

SSNA125 - Sphere digs with internal and external pressure

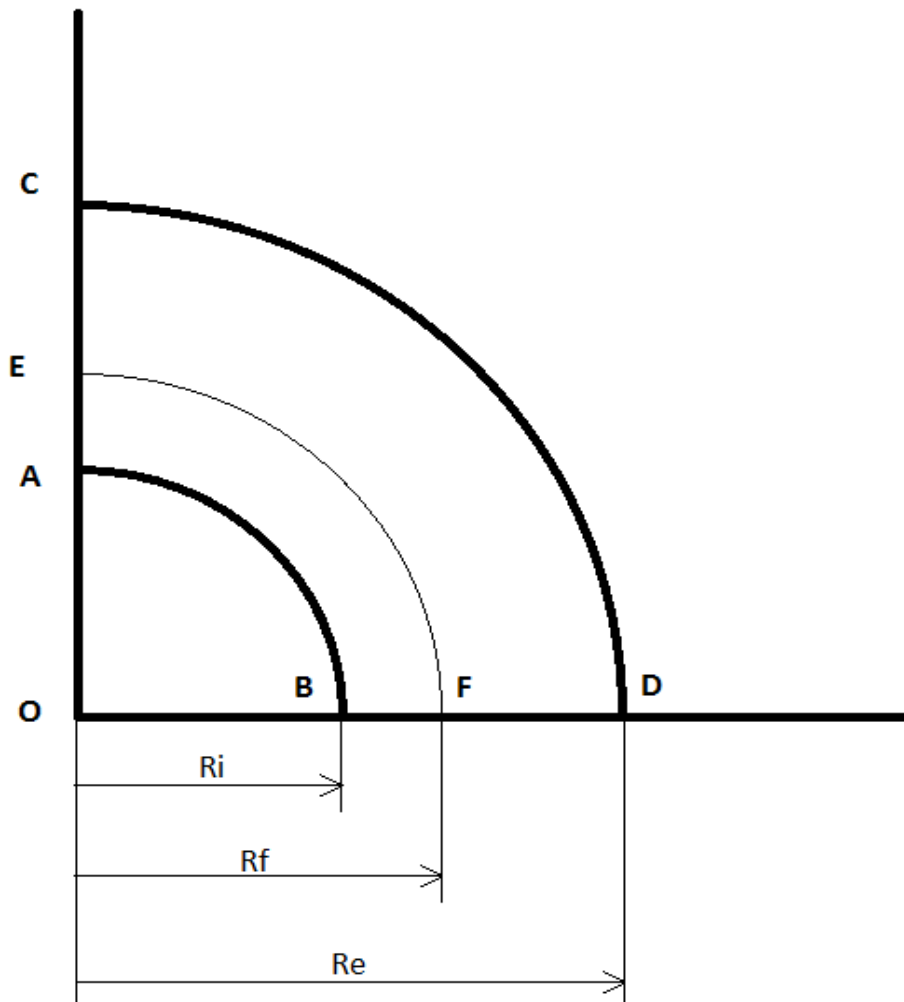
Summary:

One considers a sphere modelled in axisymetry subjected to pressures internal and external. Calculation is elastic linear. Simulation is compared with an analytical solution.

This test makes it possible to validate the elements `AXIS` with contact within the framework of approach X-FEM.

1 Problem of reference

1.1 Geometry



The sphere has an interior ray $R_i = 1\text{ m}$ and an external ray $R_e = 10\text{ m}$.

A circular interface XFEM of ray $R_f = 5,5\text{ m}$ is introduced.

1.2 Properties of materials

The characteristics of material are $E = 1\text{E}+5\text{ Pa}$ and $\nu = 0.3$. The law of behavior is linear rubber band.

1.3 Boundary conditions and loading

The following conditions are imposed:

- A pressure imposed of 300 Pa on inner face AB of ray R_i sphere $\sigma \cdot e_r(r=R_i) = P_i e_r$;

- A pressure imposed of $150 Pa$ on the external face CD of ray R_e sphere $\sigma \cdot e_r(r=R_e) = -P_e e_r$;
- Conditions of symésort: $DY=0$ on comic, $DX=0$ on AC.

Interface X-FEM is subjected to contact-friction so that the solution of the problem with interface is the solution of the continuous problem (i.e. the hollow sphere without interface). The reference solution calculated in the following paragraph is the solution of the problem of a hollow sphere subjected to an internal pressure and an external pressure. Here, one applies contact without friction.

