

SSNL501 - Beam fixed at the two ends subjected to a uniform pressure

Summary:

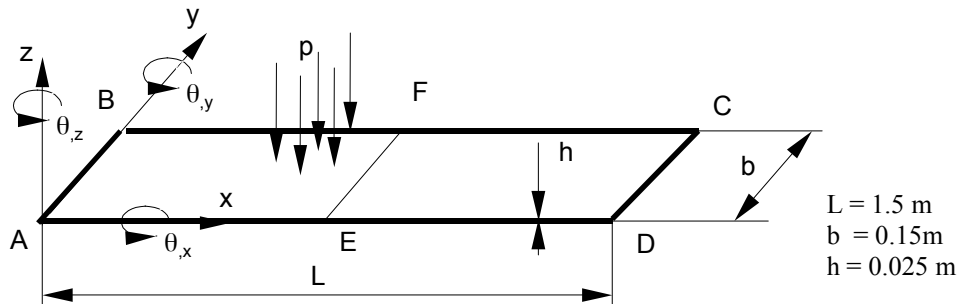
This test represents a quasi-static calculation of a fixed beam subjected to a uniform pressure, made up of a perfectly plastic elastic material. This test makes it possible to validate following modelings finite elements:

- COQUE_C_PLAN (SEG3),
- DKT (TRIA3, QUAD4),
- COQUE_3D (TRIA7, QUAD9),
- POU_D_TGM (SEG2).

The limiting pressure is compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



1.2 Properties of material

The properties of material constituting the beam are:

$E = 2.10^{11} \text{ Pa}$	Young modulus
$\nu = 0.3$	Poisson's ratio

The material follows a perfectly plastic law of elastic behavior:

$\sigma_e = 2.3510^8 \text{ Pa}$	Yield stress
$\varepsilon_e = 1.17510^{-3}$	Limiting elastic strain

1.3 Boundary conditions and loadings

- Boundary conditions: Dimensioned AB and CD embedded
- Following imposed displacement Z in E ($x = L/2$):

$$DZ_e = 6.60910^{-3} \text{ m} \left(DZ_e = \frac{q_e L^4}{384EI} \right)$$

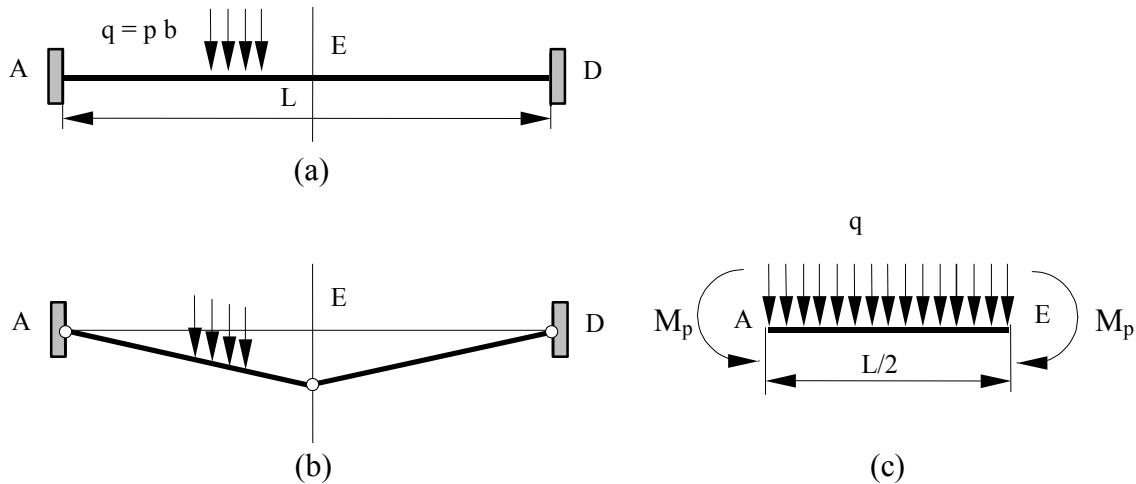
$$DZ(E) \text{ vary } 0 \text{ with } 30 DZ_e$$

