FORMA10 - Practical works of the formation “advanced Use”: way of loading

Summary:

This test illustrates, on a material point, the influence of way of loading on the answer of an elastoplastic behavior. It highlights the effects of discretization in time. It takes as a starting point the test SSNP15: a material point, made up of a plastic material with linear isotropic work hardening, is subjected at the same time to a shearing and tractive effort. The principal interest of this test lies in the nonradial character of the loading.

Modeling A corresponds to calculation with imposed force, with the behavior VMIS_ISOT_LINE, and illustrates the influence of a coarse discretization in time by comparison with the solution obtained with a step of finer time. To obtain a solution with a step of finer time, one forces the recutting of the step of time on a criterion based on the increment of cumulated plasticity (modeling B).

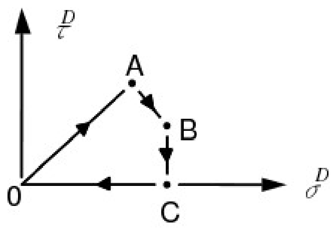
1 Problem of reference

1.1 Geometry

It is about a material point: homogeneous stress and strain state. This can be represented by only one finite element, with boundary conditions adapted, as in SSNP14 and SSNP15.

1.2 Boundary conditions and loadings

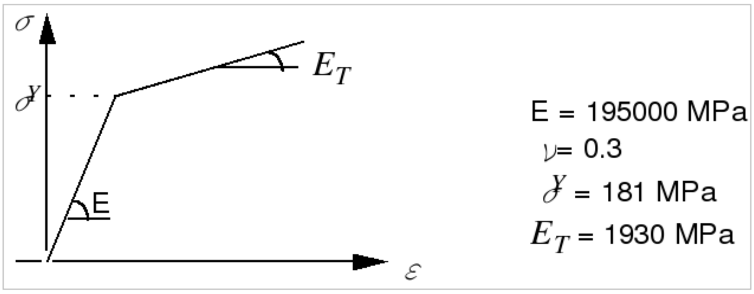
One imposes a way of loading defined by a component of traction and a component of shearing, in the following way:



	σ^D (MPa)	τ^D (MPa)
A	151.2	93.1
B	257.2	33.1
C	259.3	0

1.3 Properties of materials

The behavior is elastoplastic of Von Mises, with isotropic work hardening. Work hardening is linear.

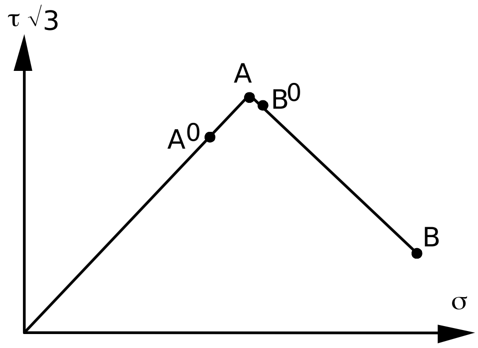


2 Reference solution

2.1 Method of calculating used for the reference solution

It is identical to that of test SSNP15.
In the plan $(\sigma, \tau\sqrt{3})$, the standard of von Mises results in the classical distance, so that one can immediately predict the phases of load and of discharge at the time of the way of loading, since it is respectively the phases where the standard grows or decrease:

Trajet de chargement



Valeurs numériques $(\sigma\ ;\ \tau)$		Phases de chargement	
A^0	(123,8 ; 76,23)	O-A^0	charge élastique
A	(151,2 ; 93,1)	A^0-A	charge plastique
B^0	(158,23 ; 89,12)	A-B^0	décharge
B	(257,2 ; 33,1)	B^0-B	charge plastique

2.1.1 Approach of resolution

Mechanically, it is about a test $0D$ controlled in constraint, the material being elastoplastic with criterion of von Mises and linear isotropic work hardening. For a loading controlled in constraint, one easily determines the cumulated plastic deformation:

$$F(\sigma, p) = \sigma_{eq} - \sigma^Y - R'p \le 0 \Rightarrow p = \frac{\sigma_{eq} - \sigma^Y}{R'} \text{ in load} \qquad \text{éq 2.1.1- 2.1.1-1}$$