2 Reference solution

2.1 Method of calculating

The field of displacement in spherical coordinates is written in the form:

$$U(r, \theta, \varphi) = u_r(r, \theta, \varphi)e_r + u_\theta(r, \theta, \varphi)e_\theta + u_\varphi(r, \theta, \varphi)e_\varphi$$

In our case, a pressure is applied only according to the direction e_r . Therefore, one can write the field of displacement in the form:

$$U(r) = u_r(r)e_r$$

The equation of Lamé Navier for this problem is written:

$$\rho f = (\lambda + 2\mu)\nabla \wedge (\nabla \wedge u) + \rho b$$

One a: $f=0$ and $b=0$.

The field of solution displacement of this equation is form:

$$U(r) = (C_1 r + C_2 / r^2)e_r$$

The boundary conditions make it possible to obtain the expression of the constants:

$$C_2 = \frac{(-P_i + P_e)R_e^3 R_i^3}{4\mu(R_i^3 - R_e^3)}$$

$$C_1 = \frac{1}{3\lambda + 2\mu} \left(-P_i + \frac{4\mu C_2}{R_i^3} \right)$$

The tensor of deformation is written,

$$\epsilon_{rr} = C_1 - \frac{2C_2}{r^3}$$

$$\epsilon_{\theta\theta} = C_1 + \frac{C_2}{r^3}$$

$$\epsilon_{\varphi\varphi} = C_1 + \frac{C_2}{r^3}$$

$$\epsilon_{\theta\varphi} = \epsilon_{\theta r} = \epsilon_{r\varphi} = 0$$

and the tensor of the constraints is written:

$$\sigma_{rr} = 3C_1\lambda + 2\mu \left(C_1 - \frac{2C_2}{r^3} \right)$$

$$\sigma_{\theta\theta} = 3C_1\lambda + 2\mu \left(C_1 + \frac{C_2}{r^3} \right)$$

$$\sigma_{\varphi\varphi} = 3C_1\lambda + 2\mu \left(C_1 + \frac{C_2}{r^3} \right)$$

$$\sigma_{\theta\varphi} = \sigma_{\theta r} = \sigma_{r\varphi} = 0$$

Boundary conditions which makes it possible to obtain the coefficients C_1, C_2 are then:

$$\begin{aligned}\sigma_{rr}(R_i) &= -P_i \\ \sigma_{rr}(R_e) &= -P_e\end{aligned}$$

2.2 Sizes and results of reference

Displacements and contact pressure on the level of the interface are used for the validation of the approach for all four modeling presented in this case test.

For the value of the field of radial displacement tested in the form of formula in $r = R_f$, we have:

$$U_r(R_f) = \frac{1}{3\lambda + 2\mu} \left(\frac{-P_i R_i^3 + P_e R_e^3}{R_i^3 - R_e^3} \right) R_f + \frac{(-P_i + P_e) R_i^3 R_e^3}{4\mu R_f^2 (R_i^3 - R_e^3)}$$

For the value of the field of pressure of contac tested in the form of formula in $r = R_f$, we have:

$$\sigma_{rr}(R_f) = \left(\frac{-P_i R_i^3 + P_e R_e^3}{R_i^3 - R_e^3} \right) - \frac{(-P_i + P_e) R_i^3 R_e^3}{R_f^3 (R_i^3 - R_e^3)}$$

All the digital values are given in unit IF.

Test	Identification	Analytical value of reference
$\sigma_{rr}(R_f)$	Min, max LAGS_C for all the nodes of postprocessing of the interface	-1.5046
$U_r(R_f)$	Min, max UR for all the nodes of postprocessing of the interface	7.1133E-05

2.3 Uncertainties on the solution

There is no uncertainty on the solution, this one being analytical.

2.4 Bibliographical references

A.F. Bower, *Applied Mechanics of Solids*, Taylor and Francis, 2010.

