SSNP128 - Validation of the element with discontinuity on a plane plate

Summary:

The goal of this test is to display an analytical solution in order to validate the quality of the element with discontinuity (see documentation [R7.02.12] for details on this element). The objective of this test is to check that this model led to a good prediction of the value of the jump of displacement along a crack. With this intention, one seeks an analytical solution presenting a nonconstant jump along a discontinuity which one compares with the solution obtained numerically. In addition when one seeks to validate a digital method it is preferable to make sure of the unicity of the required solution. We will see that it is the case for the analytical solution presented if a condition relating to the maximum size of the field studied according to the parameters of the model is checked.

1 Problem of reference

1.1 Geometry

In the Cartesian frame of reference (x, y), let us consider an elastic rectangular plane plate noted $\Omega =]0,L[\times]0,H[$ (see [Figure 1.1-a]). Let us note $\Gamma_0=\{0\}\times]0,H[$ the left face of the field and $\partial\Omega\setminus\Gamma_0$ the part complementary to the edge.

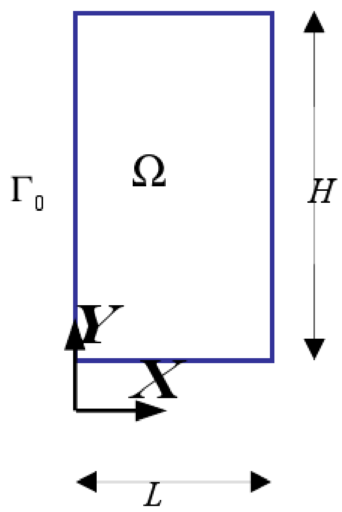

Figure 1.1-a.1-a: Diagram of the plate

Dimensions of the field Ω :

$$L=1\text{mm}, H=2\pi\text{ mm}$$

1.2 Material properties

The material is elastic with a critical stress and a tenacity arbitrarily chosen:

$$E=10\text{ MPa}, \nu=0, \sigma_c=1.1\text{ MPa}, G_c=0.9\text{ N.mm}^{-1}$$

1.3 Boundary conditions and loadings

The boundary conditions are determined by the analytical solution presented in the following part so that they lead to a crack having a nonconstant jump along Γ_0. The loading corresponds to a displacement imposed on the edges of the plate: (see [Figure 1.3-a]).

$$u=U(x,y) \quad \text{on } \Omega/\Gamma_0$$
$$u=U_0\delta(y)-(y) \quad \text{on } \Gamma_0$$

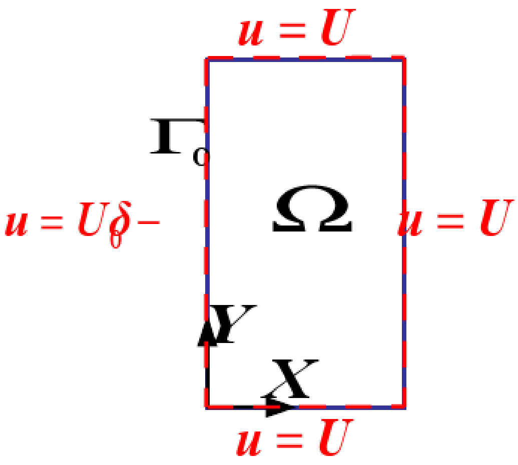

Figure 1.3-a: Diagram of the loading

Values U, U_0 and δ are defined during the construction of the reference solution in the following part.

2 Reference solution

In this part one displays an analytical solution with a nonconstant jump along Γ_0, then a condition of unicity of the solution is given.

2.1 Analytical solution

The function of Airy $\Phi(x, y)$ controlled by the equation $\Delta\Delta\Phi = 0$ known Ω R, if the external efforts are worthless, leads to constraints satisfying the compatibility and equilibrium equations in elasticity (see Fung [bib1]). Components of the constraint σ_{xx}, σ_{yy} and σ_{xy} derive from $\Phi(x, y)$ following way:

$$\sigma_{xx} = \frac{\partial^2 \Phi}{\partial y^2}, \ \sigma_{yy} = \frac{\partial^2 \Phi}{\partial x^2} \text{ and } \sigma_{xy} = \frac{\partial^2 \Phi}{\partial x \partial y} \qquad \text{éq 2.1-1}$$

Let us choose a function bi--harmonic $\Phi(x, y)$ defined by:

$$\Phi(x, y) = \beta \frac{y^3}{6} + (\alpha x + \gamma) \frac{y^2}{2} + \eta x y$$

with α, β, γ and η arbitrary real constants. One from of deduced according to [éq 2.1-1] the stress field:

$$\begin{cases} \sigma_{xx} & = & \alpha x + \beta y + \gamma \\ \sigma_{yy} & = & 0 \\ \sigma_{xy} & = & -\alpha y - \eta \end{cases} \qquad \text{éq 2.1-2}$$

