SDND105 – Shock of a material point against a wall with the plastic behavior buckling

Summary:

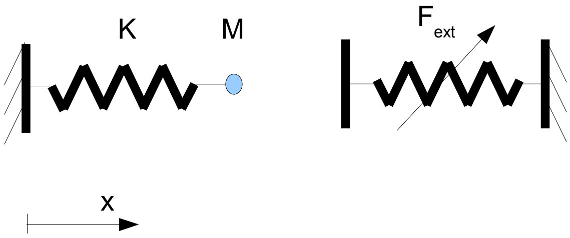
This test validates the behavior of an obstacle of type shock with possible buckling. The resolution was carried out with the operator `DYNA_VIBRA` by using the keyword `BUCKLING`.

One calculates the moment of beginning of buckling (this moment corresponds to the first moment when the force of shock exceeded the threshold of buckling), the cumulated plastic deformation and the moment of sharpening to the initial position.

The got results are in agreement with the analytical results.

1 Problem of reference

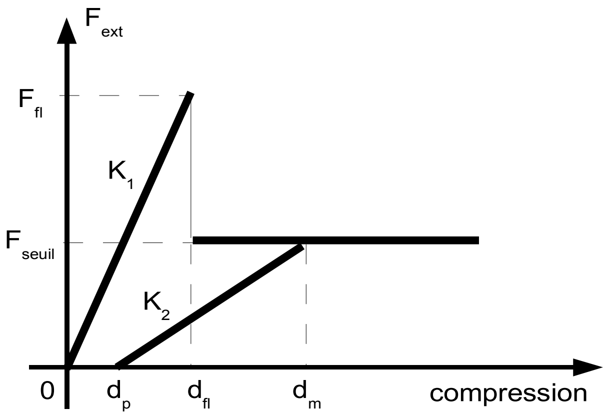
1.1 Geometry



1.2 Properties of material

$K = 10^{-7} N/m$
$M = 1\ kg$

The force of reaction F_{ext} depends on the compression of the wall which follows the law below:



With:

$K_1 = 1\ N/m$
$F_{fl} = 1\ N$
$F_{seuil} = 0.5\ N$
$K_2 = 0.5\ N/m$

1.3 Boundary conditions and loadings

One imposes the displacement of the mobile mass M along the axis of x .

1.4 Initial conditions

At the initial moment, the mobile mass is right in contact against the wall (initial game no one) with a speed initial equalizes with $2\ m/s$.
The wall did not undergo yet plastic deformation (wall not flambe).

2 Reference solution

2.1 Method of calculating

The problem consists in analyzing the answer of a mobile mass subjected at an initial speed nonworthless and coming to shock against a wall which follows a law of behavior of type buckling.

The equilibrium equation of the system is written:

$$M\frac{\partial^2 x}{\partial t^2}+Kx=F_{ext} \quad (1)$$

The resolution of (1) proceeds in several phases: before buckling of the wall, loading after buckling, unloading after buckling and coasting flight.

2.1.1 First phase

Before buckling, the force of reaction is worth: $F_{ext}=-K_1 x$

The solution of the equation (1) is put in the following form: $x(t)=A_1\sin\omega_1 t+B_1\cos\omega_1 t$

With: $\omega_1^2=\frac{K+K_1}{M}$

By taking account of the initial conditions: $x(0)=0$ and $\frac{\partial x}{\partial t}(0)=v_0$

One identifies: $A_1=\frac{v_0}{\omega_1}$ and: $B_1=0$

That is to say: $x(t)=\frac{v_0}{\omega_1}\sin\omega_1 t$

One reaches the limiting threshold of buckling F_{fl} at the moment t_{fl}, such as: $x(t_{fl})=d_{fl}=\frac{F_{fl}}{K_1}$.

From where: $t_{fl}=\frac{1}{\omega_1}Arc\sin(\frac{F_{fl}\omega_1}{K_1 v_0})$

and $\frac{\partial x}{\partial t}(t_{fl})=v_{fl}$

2.1.2 Second phase

The following phase is the phase of loading after buckling; the force of reaction is constant as long as speed remains positive. This force of reaction is worth: $F_{ext}=-F_{seuil}$ and the solution of the equation (1) is written: $x(t)=A_2\sin\omega_0 t+B_2\cos\omega_0 t-\frac{F_{seuil}}{M\omega_0^2}$

With: $\omega_0^2=\frac{K}{M}$

Like $\omega_0^2\ll 1$, one can carry out a limited development of the goniometrical functions.

