SDNS107 – Transitory answer of a reinforced concrete flagstone: model with GRILLE_EXCENTRE

Summary:

This test validates in transitory linear dynamics the modeling of square reinforced concrete plate using for the concrete a model of plate DKT and for the reinforcements elements of grid-membrane GRILLE_EXCENTRE. One checks the frequencies of the clean modes, the temporal answers in displacement, the reactions, and the kinetic energy, for a sinusoidal loading.

1 Problem of reference

1.1 Geometry

The geometry used in this case test is a concrete plate reinforced thickness $e=0.1\,\mathrm{m}$ and length $l=1\,\mathrm{m}$.

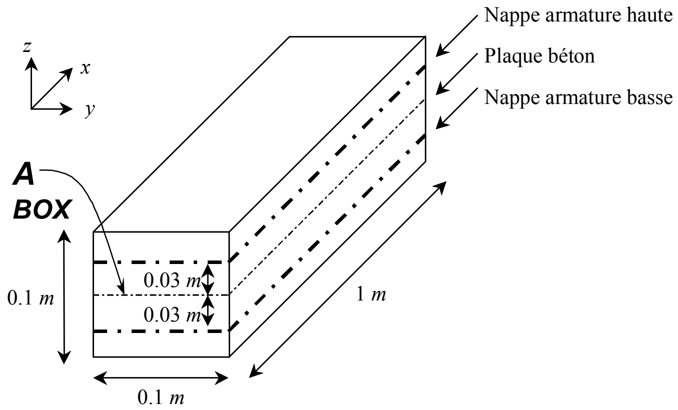

Figure 1.1-a : Studied geometry

The characteristics of the steel tablecloths of the concrete plate reinforced are:

- Higher tablecloth: section per linear meter $=0.05\,m^2/ml$; offsetting compared to the average layer: $+0.03\,m$,
- Lower tablecloth: section per linear meter $=0.05\,m^2/ml$; offsetting compared to the average layer: $-0.03\,m$.

1.2 Properties of materials

The characteristics material for multi-layer modeling concrete with steel reinforcements (DKT and GRILLE_EXCENTRE) are summarized in the table which follows.

Modeling	Young modulus N/m^2	Poisson's ratio	Density kg/m^3
Concrete (plate DKT)	$1.\,10^{10}$	0.0	2500
Steel (GRILLE_EXCENTRE)	$1.\,10^{11}$	0.0	7800

1.3 Boundary conditions and loadings

On the side A ($B0X$) plate one embeds displacements $u_x=u_y=u_z=0$, as well as rotations $\theta_x=\theta_y=\theta_z=0$. During calculation modal, displacement $u_y=0$ is blocked everywhere on the plate. A linear force is applied to the side $B1X$ (side opposed to $B0X$) in the direction $(0.0,0.0,1,0)$ and depends is worth $F_0=10^6\,N$.

In the case of dynamic calculation, a linear force of sinusoidal form is applied to the side $B1X$ in the direction $(0.0,0.0,1.0)$. The frequency of sinusoidal is of $20\,Hz$. The duration of the request is of $0{,}1\,s$.

1.4 Initial conditions

WITH the initial state, displacements and speeds are worth zero everywhere.

2 Reference solution

2.1 Method of calculating

It is possible to calculate the Eigen frequencies of the first and of the second modes of vibration of inflection of the plate because it functions like a beam-console.

The frequency f_1 first clean mode is written:

$$f_1 = \frac{3{,}5156}{2\pi L^2}\sqrt{\frac{EI}{\rho}}$$

with L the length of the console (1 m here), EI LE produced of the inertia of inflection by the Young modulus for the complete structure and ρ mass of the structure per unit of length.

Same manner, the frequency f_2 of the second clean mode is written:

$$f_2 = \frac{22{,}0336}{2\pi L^2}\sqrt{\frac{EI}{\rho}}$$

To calculate f_1 and f_2, one breaks up the parts related on the concrete and the reinforcements:

$$(EI) = (EI)_{beton} + (EI)_{acier}$$

where

$$(EI)_{acier} = 2\, E_{acier}\,(sL_2)\, e_{exc}^2$$

with E_{acier} the Young modulus of steel, s the section of the reinforcements per linear meter and e_{exc} the offsetting of the tablecloths of reinforcements compared to the average layer, and:

$$(EI)_{beton} = E_{beton}\, L_2 \frac{e^3}{12}$$

where E_{beton} is the Young modulus of the concrete.

