TTLL100 - Thermal shock on a wall plan with condition of exchange

Summary:

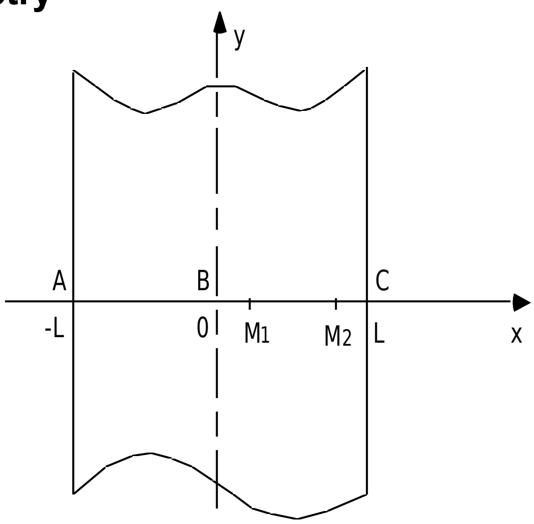
This test of transitory linear thermics consists in imposing a cold thermal shock on a wall infinite plan using a limiting condition of exchange. The shock is modelled by a linear slope $\Delta T=-100°C$ in $10^{-2} s$.

With the problem is dealt in plan.
The reference solution is analytical.
The test is carried out on 2 modelings: (TRIA3, QUAD4) and (TRIA6, QUAD9).

One tests the algorithm of linear thermics transitory when the matrix of mass is diagonalisée (modeling `PLAN_DIAG` with ‘’ farmhouse lumping' ‘).

1 Problem of reference

1.1 Geometry



$\bar{AB}=\bar{BC}=L=0.1\,m$
$x(M1)=0.02\,m$
$x(M2)=0.08\,m$

1.2 Material properties

$\lambda=1\,W/m\,°C$
$\rho C_P=1000\,J/m^3\,°C$

1.3 Boundary conditions and loadings

exchange $\lambda \left.\frac{\partial T}{\partial n}\right|_{x=\pm L}=h(T_{ext}-T(x,t))$

with: $h=100.\,W/m^2\,°C$

$$\begin{cases} T_{ext}(C)_{t=0}=100. \\ T_{ext}(C)_{t=10^{-2}}=0. \end{cases}$$

1.4 Initial conditions

$T(x,0)=100\,°C$ for all x

Discretization in time (T):

10 pas for	$[0.,1.D-2]$	that is to say	$\Delta t=10^{-3}s$
9 pas for	$[1D-2,1.D-1]$	that is to say	$\Delta t=10^{-2}s$
9 pas for	$[1.D-1,1.]$	that is to say	$\Delta t=10^{-1}s$
5 pas for	$[1.,2.]$	that is to say	$\Delta t=2.10^{-1}s$

8 pas for	$[2., 10.]$	that is to say	$\Delta t = 1\,s$

2 Reference solution

2.1 Method of calculating used for the reference solution

$$\frac{T(x,t)-T_p}{T_0-T_p}=\sum_{n=1}^{\infty} A_n \exp\left(-\xi_n^2 \frac{\lambda}{\rho C_p L^2} t\right)\cos\left(\xi_n \frac{x}{L}\right)$$

$x=$ X-coordinate
$t=$ Time
$T_0=$ Initial temperature
$T_p=$ Imposed temperature
$n=$ $1,2,3,\ldots$

With ξ_n positive roots of $\xi_n \tan \xi_n = hL/\lambda = 10.$
and $A_n=\dfrac{4\sin\xi_n}{2\xi_n+\sin(2\xi_n)}$

2.2 Results of reference

Temperatures at the points $M1$ ($x=0.02$) and $M2$ ($x=0.08$),
and at various moments ($t=0.1$, 0.5 , 2.0 and 10.0).

The values of reference are obtained by calculating the first 30 terms of the series (Mathematica).

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

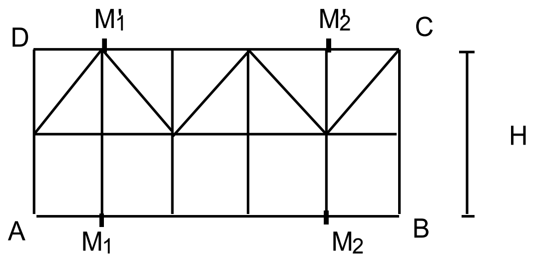
• INCROPERA F.P., OF WITT D.P., Fundamentals of heat and farmhouse transfer. Third Edition. 1990.