That is to say: $x(t)=A_{21}t^2+B_{21}t+C_{21}$

By taking account of the initial conditions $x(t_{fl})=d_{fl}$ and $\frac{\partial x}{\partial t}(t_{fl})=v_{fl}$, one obtains:

$$A_{21}=(\frac{v_{fl}t_{fl}}{2}-\frac{d_{fl}}{2}-\frac{F_{seuil}^2}{4M})\omega_0^2-\frac{F_{seuil}}{2M}$$

$$B_{21}=v_{fl}+\frac{F_{seuil}t_{fl}}{M}+(d_{fl}t_{fl}-\frac{v_{fl}t_{fl}^2}{2})\omega_0^2$$

$$C_{21}=d_{fl}-\frac{F_{seuil}t_{fl}^2}{2M}-v_{fl}t_{fl}-\frac{d_{fl}t_{fl}^2}{2}\omega_0^2$$

The moment to which speed is cancelled is: $t_m=-\frac{B_{21}}{2A_{21}}$

By neglecting the terms in ω_0^2, one deduces: $t_m=t_{fl}+\frac{M\ v_{fl}}{F_{seuil}}$,

and maximum displacement is worth: $x(t_m)=d_m$

That makes it possible to obtain the cumulated plastic deformation: $d_p=d_m-\frac{F_{seuil}}{K_2}$

2.1.3 Third phase

This phase corresponds to unloading, the force of reaction is worth: $F_{ext}=-K_2(x-d_p)$

The solution of the equation (1) is written: $x(t)=A_3\sin\omega_2 t+B_3\cos\omega_2 t+\frac{K_2 d_p}{K+K_2}$

With: $\omega_2^2=\frac{K+K_2}{M}$

By taking of account initial conditions $x(t_m)=d_m$ and $\frac{\partial x}{\partial t}(t_m)=0$,

one obtains: $A_3=(d_m-\frac{K_2 d_p}{K+K_2})\sin\omega_2 t_m$ and $B_3=(d_m-\frac{K_2 d_p}{K+K_2})\cos\omega_2 t_m$

The force of reaction is cancelled when the displacement of the material point reaches the value of the cumulated plastic deformation d_p.

Like $K\ll K_2$, the following approximation is made: $\frac{K_2 d_p}{K+K_2}\approx d_p$

Thus, the moment t_d who corresponds to the cancellation of the force of reaction is such as:
$x(t_d)=d_p=A_3\sin\omega_2 t_d+B_3\cos\omega_2 t_d+d_p$

That is to say: $\sin\omega_2 t_m\sin\omega_2 t_d+\cos\omega_2 t_m\cos\omega_2 t_d=\cos\omega_2(t_d-t_m)=0$

From where: $t_d = t_m + \frac{\pi}{2\omega_2}$

2.1.4 Fourth phase

The following phase corresponds to the phase of coasting flight $F_{ext} = 0$.

The solution of the equation (1) is written: $x(t) = A_4 \sin \omega_0 t + B_4 \cos \omega_0 t$

The initial conditions are:

$x(t_d) = d_p$ and: $\frac{\partial x}{\partial t}(t_d) = v_d = \omega_2 (x_m - \frac{K_2 d_p}{K + K_2}) \sin \omega_2 (t_m - t_d)$

What gives:

$$A_4 = d_p \sin \omega_0 t_d + \frac{v_d}{\omega_0} \cos \omega_0 t_d$$

$$B_4 = d_p \cos \omega_0 t_d - \frac{v_d}{\omega_0} \sin \omega_0 t_d$$

Like $\omega_0^2 \ll 1$, by carrying out a limited development of the goniometrical functions until order 2, the solution is put in the following form:

$$x(t) = A_{41} t^2 + B_{41} t + C_{41}$$

With:

$$A_{41} = \frac{\omega_0^2}{2} [v_d t_d - d_p (1 - \frac{\omega_0^2 t_d^2}{2})]$$

$$B_{41} = d_p t_d \omega_0^2 + v_d (1 - \frac{\omega_0^2 t_d^2}{2})$$

$$C_{41} = d_p (1 - \frac{\omega_0^2 t_d^2}{2}) - v_d t_d$$

By neglecting the terms in ω_0^2 , one obtains:

$$x(t) = v_d t + d_p - v_d t_d$$

And one deduces the moment t_0 of passage to the initial position ($x = 0$).