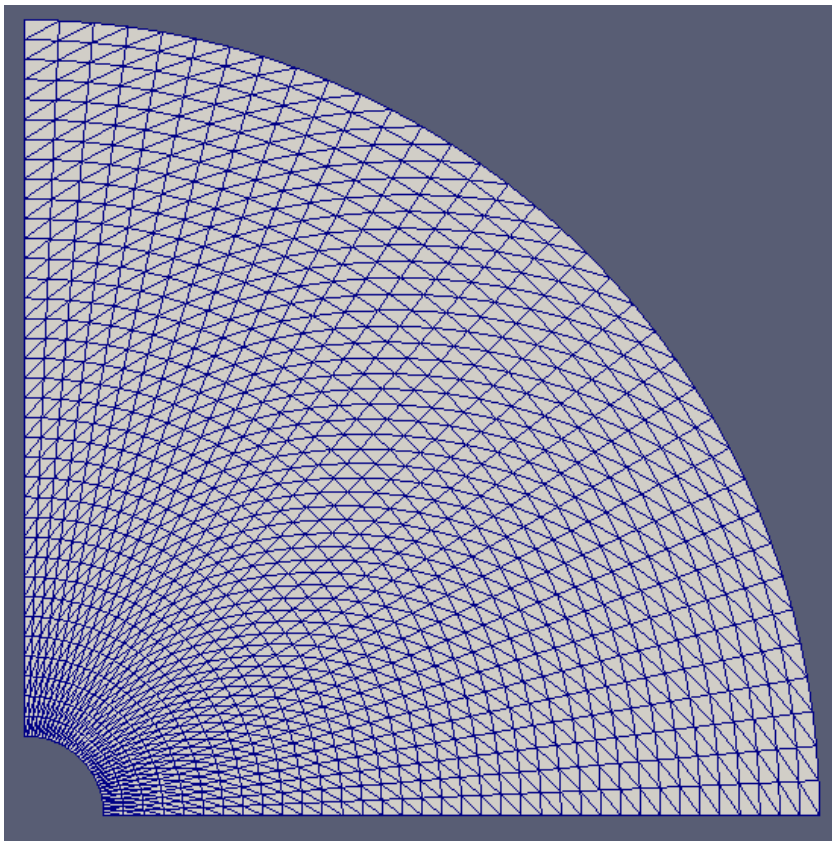
3 Modeling A

3.1 Characteristics of modeling

It is about a modeling X-FEM , in axisymetry (AXIS). The sphere is with a grid with triangular and linear elements.

3.2 Characteristics of the grid

The grid comprises 2592 meshes of the type TRIA3.



3.3 Sizes tested and results

One tests the values of the normal pressure and contact and displacement. One tests all the points of contact on the interface. It is checked that one finds the analytically determined values well. To test all the nodes in only once, one tests the minimum and the maximum on all the points of postprocessing of the interface.

Identification	Reference	Tolerance (%)
Max UR for all the nodes of postprocessing of the interface	7.1133E-05	4
Min UR for all the nodes of postprocessing of the interface	7.1133E-05	2
Max LAGS_C for all the nodes of postprocessing of the interface	-1.5046	14
Min LAGS_C for all the nodes of postprocessing of the interface	-1.5046	27

