TPLV106 - Stationary nonlinear thermics in pointer

Summary:

This elementary test makes it possible to treat a reducible three-dimensional example with a problem a variable of space in stationary nonlinear thermics in pointer (problem of convection-diffusion).

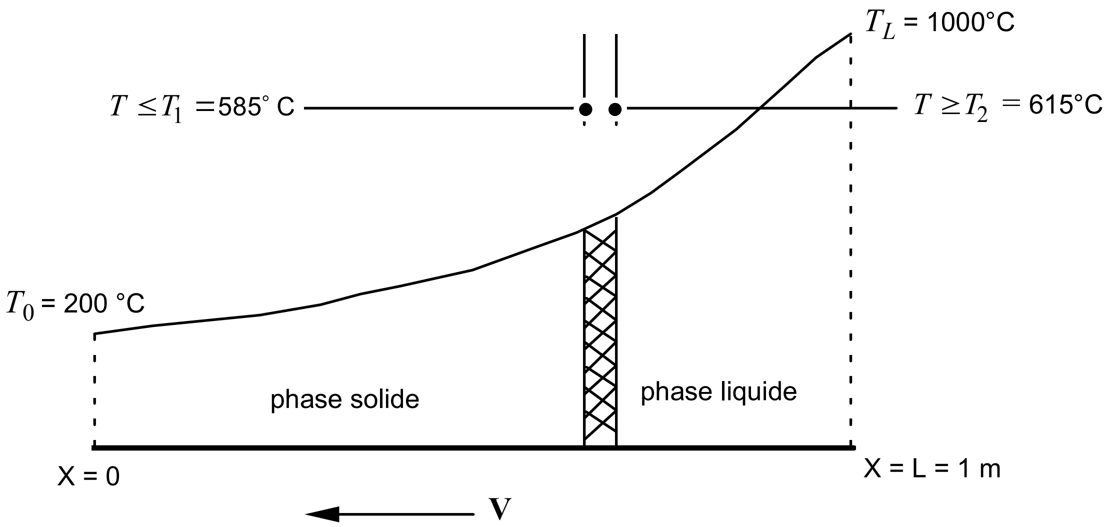
It also makes it possible to check the taking into account of a phase shift solid/liquid by *Code_Aster*.

The reference solution is analytical and the variations with the results got by *Code_Aster* are lower than 1%. The problem is modelled in the case plan.

1 Problem of reference

1.1 Geometry

That is to say a bar moving, at the speed V, with the right of conditions of temperatures imposed in $X=0$ and $X=L$ expressed in a fixed reference frame (compared to the bar moving).



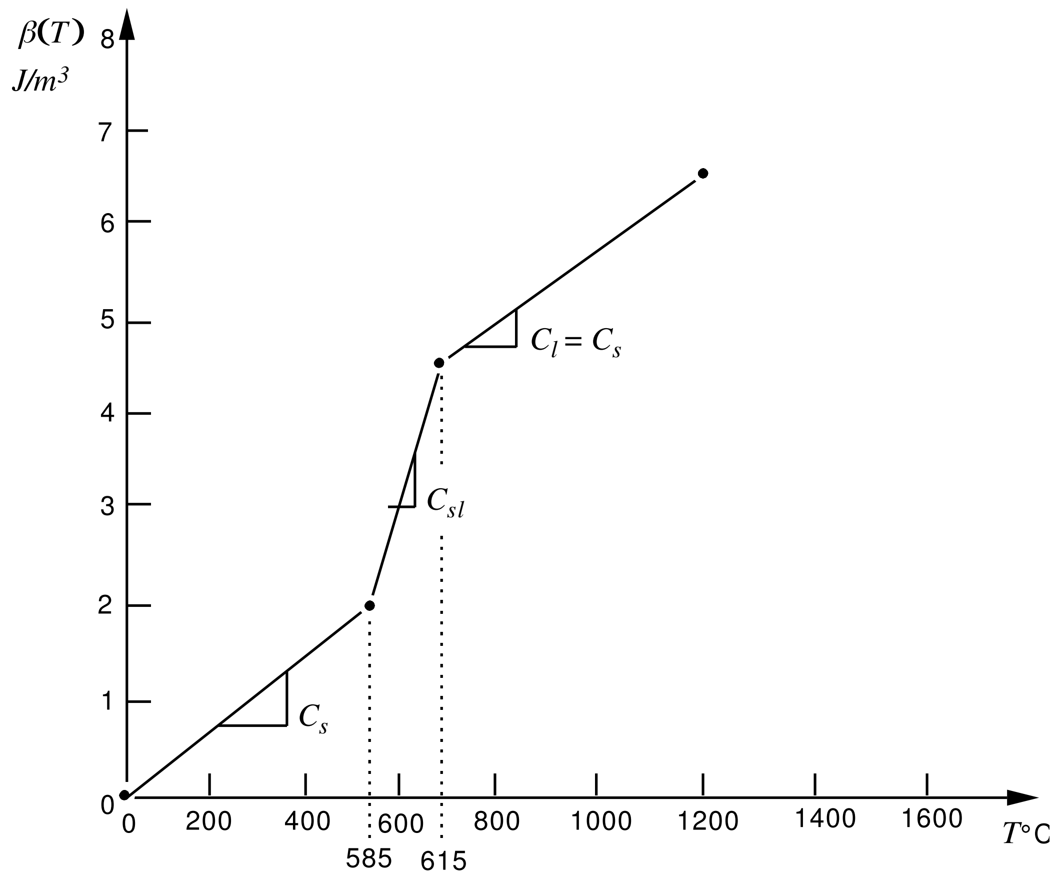
1.2 Properties of materials

- thermal conductivity is constant: $K=150\,W/m\,°C$
- the function enthalpy is such as:

$$\beta(T)=\begin{cases} C_S T & ;T\le T_1 \\ C_S T+C_{Sl}(T-T_1) & ;T_1\le T\le T_2 \\ C_S T+C_{Sl}(T_2-T_1)+C_l(T-T_2) & ;T_1\le T\le T_2 \end{cases}$$

with the following values:

$$C_S=C_l=1/3.10^7\,J/m^3\,°C$$
$$C_{Sl}=8.333\,10^7\,J/m^3\,°C$$
$$T_1=585.\,°C$$
$$T_2=615\,°C$$



1.3 Boundary conditions and loadings

Temperatures imposed at the ends

$$T_0 = 200\,°C \quad \text{for} \quad x = 0$$

$$T_L = 1000\,°C \quad \text{for} \quad x = L = 1\text{m}$$

Rate of travel of the solid: $V = 10^{-4} m/s$

2 Reference solution

2.1 Method of calculating used for the reference solution

The result of reference is of the semi-analytical type. The equation 1D to be solved is the following one:

$$\begin{cases} V\beta(T)_{,x} - K\,T_{,xx} = 0 \\ \text{avec}\, T_{(x=0)} = T_0 \,\text{et}\, T_{(x=L)} = T_L \end{cases} \quad \text{éq 2.1-1}$$

by integrating the equation [éq 2.1-1] one obtains:

$$\frac{V}{K}\beta(T) - \frac{dT}{dx} = A \quad \text{éq 2.1-2}$$

where A is a constant depending on the boundary conditions, the report $\frac{V}{K}$ and of the function enthalpy $\beta(T)$.

This constant will be analytically given.

The equation [éq 2.1-2] led to:

$$x = \int_{T_0}^{T(x)} \frac{dT}{A + \frac{V}{K}\beta(T)} \quad \text{éq 2.1-3}$$

who must check:

$$L = \int_{T_0}^{T_L} \frac{dT}{A + \frac{V}{K}\beta(T)} \quad \text{éq 2.1-4}$$

Knowing $T_{0,} T_L, L, V, t$ and $\beta(T)$, the equation [éq 2.1-4] must give the value of the constant of integration A .

However, it is difficult (even impossible) to determine this constant analytically, from where the recourse to a digital resolution of the equation [éq 2.1-4] to determine A .

With the facts of the case $(T_{0,} T_L, T_{1,} T_{2,} C_S = C_l, C_{Sl} \ldots)$, we obtained the solution (physical) of A who takes the value $A = -294.9117$.

From this constant, the analytical solution of the problem [éq 2.1-1] is analytical.

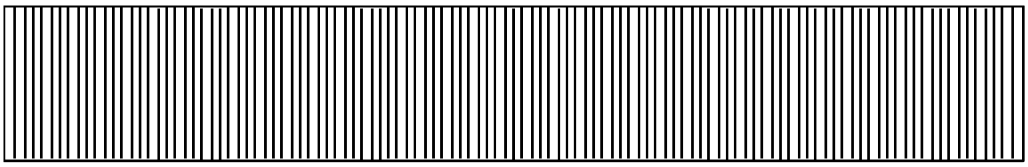
2.2 Results of reference

X-coordinate	Temperature
0.6	387.98514
0.7	451.51001
0,725	469.72232
0,750	488.97505
0,775	509.32766
0.80	530.84296
0,825	553.58738
0.85	577.63114
0.9	683.71269
0.9125	719.51615
0,925	756.32221
0.9375	794.16795
0.95	833.07971
0.9625	873.08751
0.9750	914.22222
0.9875	956.51557

3 Modeling A

3.1 Characteristics of modeling

Modeling 2D



3.2 Characteristics of the grid

80 QUAD8

3.3 Values tested

Identification Temperature	Reference
$N80(X=0.9875)$	956,515
$N79(X=0.9750)$	914,222
$N78(X=0.9625)$	873,087
$N77(X=0.9500)$	833,079
$N76(X=0.9375)$	794,167
$N75(X=0.9250)$	756,322
$N74(X=0.9125)$	719,516
$N73(X=0.9000)$	683,712
$N69(X=0.8500)$	577,631

$N67(X=0.8250)$	553,587
$N65(X=0.8000)$	530,842
$N63(X=0.7750)$	509,327
$N61(X=0.7500)$	488,975
$N59(X=0.7250)$	469,722
$N57(X=0.7000)$	451,510
$N44(X=0.6000)$	387,985

4 Summary of the results

The results are very satisfactory with variations with the reference solution lower than 1%.