For the mass per unit of length, one breaks up the mass of the concrete and the mass of steel.

By using the preceding equations, it then becomes possible to calculate the frequency of the clean modes considered. The results are:

Frequency	Reference
First mode of inflection	$54,67\,Hz$
Second mode of inflection	$342,64\,Hz$

The centre of gravity is located at the center of the console. Its coordinates are thus: $(0.5, 0.05, 0)$. Inertia along the axis of the complete structure is there:

$$I_{yy}(G)=\int_V ((x-x_G)^2+(z-z_G)^2)\,\rho\,dv$$

with (x_G, y_G, z_G) coordinates of the centre of gravity, V the volume of the structure and ρ its density. By breaking up the elements related to the concrete and those related to steel, it is possible to calculate inertia analytically:

$$I_{yy}(G)=8,611\,m^4$$

2.2 Sizes and results of reference

The results of reference are recapitulated in the table which follows.

Sizes	Reference
First mode of inflection	$54,67\,Hz$
Second mode of inflection	$342,64\,Hz$
Inertia along the axis y	$8,611\,m^4$
Next moment z with $t=0,1\,s$	$-0.000779\,N/m$
Following displacement z with $t=0,09\,s$	$-9480.0\,m$
Following displacement z with $t=0,1\,s$	$3720.0\,m$
Total kinetic energy with $t=0,1\,s$	$9.895889\,J$

2.3 Uncertainties on the solution

Analytical solutions for the clean modes.

Comparisons with EUROPLEXUS for the temporal answers in displacement, the reactions, and the kinetic energy, for a sinusoidal loading

2.4 Bibliographical references

[1] HUGHES T.J.R., COHEN MR., HAROUN, MR.: “Reduced and selective integration techniques in the finite element analysis of punts”, Nuclear Engineering and Design, vol. 46, p. 203-222 (1978).

[2] [R3.07.03] – Éléments of plate DKT, DST, DKQ, DSQ and Q4g.

3 Modeling A

3.1 Characteristics of modeling

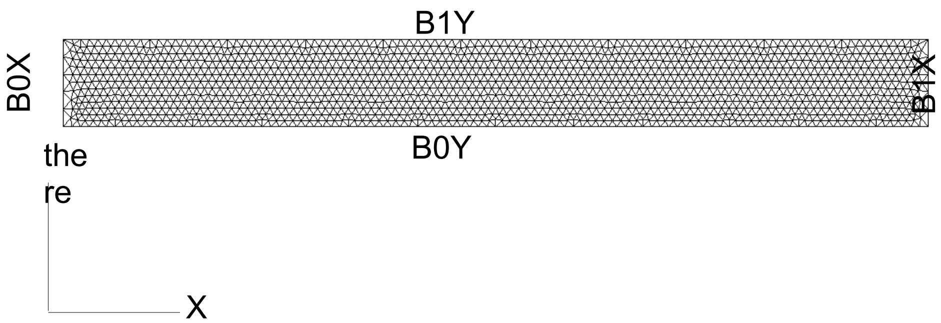

Figure 3.1-a : Grid of modeling A

Modeling: **DKTG**

Boundary conditions:

- Embedding in $B0X$,
- $DY=0.0$ on the whole of the beam.

Temporal integration:

- Diagram: `NEWMARK`, formulation: `DISPLACEMENT`,
- Pas de time: $1.10^{-3}s$ with possible subdivision until $1.10^{-5}s$.

3.2 Characteristics of the grid

Many nodes: 1536, Many meshs: elements `TRI3` : 2860, elements `SEG2` : 210.
The meshs are duplicated twice to affect the two grids of reinforcements.

3.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
Frequency (Hz) First mode	54.67	54,582	0,160
Frequency (Hz) Third mode	342.64	338,609	1,176
Position centre of gravity G (m)	0.05	0.05	0.
Inertia I_{yy} (G)	8,611	8.6038	0,083

For the transitory analysis, one tests in various moments (test of not-regression):

- the average of vertical displacements of the points of $B1X$,
- the resultant of the nodal forces applying to $B1X$,
- vertical nodal reaction on A .

The total kinetic energy is also tested (by comparison with the results provided by a loop Python).

