-regression

### 2.4 Bibliographical references

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[1] STRANG & FIX: Year analysis of the finite element method, Prentice hall, 1976.

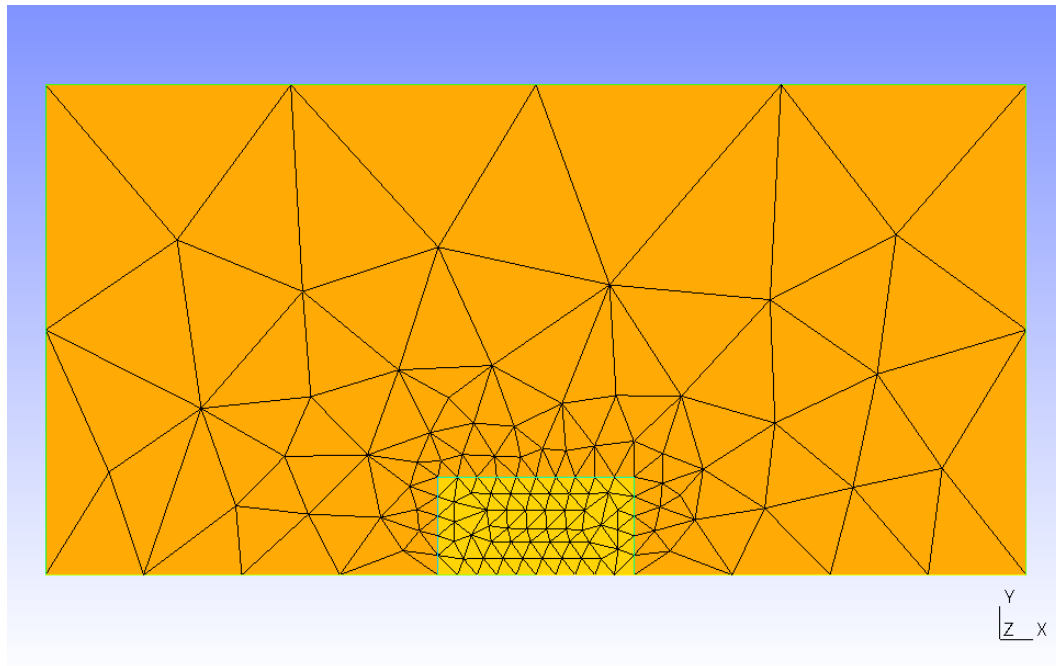
## 3 Modeling A

### 3.1 Characteristics of modeling

Modeling is two-dimensional rubber band with plane constraints C\_PLAN.

### 3.2 Characteristics of the grid

Many nodes	136	
Number of SEG2		46
Number of TRIA3		244



## 3.3 Sizes tested and results

Field SING\_ELEM :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	1.0	1.0E-04%
DEGREE MIN	1.0	'ANALYTICAL'	0.5	1.0E-04%
REPORT MAX	1.0	'NON_REGRESSION'	7.3626	0.20%
REPORT MIN	1.0	'NON_REGRESSION'	1.0310	0.20%
SIZE MAX	1.0	'NON_REGRESSION'	1.6918	0.20%
SIZE MIN	1.0	'NON_REGRESSION'	0.0295	0.20%

Identification	Moment	Type	Reference	Tolerance
M142 DEGREE	1.0	'NON_REGRESSION'	0,687	0.20%
M143 DEGREE	1.0	'NON_REGRESSION'	0.5	1.0E-04%
M144 DEGREE	1.0	'NON_REGRESSION'	0.5	1.0E-04%
M145 DEGREE	1.0	'NON_REGRESSION'	0.5	1.0E-04%
M146 DEGREE	1.0	'NON_REGRESSION'	0.5842	1.5%
M189 DEGREE	1.0	'NON_REGRESSION'	0.6870	0.20%
M190 DEGREE	1.0	'NON_REGRESSION'	0.6870	0.20%
M191 DEGREE	1.0	'NON_REGRESSION'	0.5842	1.5%

