

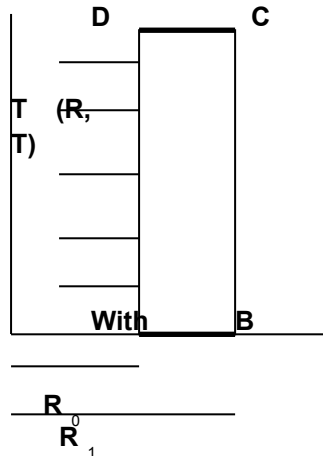
SSNA106 - Subjected hollow roll with a behavior thermoviscoelastic

Summary:

This CAS-test makes it possible to validate the law of LEMAITRE established in *Code_Aster* in the case of linear behavior thermoviscoelastic. The found results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



$$\begin{array}{ll} R_0 & 1\text{ m} \\ R_1 & 2\text{ m} \end{array}$$

1.2 Properties of materials

Young modulus: $E = 1\text{ MPa}$

Poisson's ratio: $\nu = 0.3$

Dilation coefficient: $\alpha = 0.7$

Law of LEMAITRE:

$$g(\sigma, \lambda, T) = \left(\frac{1}{K} \frac{\sigma}{\lambda^{\frac{1}{m}}} \right)^n \text{ with } \frac{1}{K} = 1, \frac{1}{m} = 0, n = 1$$

1.3 Boundary conditions and loading

Boundary conditions:

The cylinder is blocked in DY on the sides $[AB]$ and $[CD]$.

Loading:

The cylinder is subjected to a field of temperature $T(r, t) = t r^2$

2 Reference solutions

2.1 Method of calculating used for the reference solutions

The whole of this demonstration can be read with more details in the document [bib1].

In the case of a linear viscoelastic isotropic material, one can describe the behavior in the course of time using two functions $I(t)$ and $K(t)$ so that strains and stresses can be written:

$$\varepsilon(t) = (I + K) * \frac{d\sigma(t)}{dt} - K * \frac{d(\text{Tr}(\sigma(t)))}{dt} \mathbf{I}_3 + \alpha T(r, t) \mathbf{I}_3$$

where \mathbf{I}_3 indicate the matrix identity of row 3

and $*$ the product of convolution: $(f * g)(t) = \int_0^t f(t - \tau)g(\tau)d\tau$

The thermoelastic problem are equivalent, via the transform of Laplace is:

$$\varepsilon^+ = (I^+ + K^+) \sigma^+ - K^+ \text{Tr}(\sigma^+) \mathbf{I}_3 + \frac{\alpha r^2}{p} \mathbf{I}_3$$

$$\sigma_r^+ = \frac{d\sigma_r^+}{dr} = \frac{1}{r} (\sigma_\theta^+ - \sigma_r^+)$$

$$\varepsilon_z^+ = 0$$

$$(r \varepsilon_\theta^+) = \varepsilon_r^+$$

By eliminating the sign "+":

$$\sigma_r^+ + \frac{1}{r} (\sigma_r - \sigma_\theta) = 0$$

$$(I + K) \sigma_z - K(\sigma_r + \sigma_\theta + \sigma_z) + \frac{\alpha r^2}{p} = 0$$

$$r \left((I + K) \sigma_\theta - K(\sigma_r + \sigma_\theta + \sigma_z) + \frac{\alpha r^2}{p} \right) = (I + K) \sigma_r - K(\sigma_r + \sigma_\theta + \sigma_z) + \frac{\alpha r^2}{p}$$

maybe,

$$\sigma_r^+ + \frac{1}{r} (\sigma_r - \sigma_\theta) = 0$$

$$\sigma_z = \frac{K}{I} (\sigma_r + \sigma_\theta) - \frac{\alpha r^2}{pI}$$

$$r \left((I + K) \sigma_\theta - \frac{(I + K)K}{I} (\sigma_r + \sigma_\theta) + \frac{\alpha r^2}{p} \right) = (I + K) \sigma_r - \frac{(I + K)K}{I} (\sigma_r + \sigma_\theta) + \frac{(I + K) \alpha r^2}{I p}$$

$$(I + K)\sigma_\theta + r \left[(I + K)\sigma_\theta - \frac{(I + K)K}{I}(\sigma_r + \sigma_\theta) + \frac{(I + K)}{I} \frac{\alpha r^2}{p} \right] = (I + K)\sigma_r$$

