

SSNL138 - Validation of the algorithm of optimization under constraint of inequalities of the option DDL_STAB

Summary:

This test allows the validation of the option DDL_STAB of CRIT_STAB, which evaluates the stability of the states of balance found by the digital simulation of the nonconservative problems like the problems of damage. What requires to apply an algorithm of optimization under constraints of inequalities. The option can be applied to a list containing any degree of freedom available in *Code_Aster*.

1 Problem of reference

1.1 Tally theoretical

One defines the unicity of the solution of a problem of discretized damage, by the positivity of the following quotient, written starting from the tangent operator K :

$$\text{Min}_{x=(u,a)} \left(\frac{x^T \cdot Kx}{x^T \cdot x} \right) > 0 \quad \text{Equation 1.1}$$

where u indicate the degrees of freedom of displacement and has degrees of freedom of damage. When this criterion is not checked any more, it can exist several solutions with the problem discretized by the finite element method, which consists in checking the conditions first of balance. It is then necessary to discuss the stability of the solution, by checking the positivity of the derivative second of energy, in the direction of the increasing damages (condition of irreversibility on the damage):

$$\text{Min}_{x=(u,a \geq 0)} \left(\frac{x^T \cdot Kx}{x^T \cdot x} \right) \geq 0 \quad \text{Equation 1.2}$$

The unicity and the stability of the homogeneous solution of a bar in traction were studied analytically (Pham, Amor, Marigo and Maurini, "Gradient ramming models and to their uses to approximate brittle fracture", 2009) and the criteria were written like relationship between the damage has bar, its length L and the internal length l . One is interested here more particularly in the following energy formulation, which corresponds to the law of behavior ENDO_CARRE for modeling GVNO :

$$\phi = \frac{1}{2}(1-a)^2 E_0 \varepsilon(u)^2 + \frac{\sigma_M^2}{E_0} a + \frac{E_0 l^2}{2} \nabla a \cdot \nabla a \quad \text{Equation 1.2}$$

where E_0 and σ_M are respectively the healthy rigidity of material and the ultimate stress and where $\varepsilon(u)$ is the deformation of the bar associated with displacement u .

The criterion of unicity is defined then by the inequality:

$$L^2 < \frac{2\pi^2 E_0^2 (1-a)}{6\sigma_M^2} l^2 \quad \text{Equation 1.4}$$

and the stability criterion, by the inequality:

$$L^2 \leq \frac{128\pi^2 E_0^2 (1-a)}{216\sigma_M^2} l^2 \quad \text{Equation 1.5}$$

By observing the two inequalities presented (equations 1.4 and 1.5), one sees that the loss of unicity occurs before the loss of stability. The objective of the case test is then to estimate the criterion of stability, initially between the loading of loss of unicity and that of loss of stability (the value of the minimum of the quotient of Rayleigh (1.2) must then be positive), and in the second time after the loading of loss of stability (the calculated minimum must then be negative).

1.2 Geometry

A bar is considered 2D of length $L=100\text{ m}$ is height $h=1\text{ m}$.



Figure 1 : Representation of the problem

1.3 Properties of material

1.3.1 Parameters material

Elastic characteristics:

$$E = 1 \text{ Pa}$$

$$\nu = 0$$

Characteristic of the law of damage:

$$\sigma_M = 0.01 \text{ Pa}$$

Non-linear characteristic:

$$l = 1 \text{ m}$$

1.4 Boundary conditions and loadings

Embedding : Worthless imposed displacements $DY = 0\text{ m}$ on the nodes of the bottom of the bar ($y=0.$), like on the nodes top ($y=1.$). Displacement imposed no one $DX = 0\text{ m}$ on the left face ($x=0.$). See figure1.

Loading 1 : Imposed linear displacement $U = 2 \times t \text{ m}$ on the right face ($x=100.$).

2 Reference solution

The value of the estimate of the minimum of the quotient of Rayleigh under constraints of inequalities (1.2), obtained starting from the option `DDL_STAB` of the operator `CRIT_STAB` with like parameters: `NB_FREQ = 25` and `COEF_DIM_ESPACE = 2`, is of $3.430938 E-8$ with the first step of time for which the homogeneous solution is still stable and of $-5.598244 E-9$ with the second step of time when the solution is theoretically unstable. These values are used as references and the case test is a case of nonregression.

It is seen that one finds well with the option `DDL_STAB`, an estimate of the criterion of positive stability for the first step of time, then negative for the second. The results are thus in agreement with the theory, which validates the developed algorithm.

3 Modeling A

3.1 Characteristics of modeling

Modeling is used D_PLAN_GVNO.

3.2 Characteristics of the grid

The grid contains 1416 elements QUAD8.

3.3 Results

NUMBER	TYPE_REFERENCE	VALE_REF	SHEET
1	'NON_REGRESSION'	3.430938 E-08	5.0E-5%
2	'NON_REGRESSION'	-5.598244 E-09	5.0E-5%

Table 1: Comparison of the estimate of the criterion of stability with the value of reference

4 Summary of the results

One finds the results of reference and that allows the validation of the developments of the algorithm of optimization under constraints of inequalities available with the option DDL_STAB of CRIT_STAB.