Field SING\_ELNO :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	1.0	1.0E-04%
DEGREE MIN	1.0	'ANALYTICAL'	0.5	1.0E-04%
REPORT MAX	1.0	'NON_REGRESSION'	7.3626	0.20%
REPORT MIN	1.0	'NON_REGRESSION'	1.0310	0.20%
SIZE MAX	1.0	'NON_REGRESSION'	1.6918	0.20%
SIZE MIN	1.0	'NON_REGRESSION'	0.0295	0.20%

## 4 Modeling B

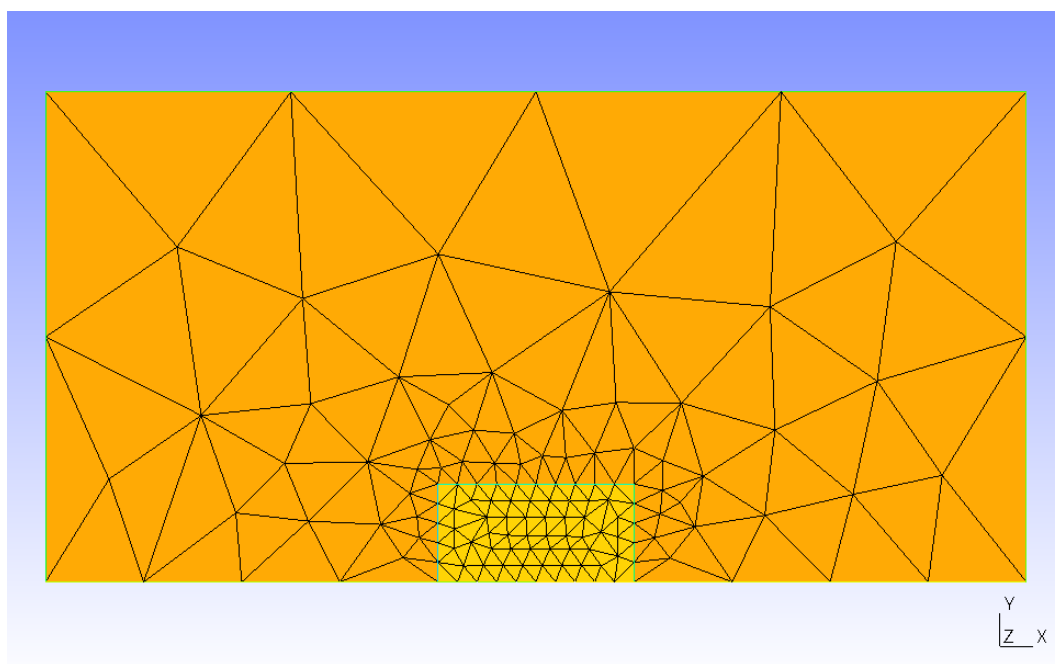
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### 4.1 Characteristics of modeling

Modeling is two-dimensional rubber band with plane constraints C\_PLAN.

### 4.2 Characteristics of the grid

Many nodes	515	
Number of SEG3		46
Number of TRIA6		244



## 4.3 Sizes tested and results

Field SING\_ELEM :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	2.0	1.0%
DEGREE MIN	1.0	'ANALYTICAL'	0.5	1.0%
REPORT MAX	1.0	'NON_REGRESSION'	2.52	2.0%
REPORT MIN	1.0	'NON_REGRESSION'	0.2150	2.0%
SIZE MAX	1.0	'NON_REGRESSION'	4.48	2.0%
SIZE MIN	1.0	'NON_REGRESSION'	0.0861	2.0%

Identification	Moment	Type	Reference	Tolerance
M142 DEGREE	1.0	'NON_REGRESSION'	0,750	1.0%
M143 DEGREE	1.0	'NON_REGRESSION'	0.5	1.0%
M144 DEGREE	1.0	'NON_REGRESSION'	0.5	1.0%
M145 DEGREE	1.0	'NON_REGRESSION'	0.5	1.0%
M146 DEGREE	1.0	'NON_REGRESSION'	0,550	2.0%
M189 DEGREE	1.0	'NON_REGRESSION'	0,750	1.0%
M190 DEGREE	1.0	'NON_REGRESSION'	0,750	1.0%
M191 DEGREE	1.0	'NON_REGRESSION'	0,550	2.0%
M192 DEGREE	1.0	'NON_REGRESSION'	0,550	2.0%
M193 DEGREE	1.0	'NON_REGRESSION'	0,550	2.0%