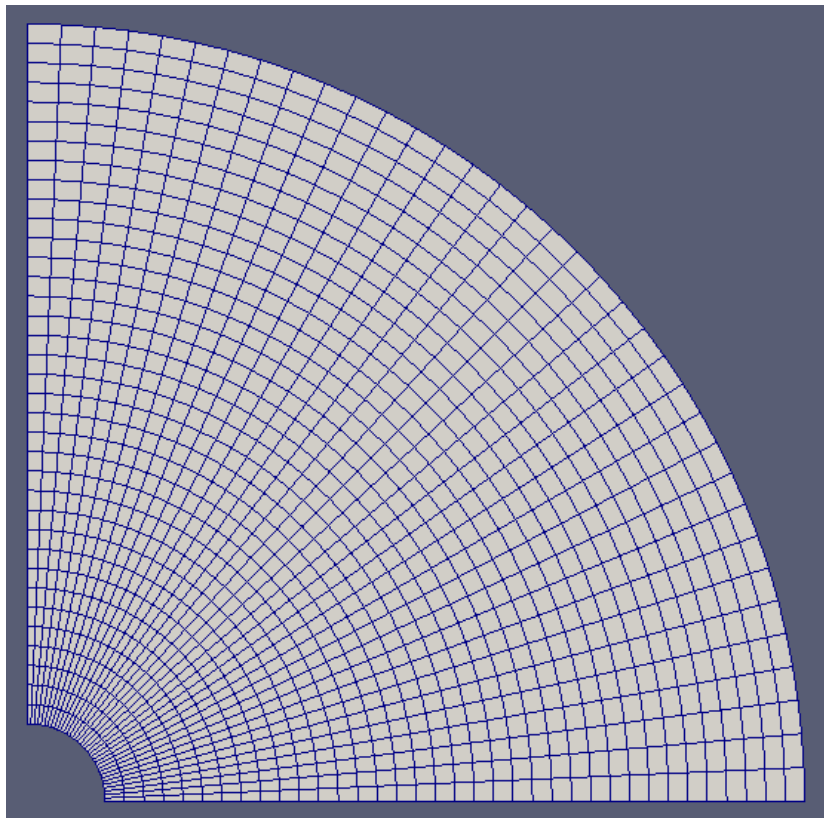
4 Modeling B

4.1 Characteristics of modeling

It is about a modeling X-FEM , in axisymetry (AXIS). The sphere is with a grid with quadrangular and linear elements.

4.2 Characteristics of the grid

The grid comprises 1296 meshes of the type QUAD4.



4.3 Sizes tested and results

One tests the values of the normal pressure and contact and displacement. One tests all the points of contact on the interface. It is checked that one finds the analytically determined values well. To test all the nodes in only once, one tests the minimum and the maximum on all the points of postprocessing of the interface.

Identification	Reference	Tolerance (%)
Max UR for all the nodes of postprocessing of the interface	7.1133E-05	3
Min UR for all the nodes of postprocessing of the interface	7.1133E-05	1
Max LAGS_C for all the nodes of postprocessing of the interface	-1.5046	2
Min LAGS_C for all the nodes of postprocessing of the interface	-1.5046	6

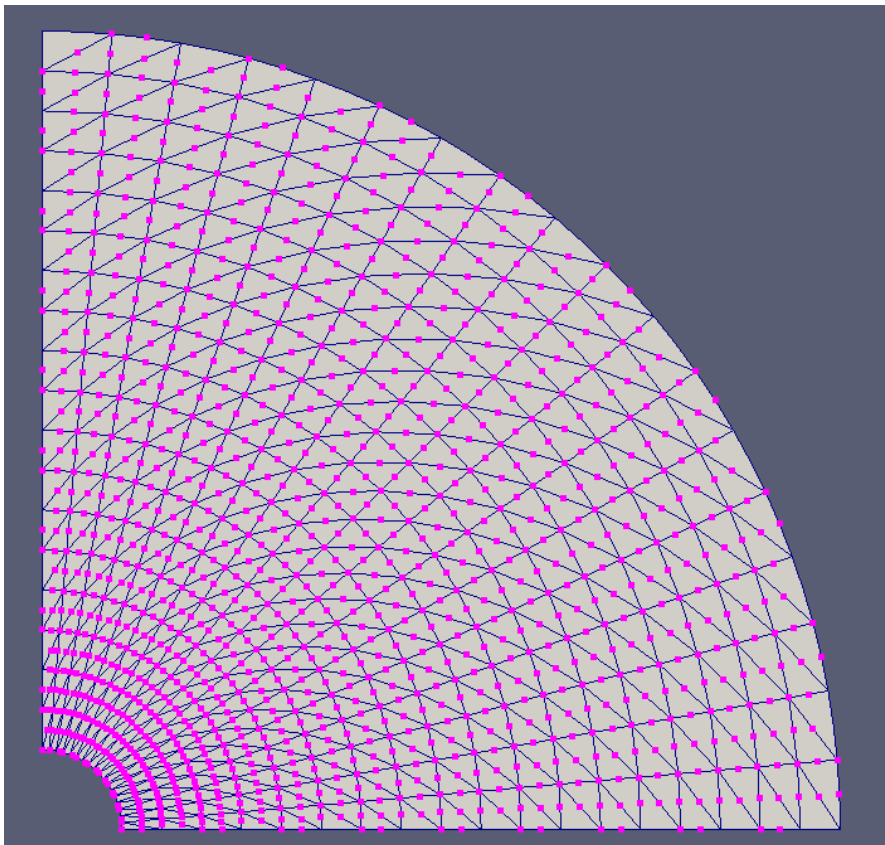
5 Modeling C

5.1 Characteristics of modeling

It is about a modeling X-FEM , in axisymetry (AXIS). The sphere is with a grid with triangular and quadratic elements.