The integration of the plastic deformation is of course more delicate. The equation of flow is written:

$$\dot{\varepsilon}^p = \frac{3}{2}\dot{p}\frac{\tilde{\sigma}}{\sigma_{eq}} \Rightarrow \dot{\varepsilon}^p = \frac{3}{2R'}\frac{\dot{\sigma}_{eq}}{\sigma_{eq}}\tilde{\sigma} \text{ in load} \qquad \text{éq 2.1.1- 2.1.1-2}$$

Lastly, one will deduce the deformation via the relation from state:

$$\varepsilon = \varepsilon^p + E^{-1} : \sigma \Rightarrow \varepsilon_{xx} = \varepsilon_{xx}^p + \frac{\sigma}{E} \text{ and } \varepsilon_{xy} = \varepsilon_{xy}^p + \frac{\tau}{2\mu} \qquad \text{éq 2.1.1- 2.1.1-3}$$

2.1.2 Treatment of the phase of radial loading

Let us notice that in phase of radial loading, the law of flow [éq 2.1.2-1] is integrated directly:

$$\boldsymbol{\varepsilon}^{\mathbf{p}}=\frac{3}{2}p\frac{\tilde{\boldsymbol{\sigma}}}{\sigma_{eq}} \qquad \text{éq 2.1.2-1}$$

The cumulated plastic deformation is then given by [éq 2.1.1-1], the plastic deformation by [éq 2.1.2-1] and the total deflection by [éq 2.1.1-3]. With:

E	$=195\ 000\ MPa$	2μ	$=150\ 000\ MPa$	R'	$=1\ 949,29\ Mpa$

One obtains:

$p(A)$	$=2,0547\ 10^{-2}$	$\varepsilon_{xx}^{p}(A)$	$=1,4054\ 10^{-2}$	$\varepsilon_{xx}(A)$	$=1,4830\ 10^{-2}$
		$\varepsilon_{xy}^{p}(A)$	$=1,2981\ 10^{-2}$	$\varepsilon_{xy}(A)$	$=1,3601\ 10^{-2}$

2.1.3 Treatment of the phase of nonradial loading

In the phase of nonradial loading $B0-B$, one can parameterize the way of constraint by:

$$\boldsymbol{\sigma}(q)=\boldsymbol{\sigma}^{\mathbf{B^0}}+q\underbrace{\left(\boldsymbol{\sigma}^{\mathbf{B}}-\boldsymbol{\sigma}^{\mathbf{B^0}}\right)}_{\text{direction fixe}} \quad \text{with } 0\leq q\leq 1 \qquad \text{éq 2.1.3-1}$$

As the way of loading remains confined in the traction-shearing plan (σ,τ), one will may find it beneficial to represent the state of stress by a complex number:

$$\Sigma=\sigma+i\sqrt{(3)}\tau\Rightarrow\sigma_{eq}=|(\Sigma)| \quad \text{and} \quad \Sigma(q)=\Sigma^{B^0}+q\underbrace{\left(\Sigma^{B^0}-\Sigma^{B}\right)}_{\text{direction fixe}} \qquad \text{éq 2.1.3-2}$$

The integration of the law of flow [éq 2.1.1-2], followed by an integration by part, makes it possible to express the plastic deformation:

$$\frac{2R'}{3}\left[\boldsymbol{\varepsilon}^{\mathbf{p}}\right]_0^1=\int_0^1\frac{\dot{\sigma}_{eq}}{\sigma_{eq}}\tilde{\boldsymbol{\sigma}}\,dq=\left[\ln\left(\sigma_{eq}\right)\tilde{\boldsymbol{\sigma}}\right]_0^1-\frac{1}{2}\underbrace{\dot{\tilde{\boldsymbol{\sigma}}}}_{\tilde{\sigma}^B-\tilde{\sigma}^{B^0}}\int_0^1\ln\left(\sigma_{eq}^2\right)dq$$