By integrating the elastic law, if one notes E the Young modulus and ν the Poisson's ratio (which one takes no one), one from of deduced the field from displacement in Γ checking balance:

$$\boldsymbol{u} = \begin{Bmatrix} u(x, y) \\ v(x, y) \end{Bmatrix} = \begin{Bmatrix} \frac{1}{E}\left(\alpha\left(\frac{x^2}{2} - y^2\right) + x(\beta y + \gamma)\right) \\ -\frac{1}{E}\left(\beta \frac{x^2}{2} + 2\eta x\right) \end{Bmatrix} \qquad \text{éq 2.1-3}$$

Let us note respectively U_0 and U displacements on Γ_0 and $\partial\Omega \setminus \Gamma_0$ given by [éq 2.1-3]. The latter correspond to the boundary conditions leading to the stress fields [éq 2.1-2]. From these data, it is easy to build a field of displacement with a discontinuity on the edge Γ_0. Indeed, knowing the normal constraint σn on Γ_0 that one notes $F(y)$, the jump of displacement is obtained $\delta(y)$ by reversing the exponential law of behavior of Barenblatt type: `CZM_EXP` (see documentation on the elements with internal discontinuity and their behavior: [R7.02.12]):

$$\delta(y) = -\frac{G_c F(y)}{\sigma_c \|F(y)\|} \ln\left(\frac{\|F(y)\|}{\sigma_c}\right)$$

for all y in $[0, H]$. Thus, the new displacement imposed on Γ_0 generating such a jump is equal to $U_0 - \delta$. One thus built an analytical solution of the plane plate checking the equations of balance and compatibility with a discontinuity in Γ_0 along which the jump of displacement δ is not constant. Let us point out the boundary conditions of the problem:

$$\begin{cases} \boldsymbol{u}=U(x,y) & sur \quad \partial\Omega \setminus \Gamma_0 \\ \boldsymbol{u}=U_0(y)-\delta(y) & sur \quad \Gamma_0 \end{cases}$$ éq 2.1-4

2.2 Unicity of the solution

After having built an analytical solution it is important to make sure that the latter is single to be able to compare it with the digital solution. One shows, to see [bib2], that unicity is guaranteed as soon as the following condition, on the geometry of the field like on the parameters material, is checked:

$$L<2\mu\frac{G_c}{\sigma_c^2}.$$ éq 2.2-1

Dimensions of the plate and the parameters material previously given check this condition.

2.3 References bibliographical

[1] FUNG Y.C. : Foundation of Solid Mechanics, Prentice-Hall, (1979).

[2] LAVERNE J.: Energy formulation of the rupture by models of cohesive forces: digital considerations theoretical and establishments, Doctorate of the University Paris 13, November 2004.

3 Modeling A

3.1 Characteristics of modeling

The idea is to carry out a digital simulation corresponding to the problem presented in the preceding part and to compare the got results. The elements with discontinuity make it possible to represent the crack along Γ_0 . The latter have as a modeling PLAN_ELDI and a behavior CZM_EXP. The other elements of the grid are elastic QUAD4 in modeling D_PLAN.

The values of the parameters of the function of Airy for the construction of the analytical solution are taken arbitrarily:

$$\alpha=0\,\mathrm{MPa.mm^{-1}},\beta=1/4\pi\,\mathrm{MPa.mm^{-1}},\gamma=1/2\,\mathrm{MPa}\quad \mathrm{et}\quad \eta=0\,\mathrm{MPa}$$

3.2 Characteristics of the grid

One carries out a grid of the plate structured in quadrangles with 20 meshs in the width and 50 in the height. One has the elements with discontinuity along with dimensions one Γ_0 with the normal directed according to $-X$. This is carried out using the keyword CREA_FISS of CREA_MAILLAGE (see documentation [U4.23.02]).

3.3 Sizes tested and results

Size tested	Type	Reference	Tolerance ($\%$)
Variable threshold: $VI1$ On the element $MJ15$	'ANALYTICAL'	0.49315	0.10
Variable threshold: $VI1$ On the element $MJ45$	'ANALYTICAL'	1,075	0.10
Normal constraint: $VI6$ On the element $MJ30$	'ANALYTICAL'	0.4489	0.10

4 Summary of the results

These results enable us to conclude that the element with internal discontinuity led to a good approximation from the analytical solution. Moreover, one study on the dependence with the grid was carried out in [bib2]. It is noted that the error made on the jump of displacement decrease when the grid is refined. That makes it possible to conclude that, in spite of a constant jump by element, this model makes it possible to correctly reproduce a crack with a nonconstant jump by refining the grid.