Identification	Reference	*Aster*	% difference
Average of vertical displacements on $B1X$ (with the sequence number 100)	– 7.79 10^{-4}	– 7.7917 10^{-4}	0,022
Vertical resultant of the forces applied to $B1X$ (with the sequence number 90)	– 9.48 10^{+3}	– 9.4833 10^{+3}	0,035
Vertical nodal reaction on A (with the sequence number 100)	3.72 10^{+3}	3.7139 10^{+3}	– 0,161
Total kinetic energy (with the sequence number 100)	9.89588	9,902	0,062

4 Modeling B

4.1 Characteristics of modeling

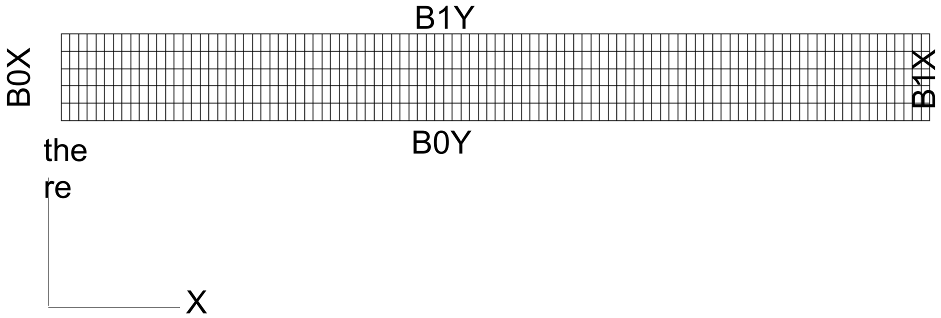

Figure 4.1-a : Grid of modeling B

The characteristics of modeling B are identical to those of modeling A, only the nature of the elements change (QUAD4 instead of TRIA3).

Temporal integration:

- Diagram: NEWMARK, formulation: DISPLACEMENT,
- Pas de time: $1.10^{-3}s$ with possible subdivision until $1.10^{-5}s$.

4.2 Characteristics of the grid

Many nodes: 606, Many meshs: elements QUA4 : 500, elements SEG2 : 210. The meshs are duplicated twice to affect the two grids of reinforcements.

4.3 Sizes tested and results

Identification	Reference	Aster	% difference
Frequency (Hz) First mode	54.67	54,579	0,166
Frequency (Hz) Third mode	342.64	338,511	1,205

For the transitory analysis, one tests in various moments (values compared with modeling B):

the average of vertical displacements of the points of $B1X$
the resultant of the nodal forces applying to $B1X$
vertical nodal reaction on A

The total kinetic energy is also tested (by comparison with the results provided by a loop Python).

Identification	Reference	Aster	% difference
Average of vertical displacements on $B1X$ (with the sequence number 100)	$-7.79\ 10^{-4}$	$-7.8112\ 10^{-4}$	0,272
Vertical resultant of the forces applied to $B1X$ (with the sequence number 90)	$-9.48\ 10^{+3}$	$-9.4813\ 10^{+3}$	0,014
Vertical nodal reaction on A (with the sequence number 100)	$3.72\ 10^{+3}$	$3.7413\ 10^{+3}$	0,574
Total kinetic energy (with the sequence number 100)	9.89588	9.9020	0.06

5 Summary of the results

One finds a light difference between the solutions obtained by the two computer codes. He is regarded as reasonable and the results are considered to be satisfactory. Even if in this test one wanted to approach modeling Code_Aster as much as possible of that of the code Europlexus, one is conscious, in particular, of the difference on the level of calculation of the matrix in mass. The matrix of mass implemented in Europlexus follows the method suggested in [1], which is adapted better for a computation software of fast dynamics explicit. Code_Aster use a more standard approach, explained in [2]. The difference relates to in particular the calculation of inertias (component of the matrix of mass corresponding to the degrees of freedom of rotation), which are not very important in the case of slow dynamics. On the other hand, in fast dynamics, it is they which influence more the critical step of time. For example, by neglecting them the matrix of mass becomes singular and explicit integration becomes unconditionally unstable.

Various simulations suggested validate the use of modeling `GRILLE_EXCENTRE` for modal calculations, of explicit dynamics and implicit dynamics.

The values obtained are in agreement with the analytical solutions, when those are available.