The adoption of the complex plan allows an easy calculation of the last integral:

$$\int_0^1\ln\left(\sigma_{eq}^2\right)dq=\int_0^1\ln\left(\Sigma\,\overline{\Sigma}\right)dq=\int_0^1\ln(\Sigma)\,dq+\int_0^1\ln\left(\overline{\Sigma}\right)dq=2\Re\left[\int_0^1\ln(\Sigma)\,dq\right]=2\Re\left[\frac{\Sigma\ln(\Sigma)-\Sigma}{\Sigma^B-\Sigma^{B^0}}\right]_0^1$$

Finally, the increment of plastic deformation on the way $B0-B$ is worth:

$$\left[\boldsymbol{\varepsilon}^{\mathbf{p}}\right]_{B^0}^{B}=\frac{3}{2R'}\left[\ln\left(\sigma_{eq}\right)\tilde{\boldsymbol{\sigma}}\right]_{B^0}^{B}-\frac{3}{2R'}\Re\left[\frac{\Sigma\ln(\Sigma)-\Sigma}{\Sigma^B-\Sigma^{B^0}}\right]_{B^0}^{B}\left(\tilde{\sigma}^B-\tilde{\sigma}^{B^0}\right) \qquad \text{éq 2.1.3-4}$$

2.2 Results of reference

By calculating the plastic deformation cumulated by [éq 2.1.1-1], the plastic deformation by [éq 2.1.3-4] and the total deflection by [éq 2.1.1-3], one obtains:

$p(B)$	$=4,2329\ 10^{-2}$	$\varepsilon_{xx}^{p}(B)$	$=3,3946\ 10^{-2}$	$\varepsilon_{xx}(B)$	$=3,5265\ 10^{-2}$
One obtains:		$\varepsilon_{xy}^{p}(B)$	$=2,0250\ 10^{-2}$	$\varepsilon_{xy}(B)$	$=2,0471\ 10^{-2}$

One will be interested in the values of the constraints, the deformations and the plastic deformation cumulated at the points A and B way of loading.

2.3 Bibliographical references

1) French company of the Mechanics. Guide of validation of the software packages of structural analysis (VPCS). Technical AFNOR, 1990.

3 Modeling A

3.1 Characteristics of modeling

It is about a test on material point. One uses for that the order SIMU_POINT_MAT, which allows calculation on only one element, with only one point of integration.

The way of loading enters the point A and the point B is discretized in 5 pas de time.

3.2 Sizes tested and results

Identification	Moments	Reference	*Aster*	$\%$ difference
ε_{xx}	A	$1.4830\ 10^{-2}$	$1.48297\ 10^{-2}$	– 0,002
ε_{xy}	A	$1.3601\ 10^{-2}$	1.360110^{-2}	0,003
ε_{xx}	B	$3.5265\ 10^{-2}$	$3.5686\ 10^{-2}$	1.2
ε_{xy}	B	$2.0471\ 10^{-2}$	$1.9577\ 10^{-2}$	– 4.3

4 Modeling B

4.1 Characteristics of modeling

It is about a test on material point. One uses for that the order `SIMU_POINT_MAT`, which allows calculation on only one element, with only one point of integration.

The way of loading enters the point A and the point B is discretized in 1 pas de time. But one uses a rather recent functionality which makes it possible Re-to cut out the step of time if an additional criterion is not checked with convergence. Here, one proposes to check that with each convergence, the increment of cumulated plastic deformation does not exceed by 0.2 10^{-2}. As soon as this criterion is not satisfied, then the automatic code Re-cutting the step of time and starts again until this criterion is satisfied.
As an indication, one thus carries out 25 calculations between the point A and the point B , this temporal discretization is thus described as fine, compared with that used with modeling A.
It is pointed out that the point A corresponds to the moment $t=1\text{s}$ and the point B corresponds to the moment $t=2\text{s}$.