3 Modeling A

3.1 Characteristics of modeling

TRIA3, QUAD4

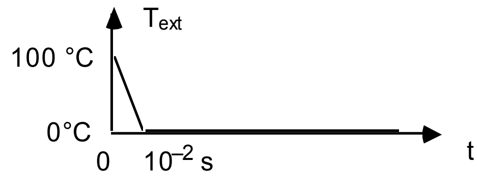
By reason of symmetry, one nets only the half of thickness of the wall. Modeling is made on a height $H=0.1\,m$ with 2 layers of elements.



Conditions limites

sur [AB], [AD] et [CD] : flux nul

sur [BC] : échange h, T_{ext}



points	nœuds	x	y
M1	N16	0.02	0.0
M2	N6	0.08	0.0
M'1	N14	0.02	0.1
M'2	N4	0.08	0.1

Conditions initiales
T = 100 °C

3.2 Characteristics of the grid

Many nodes: 18
Many meshs and types: 5 QUAD4, 10 TRIA3

3.3 Values tested

Identification	Reference	*Aster*	% difference
$M1(x=0.02)\,N16$			
$t=0.1$	100.00	99,998	+0.00
$t=0.5$	99,408	99,042	– 0.37
$t=2.0$	79,859	79,794	– 0.08
$t=10.0$	15,717	16,138	+2.68
$M2(x=0.08)\,N6$			
$t=0.1$	93,666	93,380	– 0.31
$t=0.5$	63,500	63,813	+0.49
$t=2.0$	35,717	35,667	– 0.14
$t=10.0$	6.7948	6.9326	+2.03
$M'1(x=0.02)\,N14$			
$t=0.1$	100.00	99,998	+0.00

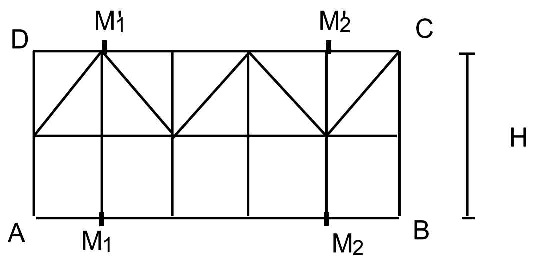
$t=0.5$	99,408	99,077	– 0.33
$t=2.0$	79,859	80,002	+0.18
$t=10.0$	15,717	16,211	+3.14
$M'2(x=0.08)N4$			
$t=0.1$	93,666	92,895	– 0.82
$t=0.5$	63,500	61,882	– 2.55
$t=2.0$	35,717	35,331	– 1.08
$t=10.0$	6.7948	6.8885	+1.38

4 Modeling B

4.1 Characteristics of modeling

TRIA6, QUAD9

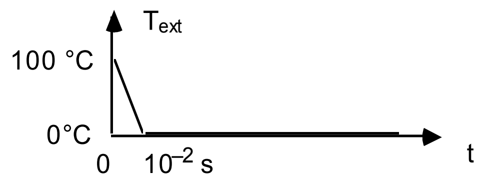
By reason of symmetry, one nets only the half of thickness of the wall. Modeling is made on a height $H=0.1\,m$ with 2 layers of elements.



Conditions limites

sur [AB], [AD] et [CD] : flux nul

sur [BC] : échange h, T_{ext}



Conditions initiales
T = 100 °C

points	nœuds	x	y
M1	N16	0.02	0.0
M2	N6	0.08	0.0
M'1	N14	0.02	0.1
M'2	N4	0.08	0.1

4.2 Characteristics of the grid

Many nodes: 55
Many meshs and types: 5 QUAD9, 10 TRIA6

4.3 Values tested

Identification	Reference	*Aster*	% difference
$M1(x=0.02)\,N18$			
$t=0.1$	100.00	100.00	+0.00
$t=0.5$	99,408	99,278	– 0.13
$t=2.0$	79,859	79,898	+0.05
$t=10.0$	15,717	16,043	+2.07
$M2(x=0.08)\,N49$			
$t=0.1$	93,666	94,077	+0.44
$t=0.5$	63,500	63,979	+0.75
$t=2.0$	35,717	35,825	+0.30
$t=10.0$	6.7948	6.9321	+2.02
$M'1(x=0.02)\,N12$			
$t=0.1$	100.00	100.00	+0.00

$t=0.5$	99,408	99,311	– 0.10
$t=2.0$	79,859	80,101	+0.30
$t=10.0$	15,717	16,093	+2.39
$M'2(x=0.08)$ $N30$			
$t=0.1$	93,666	93,469	– 0.21
$t=0.5$	63,500	62,860	– 1.01
$t=2.0$	35,717	35,641	– 0.21
$t=10.0$	6.7948	6.9068	+1.65

5 Summary of the results

Modeling `PLAN_DIAG` give rather satisfactory results. Although the grid comprises few elements in the thickness, the variation on the temperatures remains lower than 3.2 %.

Although the thermal shock is brutal, the diagonalisation of the matrix of mass makes it possible to obtain a solution in temperature which does not oscillate during the transient.