1.4 Initial conditions

Without object

2 Reference solution

2.1 Method of calculating used for the reference solution



The ruin of the beam appears when there are plastic kneecaps at the points A , D and E (figure b). The static balance of the left half of the beam, makes it possible to determine the pressure limits (figure c)

$$\sum M_A = 2M_p - q_L \left(\frac{L}{2}\right) \left(\frac{L}{4}\right) = 0 \quad \Rightarrow \quad q_L = \frac{16M_p}{L^2}$$

where:

q_L represent the limiting pressure

M_p represent the plastic moment ($M_p = \sigma_e \frac{bh^2}{4}$)

The appearance of the first plastic point on fibre external of the beam takes place at the points A and D , other fibres being in elastic mode. The pressure limits elastic is of $q_e = 2\sigma_e \frac{bh^2}{L^2}$.

2.2 Results of reference

Limiting pressure $q_L = 39\,166.67 \text{ N/m}$

2.3 Uncertainties on the solution

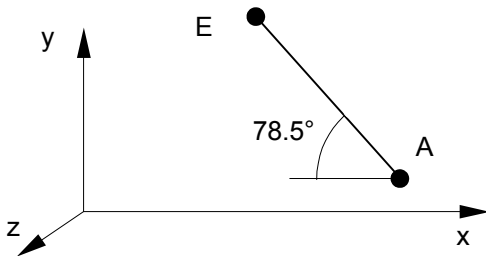
Analytical solution

2.4 Bibliographical references

- 1) WILLIAM A. NASH: Theory and problems of Strength of material, Schaum' S outline series, 2/ed, McGRAW-HILL

3 Modeling A

3.1 Characteristics of modeling



Modélisation COQUE_C_PLAN

Point A (2.1 ; 0.7) (nœud n1)

Conditions aux limites : Point A : $u = v = \theta_z = 0$

Conditions de symétrie : Point E : $\theta_z = 0$

la poutre étant inclinée à 78.5° , la valeur du déplacement imposé est alors : $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(78.5^\circ)$

3.2 Characteristics of the grid

Many nodes: 21

Number of meshes and type: 10 SEG3

3.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	Aster	% difference
10	ETA_PILOTAGE	10	1.0	1.0699	6.9
12	ETA_PILOTAGE	12	1.0	1.0699	6.9

3.4 Remarks

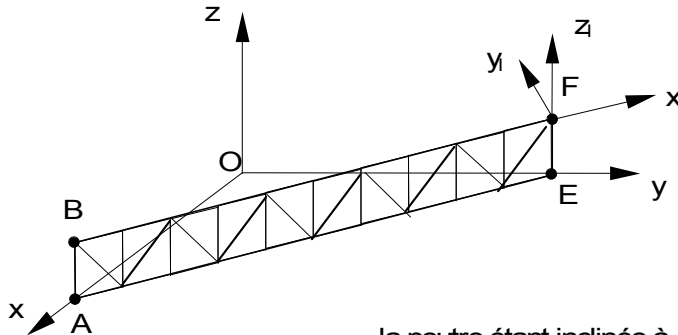
The width for modeling COQUE_C_PLAN is imposed on 1 in Code_Aster. Consequently, we multiplied by $1/b$ the loading to take account of the real width of the beam.

In this analysis one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.

The reference being taken compared to the solution beam with 20 elements, one observes an improvement of the results COQUE_C_PLAN when the grid is refined.

4 Modeling B

4.1 Characteristics of modeling



Modélisation DKT (TRIA3)

$$OE = OA = L / 2\sqrt{2}$$

AB // EF // axe Z

Conditions aux limites (repère global)

- côté AB: $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- côté EF: $u = 0$ (repère local $x_1 y_1 z_1$)

- côté EF: $\theta_z = 0$ (repère global)

la poutre étant inclinée à 45° , la valeur du déplacement imposé est alors : $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(45^\circ)$

4.2 Characteristics of the grid

Many nodes: 43

Number of meshes and type: 20 TRIA3

4.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	Aster	% difference
5	ETA_PILOTAGE	5	1.0	1.11133	11.11
25	ETA_PILOTAGE	15	1.0	1,142	11.42