According to the equilibrium equation, one has $\sigma_\theta = r\sigma_r' + \sigma_r$, one obtains:

$$(I + K)\sigma_r' + r \left[(I + K)(r\sigma_r' + \sigma_r) - \frac{(I + K)K}{I}(2\sigma_r + r\sigma_r') + \frac{(I + K)}{I} \frac{\alpha r^2}{p} \right] = 0,$$

$$\left[(2\sigma_r + r\sigma_r') + \frac{\alpha r^2}{p(I - K)} \right] = 0,$$

$$2\sigma_r + r\sigma_r' = A + \frac{\alpha r^2}{p(K - I)} \text{ what while integrating compared to R gives:}$$

$$\sigma_r = \frac{A}{2} + \frac{B}{r^2} + \frac{\alpha r^2}{4p(K - I)},$$

boundary conditions $\sigma_r(r_0) = \sigma_r(r_1) = 0$ give:

$$A = - \frac{\alpha}{2p(K - I)}(r_0^2 + r_1^2)$$

$$B = \frac{\alpha r_0^2 r_1^2}{4p(K - I)}$$

One thus has by taking again the initial notations:

$$\sigma_r^+ = \frac{\alpha}{4p(I^+ - K^+)}(r_0^2 + r_1^2 - r^2 - \frac{r_0^2 r_1^2}{r^2})$$

$$\sigma_\theta^+ = \frac{\alpha}{4p(I^+ - K^+)}(r_0^2 + r_1^2 - 3r^2 + \frac{r_0^2 r_1^2}{r^2})$$

$$\sigma_z^+ = \frac{\alpha}{p(I^+ - K^+)} \left(\frac{K^+}{I^+} \frac{(r_0^2 + r_1^2)}{2} - r^2 \right)$$

Maybe, by taking the opposite transform,

$$\sigma = \begin{pmatrix} \frac{\alpha}{2k}(1 - e^{-bt}) \left[r_0^2 + r_1^2 - r^2 - \frac{r_0^2 r_1^2}{r^2} \right] & 0 & 0 \\ 0 & \frac{\alpha}{2k}(1 - e^{-bt}) \left[r_0^2 + r_1^2 - 3r^2 + \frac{r_0^2 r_1^2}{r^2} \right] & 0 \\ 0 & 0 & \frac{\alpha}{k} \left[(1 - e^{-bt})(r_0^2 + r_1^2 - 2r^2) + \frac{r_0^2 + r_1^2}{r^2} (1 - e^{-Ekt}) \right] \end{pmatrix}$$

One from of deduced ε_V and w :

$$w(r, t) = \frac{1 - 2\nu}{Ek} \alpha r \left[(1 - e^{-bt}) \left[r_0^2 + r_1^2 - \frac{r_0^2 r_1^2}{r^2} \right] + (1 - e^{-Ekt}) \left[- \frac{(r_0^2 + r_1^2)}{4} \right] + \frac{3Ekt}{4(1 - 2\nu)} \left[\frac{r_0^2 r_1^2}{r^2} + r^2 \right] \right]$$

2.2 Results of reference

Displacement DX on the node B

2.3 Uncertainty on the solution

0% : analytical solution

2.4 Bibliographical references

PH. BONNIERES, two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note HI-71/8301

3 Modeling A

3.1 Characteristics of modeling

The problem is modelled in axisymetry

3.2 Characteristics of the grid

120 meshes QUAD4

3.3 Sizes tested and results

Identification	Moments	Reference	Tolerance %
$DX(B)$	0.24	1,110	0.1%

4 Summary of the results

Results calculated by *Code_Aster* are in agreement with the analytical solutions but very strongly depend on the refinement of the grid.