SSLX100 - Mixture 3D - Hull - Beam in inflection

Summary:

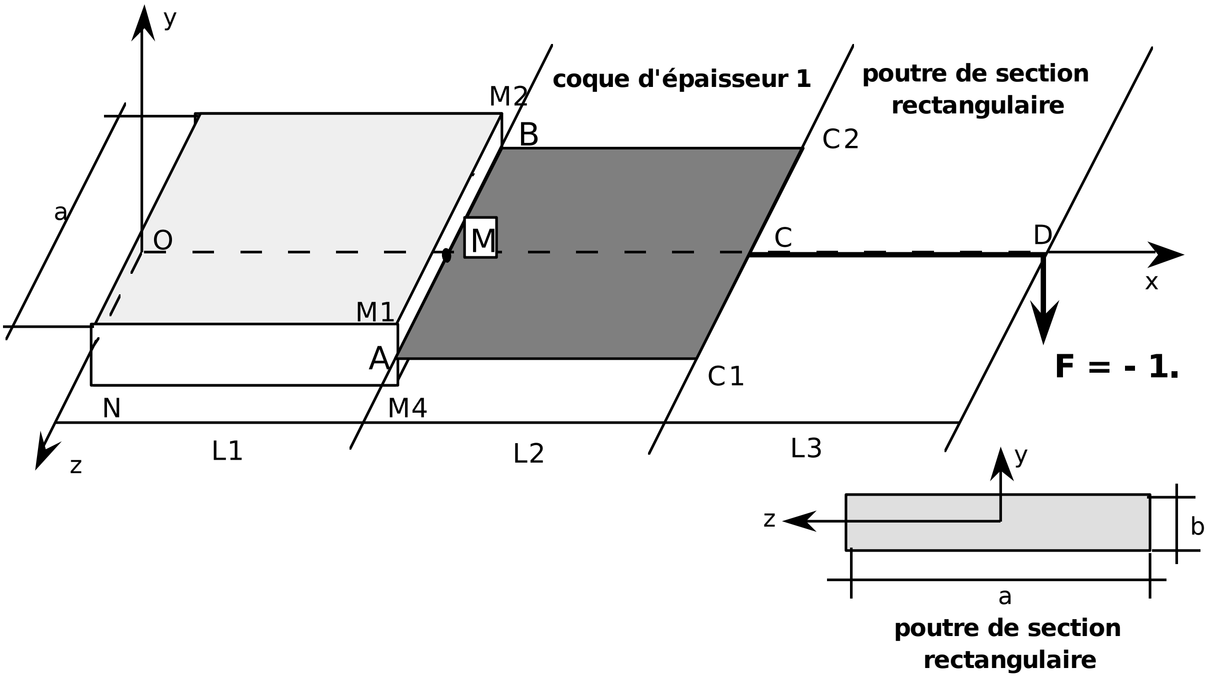
This test makes it possible to validate for a linear elastic design:

- a mixture of various mechanical models: model 3D (element HEXA20), models of hull (elements DKT, or elements COQUE_3D) and model of beam (elements POU_D_E, or elements COQUE_C_PLAN),
- linear relations between degrees of freedom, built in particular with the connections [R3.03.03] and [R3.03.06].

The test is based on the analytical solution **rubber band** of a beam in inflection; , the reduced number of elements for the different ones **models** led to a poor solution, which is however improved clearly with employment of boundary conditions appropriate to the theory of the beams.

1 Problem of reference

1.1 Geometry



L1 = L2 = L3 = 10 mm
b = 1 mm
a = 3 mm

1.2 Material properties

$E=200\,000\,MPa$

$\nu=0.3$

$\nu=0.0$ allows to avoid the variation of orthogonal curve induced by the effect Fish in the plates, which causes a difference between the theories of beams and plates, out of average fibre.

1.3 Boundary conditions and loadings

- force $F_y=-1$ (load 1) or couples $C_z=1$ (load 2)
- defined or applied to neutral fibre
- embedding of the section $x=0$
- continuity of displacements of translation on AB
- continuity of displacements of translation in C
- equality of displacements of rotation around z on $C1-C2$
- for the points M section ($M1$ $M2$ $M4$) displacements of translation $u(M)$ depend linearly on the displacement of rotation φ_z points P of AB

$$u(M)=-\varphi_z(P).y+dx(P)$$

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution, *isostatic structure*.

The elastic arrow, elastic constraints and axial deformations and bending moment in any point of X-coordinate x are given by:

- Load n°1: force $F_y=-1$

$M_y(x)=F_y.L(1-x/L)$ (= $E.I_z.u_y''(x)$ in elasticity)
$u_y(x)=F_y L.x^2.(3-x/L)/(6.E.I_z)$ (in elasticity)
$\varepsilon_{xx}(x,y)=-F_y.L(1-x/L).y/(E.I_z)$ (in elasticity)
$\sigma_{xx}(x,y)=-F_y.L(1-x/L).y/I_z$ (in elasticity)

- Load n°2: couples $C_z=1$ or rotation $dr_z=C_{z.L}/(E.I_z)$

$M_y(x)=C_z$ (= $E.I_z.u_y''(x)$ in elasticity)
$u_y(x)=C_{z.}x^2/(2.E.I_z)$ (in elasticity)
$\sigma_{xx}(x,y)=-C_z.y/I_z$ (in elasticity)
$\varepsilon_{xx}(x,y)=-C_z.y/(E.I_z)$ (in elasticity)

with:

$$L=L1+L2+L3=30\,mm$$
$$I_z=a.h^3/12=0.25\,mm^4$$
$$dr_z=0.0006$$

2.2 Results of reference

Mark with arrows, axial constraints and deformations and bending moments in 4 points of the axis of the beam.