4.4 Remarks

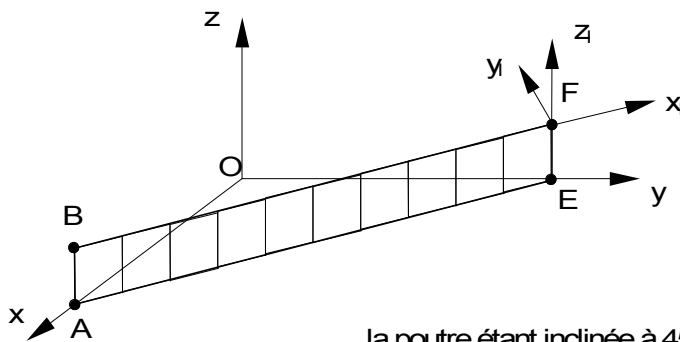
In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.

Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of the results DKT TRIA3 when the grid is refined.

5 Modeling C

5.1 Characteristics of modeling



Modélisation DKT (QUAD4)

$$OE = OA = L / 2\sqrt{2}$$

AB // EF // axe Z

Conditions aux limites (repère global)

- côté AB: $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- côté EF: $u = 0$ (repère local $x_1 y_1 z_1$)

- côté EF: $\theta_z = 0$ (repère global)

la poutre étant inclinée à 45° , la valeur du déplacement imposé est alors : $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(45^\circ)$

5.2 Characteristics of the grid

Many nodes: 43

Number of meshes and type: 10 QUAD4

5.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	Aster	% difference
5	ETA_PILOTAGE	5	1.0	1.0837	8.37
25	ETA_PILOTAGE	25	1.0	1.0998	9.98

5.4 Remarks

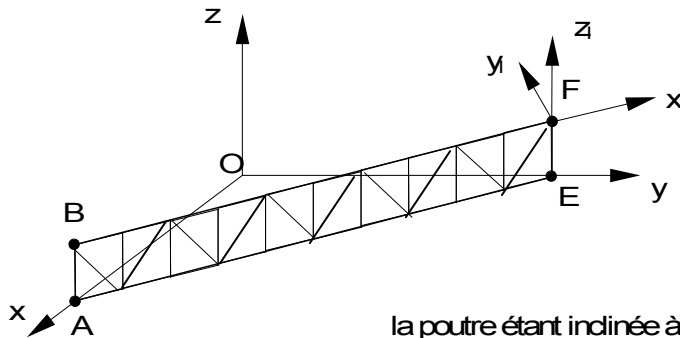
In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.

Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of the results DKT QUAD4 when the grid is refined.

6 Modeling D

6.1 Characteristics of modeling



Modélisation COQUE_3D (TRIA7)

$$OE = OA = L / 2\sqrt{2}$$

AB // EF // axe Z

Conditions aux limites (repère global)

- côté AB: $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- côté EF: $u = 0$ (repère local $x_1 y_1 z_1$)

- côté EF: $\theta_z = 0$ (repère global)

la poutre étant inclinée à 45°, la valeur du déplacement imposé est alors : $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(45^\circ)$

6.2 Characteristics of the grid

Many nodes: 83

Number of meshes and type: 20 TRIA7

6.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	Aster	% difference
5	ETA_PILOTAGE	5	1.0	1.1143	11.43
15	ETA_PILOTAGE	15	1.0	1.1682	16.82

6.4 Remarks

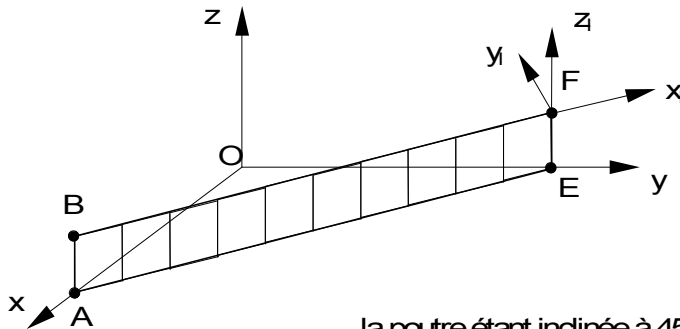
In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.