Field SING\_ELNO :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	2.0	1.0%
DEGREE MIN	1.0	'ANALYTICAL'	0.5	1.0%
REPORT MAX	1.0	'NON_REGRESSION'	2.52	2.0%
REPORT MIN	1.0	'NON_REGRESSION'	0,216	2.0%
SIZE MAX	1.0	'NON_REGRESSION'	4.48	2.0%
SIZE MIN	1.0	'NON_REGRESSION'	0.0860	0.20%



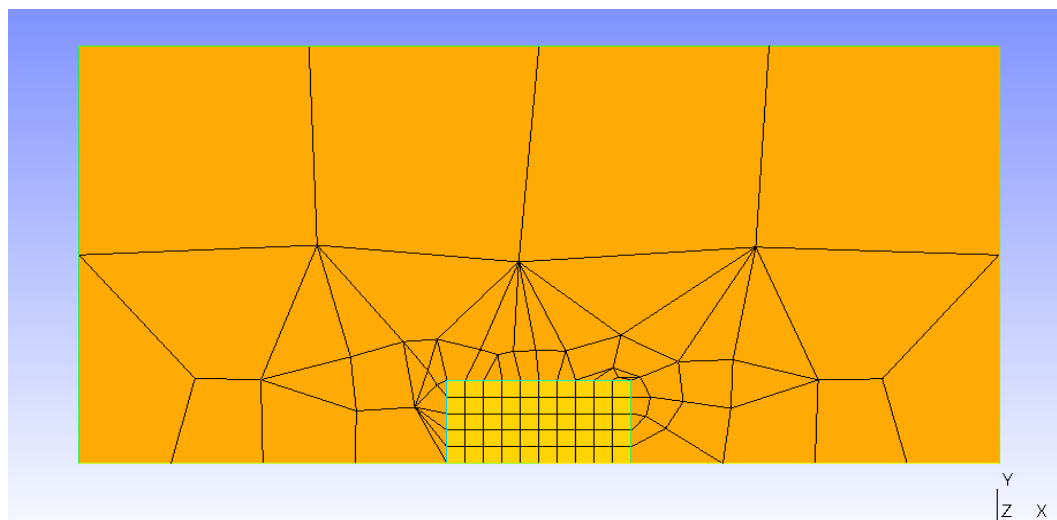
## 5 Modeling C

### 5.1 Characteristics of modeling

Modeling is two-dimensional rubber band with plane constraints C\_PLAN.

### 5.2 Characteristics of the grid

Many nodes	111	
Number of SEG2		46
Number of TRIA3		26
Number of QUAD4		84



## 5.3 Sizes tested and results

Field SING\_ELEM :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	1.0	1.0%
DEGREE MIN	1.0	'ANALYTICAL'	0.5	1.0%
REPORT MAX	1.0	'NON_REGRESSION'	2.17	2.0%
REPORT MIN	1.0	'NON_REGRESSION'	0,167	2.0%
SIZE MAX	1.0	'NON_REGRESSION'	4.59	2.0%
SIZE MIN	1.0	'NON_REGRESSION'	0.2	2.0%

Identification	Moment	Type	Reference	Tolerance
M30 DEGREE	1.0	'NON_REGRESSION'	0,750	1.0%
M31 DEGREE	1.0	'NON_REGRESSION'	0.5	1.0%
M32 DEGREE	1.0	'NON_REGRESSION'	0.5	1.0%
M55 DEGREE	1.0	'NON_REGRESSION'	0,780	1.0%
M56 DEGREE	1.0	'NON_REGRESSION'	0.6	1.0%
M57 DEGREE	1.0	'NON_REGRESSION'	0.6	1.0%