4.2 Sizes tested and results

Identification	Moments	Reference	% difference
ε_{xx}	A	1.4830 10^{-2}	– 0,002
ε_{xy}	A	1.3601 10^{-2}	0,003
ε_{xx}	B	3.5265 10^{-2}	0,39
ε_{xy}	B	2.0471 10^{-2}	1,1

One can note graphically on Figure 4.2-1 the difference on the deformation ε_{xy} between a fine temporal discretization (modeling B) and a coarse temporal discretization (5 pas de time, modeling A). This difference exists only for the nonradial part of the loading (beyond the point A). It is seen clearly that for modeling B, the step of time becomes rather small with the approach of the point B , right before the moment $t=2\text{s}$.

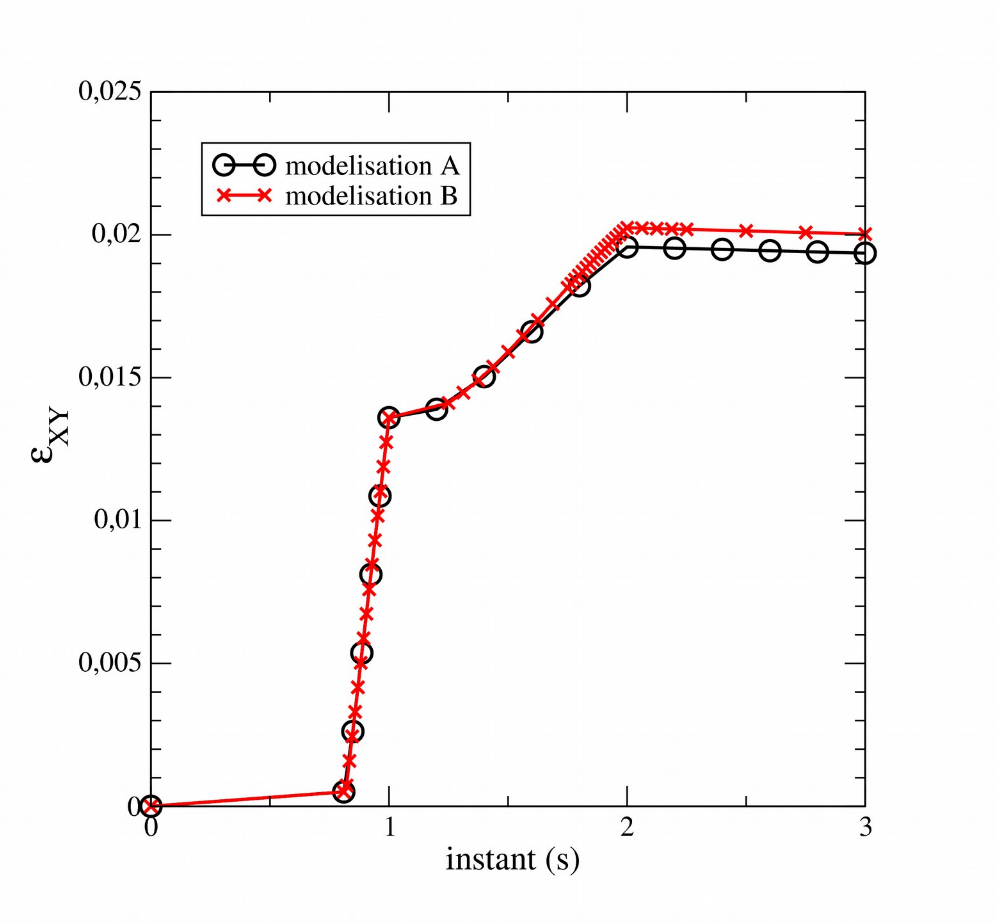


Figure 4.2-1: comparison between various temporal discretizations

5 Summary of the results

This test makes it possible to highlight, on a nonradial elastoplastic problem, the influence of the discretization in time.