Calculations were stopped when the value of the parameter `ETA_PILOTAGE` stabilized itself.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of the hulls results 3D TRIA7 when the grid is refined.

7 Modeling E

7.1 Characteristics of modeling



Modélisation COQUE_3D (QUAD9)

$$OE = OA = L / 2\sqrt{2}$$

AB // EF // axe Z

Conditions aux limites (repère global)

- côté AB: $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- côté EF: $u = 0$ (repère local $x_1 y_1 z_1$)

- côté EF: $\theta_z = 0$ (repère global)

la poutre étant inclinée à 45° , la valeur du déplacement imposé est alors : $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(45^\circ)$

7.2 Characteristics of the grid

Many nodes: 54

Number of meshes and type: 10 QUAD9

7.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	Aster	% difference
5	ETA_PILOTAGE	5	1.0	1.0978	9.78
25	ETA_PILOTAGE	25	1.0	1.1085	10.85

7.4 Remarks

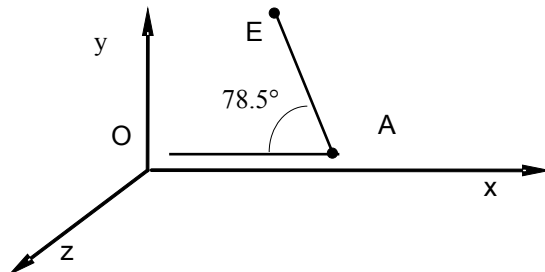
In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.

Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself.

The reference being taken compared to the solution beam with 50 elements, one observes an improvement of the hulls results 3D QUAD9 when the grid is refined.

8 Modeling F

8.1 characteristics of modeling



Modélisation POU_D_TGM

A= 2.1 0.7 0.

AE =0.75 m

Conditions aux limites (repère global)

- noeud A: $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Conditions de symétrie

- noeud E: $u = 0, \theta_z = 0$

la poutre étant inclinée à 78.5° , la valeur du déplacement imposé est alors : $DX_e = 6.609 \cdot 10^{-3} \text{ m} \cdot \sin(78.5^\circ)$

8.2 characteristics of the grid

- grid of the beam

Many nodes: 21

Number of meshes and type: 10 SEG2

- grid of the section



Many nodes: 355

Number of meshes and type: 280 QUAD4

8.3 Values tested

$DX(E)/DX_e$	Identification	Moments	Reference	% Tolerance
5	ETA_PILOTAGE	10	1.0	14.
5.24	ETA_PILOTAGE	12	1.0	14.

In this analysis, one uses to find the solution, a technique of resolution of type imposed displacement ('DDL_IMPO'). This method provides for each value of imposed displacement, a multiplying coefficient of the loading ('ETA_PILOTAGE'). The value of the loading imposed in 'AFFE_CHAR_MECA' is equal to the limiting pressure, consequently the value of reference of the parameter 'ETA_PILOTAGE' is equal to 1.

Calculations were stopped when the value of the parameter ETA_PILOTAGE stabilized itself.

The solution beam improves appreciably when the grid is refined.

9 Summary of the results

Concerning the evolution of normal displacement in the center of the plate according to the parameter of piloting, one observes that:

- modelings comprising of the quadrangles give better results, compared to the meshes triangles.

Calculations were stopped when the value of the parameter `ETA_PILOTAGE` stabilized itself, or when calculation was not possible any more. Taking into account the grids used, the got results are satisfactory. The errors observed are for modelings:

- `COQUE_C_PLAN` : 6.9% (With),
- `DKT` : 11.4% for the mesh `TRIA3` (B) and 9.9% for the mesh `QUAD4` (C),
- `COQUE_3D` : 16% for the mesh `TRIA7` (D) and 10.8% for the mesh `QUAD9` (E),
- `POUT_D_TGM` : 9% (F).

But it is noted that with a finer grid at the ends and the center of the plate, place or plasticization appears, it is possible to minimize the error compared to the reference solution.