That is to say: $t_0 = t_d - \frac{d_p}{v_d}$

2.2 Sizes and results of reference

One proposes to test the following sizes:

t_{fl} : moment of beginning of buckling

d_p : cumulated plastic deformation

t_0 : moment of sharpening to the initial position (after buckling and discharge)

Taking into account the digital values of the data input, one obtains:

$t_{fl}=\frac{\pi}{6}$ (expressed in seconds)

$d_p=3$ (expressed in meters)

$t_0=\frac{\pi}{6}+2\sqrt{3}+\frac{\pi+6}{\sqrt{2}}$ (expressed in seconds)

2.3 Uncertainties on the solution

The reference solution is analytical (with the second order near).

3 Modeling A

3.1 Characteristics of modeling

One models the system with a material point and an obstacle of the type PLAN_Y.

One evaluates the sizes obtained following buckling due to the shock by using the keyword BUCKLING of the operator DYNA_VIBRA.

One also checks the various methods of resolution (EULER, ADAPT_ORDRE2 and DEVOGE). With the diagram in adaptive time ADAPT_ORDRE2 one defines (in seconds):

- the step of initial time: NOT = 0.0002,
- the maximum value of the step of time: PAS_MAXI = 0,001,
- the minimal value of the step of time: PAS_MINI = 2.E-8.

3.2 Characteristics of the grid

Many nodes: 2
Number of mesh: 1 **SEG2**

3.3 Sizes tested and results

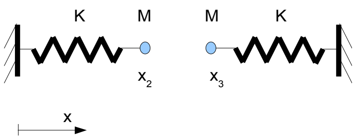
One tests the values of the sizes related to the buckling of the wall.

Identification	Reference	*Aster*	variation
t_{fl}	$\frac{\pi}{6} s$	0.52411 S	0.097%
d_p	3 m	2.998584 m	0.047%
$x(t_0)=x(\frac{\pi}{6}+2\sqrt{3}+\frac{\pi+6}{\sqrt{2}})$	0 m	-2,154 E-3 m	2.2 E-3 m

4 Modeling B

4.1 Characteristics of modeling

For this modeling, one considers two mobile material points according to the following diagram:



The system is perfectly symmetrical. One obtains the same formulation as modeling A if one chooses normal rigidities of shock equal to half of the rigidities chosen for modeling A.

Indeed, if one notes: $x_2=-x_3=x$
The force of reaction F_{ext} puts itself in the following form:

During the first phase: $F_{ext}=-K_1(x_2-x_3)=-2K_1 x$
During the second phase: $F_{ext}=-F_{seuil}$
During the third phase: $F_{ext}=-K_2(x_2-x_3-2d_p)=-2K_2(x-d_p)$
During the fourth phase: $F_{ext}=0$

One models the problem with an obstacle of the type BI_PLAN_Y.

At the initial moment, the two material points are in contact with an initial speed equal to $2m/s$.

One evaluates the sizes obtained following buckling due to the shock by using the keyword BUCKLING of the operator DYNA_VIBRA, with the diagrams in time EULER and ADAPT_ORDRE2.
With the diagram in adaptive time ADAPT_ORDRE2 one defines (in seconds):
- the step of initial time: NOT = 0,001,
- the maximum value of the step of time: PAS_MAXI = 0,005.

4.2 Characteristics of the grid

Many nodes: 4
Many meshs: 2 **SEG2**

4.3 Sizes tested and results

One tests the values of the sizes related to the behavior of buckling during the shock.

Identification	Reference	*Aster*	variation
t_{fl}	$\frac{\pi}{6}s$	0,524 S	0.077%
d_p	3 m	2.9984 m	0.052%
$x(t_0)=x(\frac{\pi}{6}+2\sqrt{3}+\frac{\pi+6}{\sqrt{2}})$	0 m	-1,930 E-3 m	1.93 E-3 m

5 Summary of the results

The differences between the solutions obtained with Aster and the analytical solutions are very weak.