Field SING\_ELNO :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	1.0	1.0%
DEGREE MIN	1.0	'ANALYTICAL'	0.5	1.0%
REPORT MAX	1.0	'NON_REGRESSION'	2.17	2.0%
REPORT MIN	1.0	'NON_REGRESSION'	0,167	2.0%
SIZE MAX	1.0	'NON_REGRESSION'	4.59	2.0%
SIZE MIN	1.0	'NON_REGRESSION'	0,205	2.0%

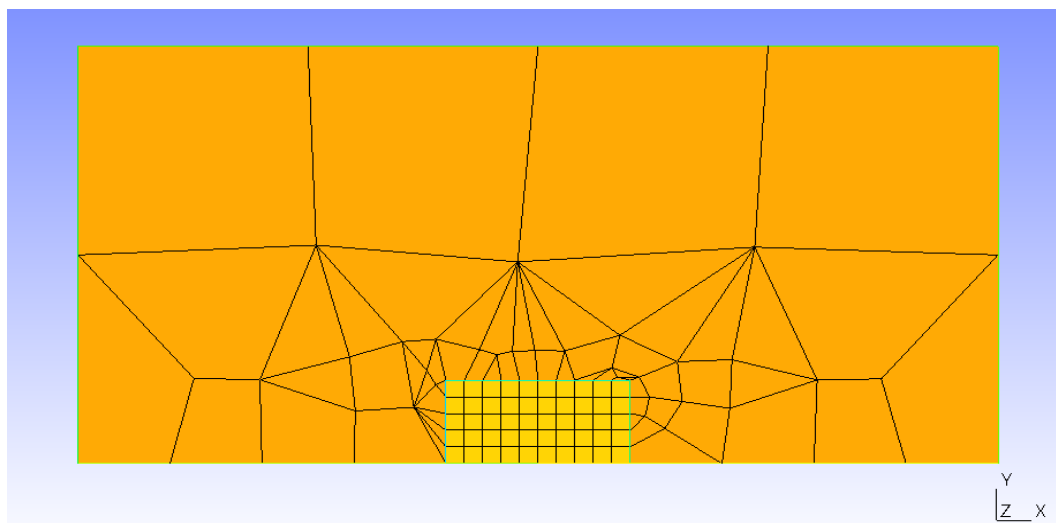
## 6 Modeling D

### 6.1 Characteristics of modeling

Modeling is two-dimensional rubber band with plane constraints C\_PLAN.

### 6.2 Characteristics of the grid

Many nodes	331	
Number of SEG3		46
Number of TRIA6		26
Number of QUAD8		84



## 6.3 Sizes tested and results

Field SING\_ELEM :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	2.0	1.0%
DEGREE MIN	1.0	'NON_REGRESSION'	0.51	1.0%
REPORT MAX	1.0	'NON_REGRESSION'	1.15	2.0%
REPORT MIN	1.0	'NON_REGRESSION'	0.16	2.0%
SIZE MAX	1.0	'NON_REGRESSION'	4.56	2.0%
SIZE MIN	1.0	'NON_REGRESSION'	0,259	2.0%

Identification	Moment	Type	Reference	Tolerance
M30 DEGREE	1.0	'NON_REGRESSION'	0.81	1.0%
M31 DEGREE	1.0	'NON_REGRESSION'	0.51	1.0%
M32 DEGREE	1.0	'NON_REGRESSION'	0.51	1.0%
M33 DEGREE	1.0	'NON_REGRESSION'	0,650	1.0%
M55 DEGREE	1.0	'NON_REGRESSION'	0.84	1.0%
M56 DEGREE	1.0	'NON_REGRESSION'	0.6	1.0%
M57 DEGREE	1.0	'NON_REGRESSION'	0.6	1.0%
M58 DEGREE	1.0	'NON_REGRESSION'	0.7	2.0%