5.2 Characteristics of the grid

The grid comprises 648 meshes of the type TRIA6.



5.3 Sizes tested and results

One tests the values of the normal pressure and contact and displacement. One tests all the points of contact on the interface. It is checked that one finds the analytically determined values well. To test all the nodes in only once, one tests the minimum and the maximum on all the points of postprocessing of the interface.

Identification	Reference	Tolerance (%)
Max UR for all the nodes of postprocessing of the interface	7.1133E-05	2
Min UR for all the nodes of postprocessing of the interface	7.1133E-05	2
Max LAGS C for all the nodes of postprocessing of the interface	-1.5046	2
Min LAGS C for all the nodes of postprocessing of the interface	-1.5046	2

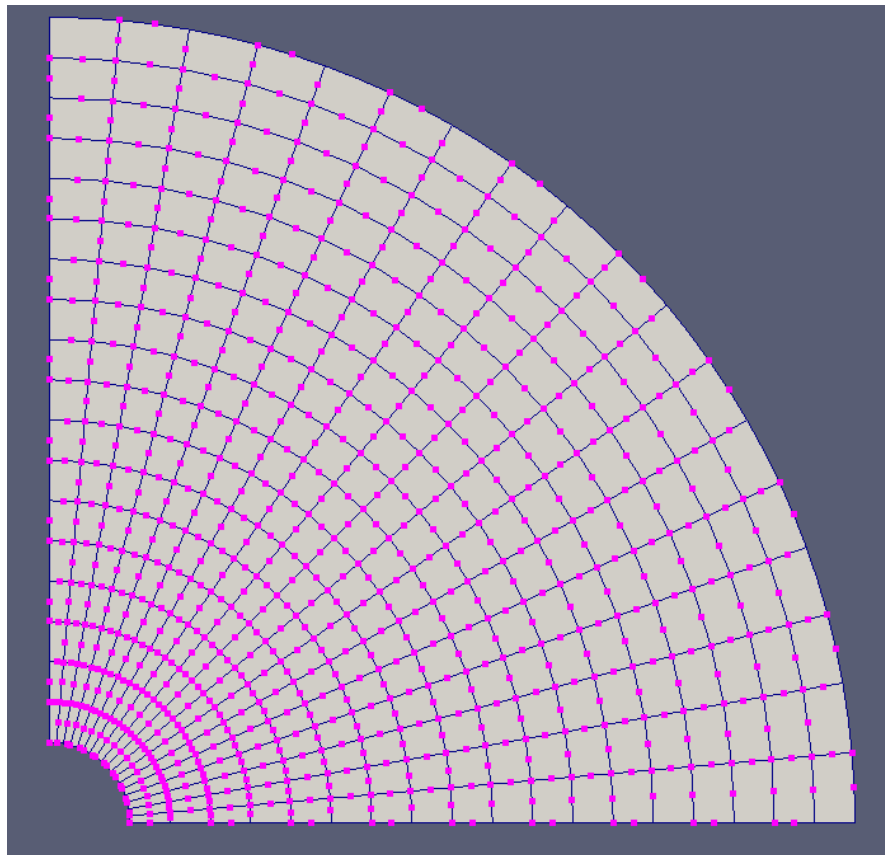
6 Modeling D

6.1 Characteristics of modeling

It is about a modeling X-FEM , in axisymetry (AXIS). The sphere is with a grid with quadrangular and quadratic elements.

6.2 Characteristics of the grid

The grid comprises 324 meshes of the type QUAD8.



6.3 Sizes tested and results

One tests the values of the normal pressure and contact and displacement. One tests all the points of contact on the interface. It is checked that one finds the analytically determined values well. To test all the nodes in only once, one tests the minimum and the maximum on all the points of postprocessing of the interface.

Identification	Reference	Tolerance (%)
Max UR for all the nodes of postprocessing of the interface	7.1133E-05	2
Min UR for all the nodes of postprocessing of the interface	7.1133E-05	2
Max LAGS_C for all the nodes of postprocessing of the interface	-1.5046	2
Min LAGS_C for all the nodes of postprocessing of the interface	-1.5046	2

7 Synthesis

Various modelings of this case test validate the elements `AXIS` with contact in X-FEM.