Field SING\_ELNO :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	2.0	1.0%
DEGREE MIN	1.0	'NON_REGRESSION'	0.51	1.0%
REPORT MAX	1.0	'NON_REGRESSION'	1.15	2.0%
REPORT MIN	1.0	'NON_REGRESSION'	0.16	2.0%
SIZE MAX	1.0	'NON_REGRESSION'	4.56	2.0%
SIZE MIN	1.0	'NON_REGRESSION'	0,259	2.0%

## 7 Modeling E

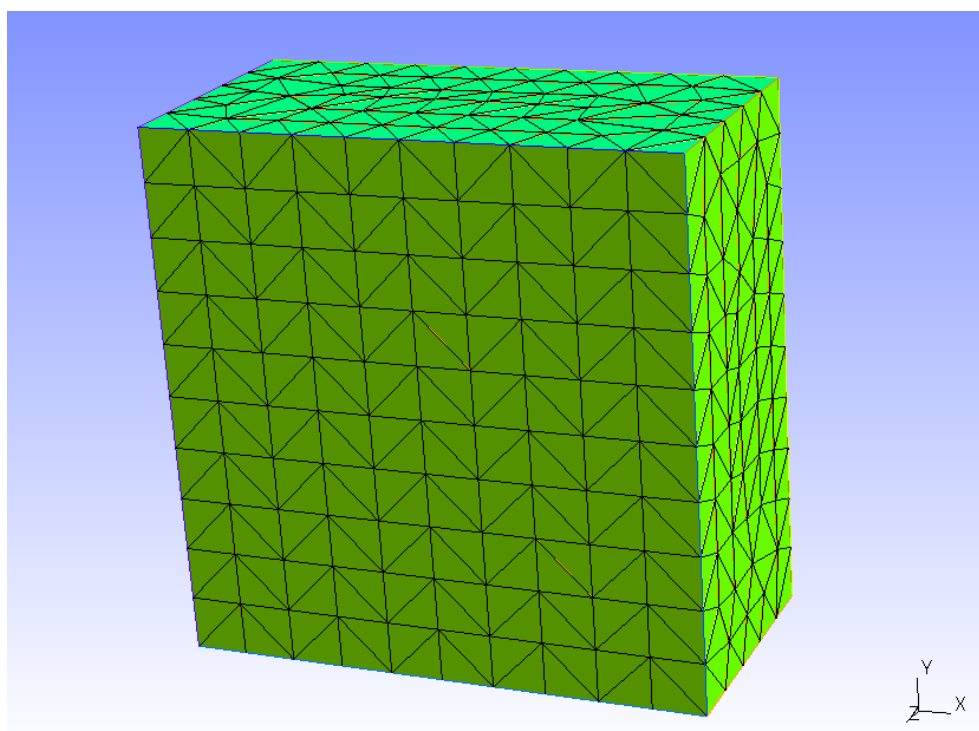
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### 7.1 Characteristics of modeling

Modeling is three-dimensional rubber band 3D.

### 7.2 Characteristics of the grid

Many nodes	796	
Number of SEG2		120
Number of TRIA3		868
Number of TETRA4		3261



## 7.3 Sizes tested and results

Field SING\_ELEM :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	1.0	1.0%
DEGREE MIN	1.0	'NON_REGRESSION'	0.56	1.0%
REPORT MAX	1.0	'NON_REGRESSION'	2.77	2.0%
REPORT MIN	1.0	'NON_REGRESSION'	0,155	2.0%
SIZE MAX	1.0	'NON_REGRESSION'	9.1	2.0%
SIZE MIN	1.0	'NON_REGRESSION'	0.44	2.0%

Field SING\_ELNO :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	1.0	1.0%
DEGREE MIN	1.0	'NON_REGRESSION'	0.56	1.0%
REPORT MAX	1.0	'NON_REGRESSION'	2.77	1.0%
REPORT MIN	1.0	'NON_REGRESSION'	0,155	1.0%
SIZE MAX	1.0	'NON_REGRESSION'	2.77	1.0%
SIZE MIN	1.0	'NON_REGRESSION'	0,155	1.0%

## 8 Modeling F

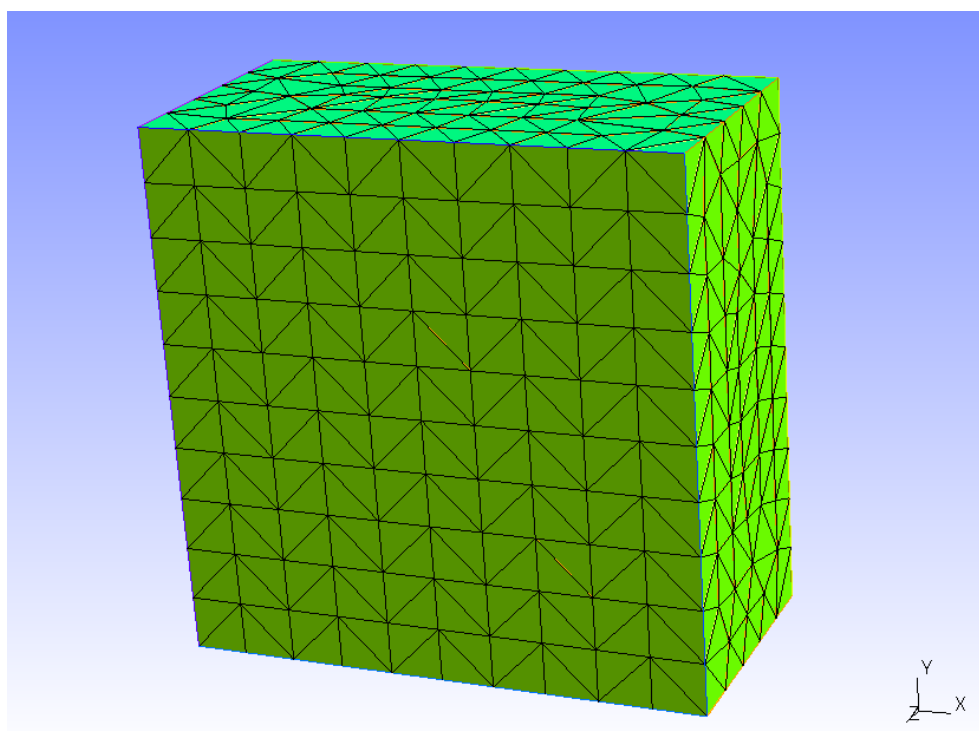
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### 8.1 Characteristics of modeling

Modeling is three-dimensional rubber band 3D.

### 8.2 Characteristics of the grid

Many nodes	5286	
Number of SEG3		120
Number of TRIA6		868
Number of TETRA10		3261



## 8.3 Sizes tested and results

Field SING\_ELEM :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	2.0	1.0E-04%
DEGREE MIN	1.0	'NON_REGRESSION'	0.5604	1.0E-04%
REPORT MAX	1.0	'NON_REGRESSION'	2.65	1.0E-04%
REPORT MIN	1.0	'NON_REGRESSION'	0.2169	1.0E-04%
SIZE MAX	1.0	'NON_REGRESSION'	6.5176	1.0E-04%
SIZE MIN	1.0	'NON_REGRESSION'	0.4940	1.0E-04%

Field SING\_ELNO :

Identification	Moment	Type	Reference	Tolerance
DEGREE MAX	1.0	'ANALYTICAL'	2.0	1.0E-04%
DEGREE MIN	1.0	'NON_REGRESSION'	0.5604	1.0E-04%
REPORT MAX	1.0	'NON_REGRESSION'	2.6526	1.0E-04%
REPORT MIN	1.0	'NON_REGRESSION'	0.2169	1.0E-04%
SIZE MAX	1.0	'NON_REGRESSION'	6.5176	1.0E-04%
SIZE MIN	1.0	'NON_REGRESSION'	0.4940	1.0E-04%



## 9 Summary of the results

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The got results are in coherence with the analytical solutions. They make it possible to validate this calculation in 2D and in 3D for linear and quadratic elements.