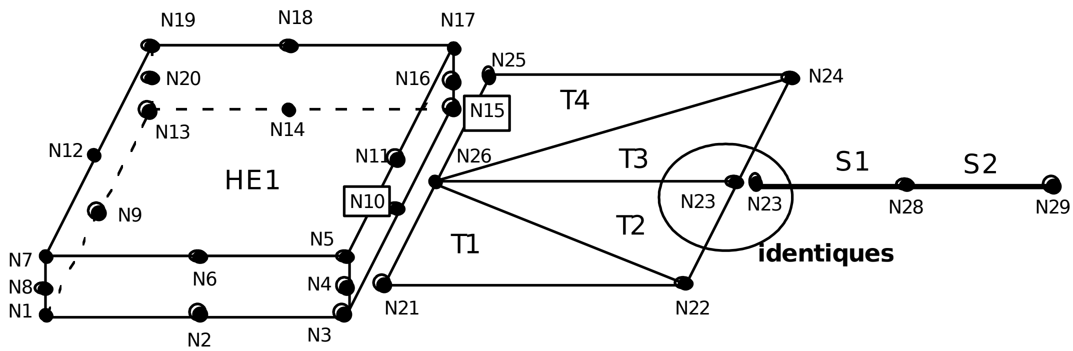
2.3 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling

Modélsiation mixed: 3D, DKT and POU_D_E



Point N --> N1
Point A --> N4
Point C --> N23
Point D --> N29

Load n°1: force F_y
Total embedding on the section in O

3.2 Characteristics of the grid

Many nodes: 28

Many meshs and types: 1 HEXA20 , 4 TRIA3 / DKT , 2 SEG2/ POU_D_E

3.3 Sizes tested and results

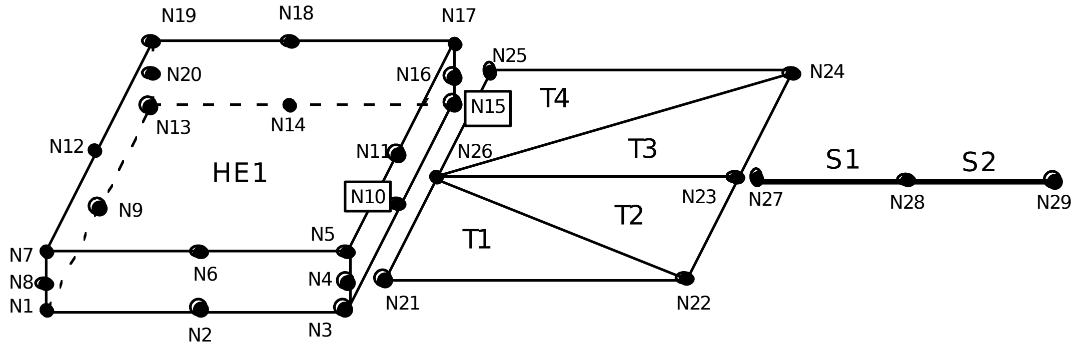
- Load n°1: force F_y

Identification	Type of Reference	Reference	Tolerance (%)
uN (node $N1$)	ANALYTICAL	0.	1,00E-006
uM (node $N26$)	ANALYTICAL	– 0.0267	14.0
uA (node $N4$)	ANALYTICAL	– 0.0267	14.0
$uC1$ (node $N22$)	ANALYTICAL	– 0.0933	8.40
uC (node $N23$)	ANALYTICAL	– 0.0933	8.30
uD (node $N29$)	ANALYTICAL	– 0.18	6.40

4 Modeling B

4.1 Characteristics of modeling

Mixed modeling: `3D`, `DKT`, `POU_D_E` and `DIS_TR`



Point N --> N1
Point A --> N4
Point C --> N23
Point D --> N29

Load n°1: force F_y
Embedding on the section in O realized by a connection `3D_POUTRE` between the face $N1\,N13\,N19\,N7$ and a discrete element located on the origin.

Additional relation, compared to modeling A, enters C_1 C_2 and C, introduced by `LIAISON_ELEM: 'COQ_POU'`.

4.2 Characteristics of the grid

Many nodes: 29

Many meshs and types: 1 `HEXA20` , 4 `TRIA3` / `DKT` , 2 `SEG2`/ `POU_D_E` , 1 `POI1`/`DIS_TR` , 1 `QUAD8` , 2 `SEG2`/`BORD_DKT`

4.3 Sizes tested and results

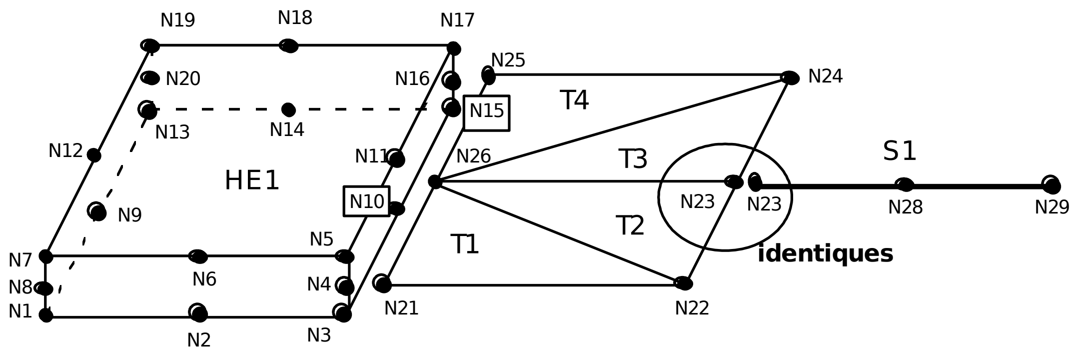
- Load n°1: force F_y

Identification	Type of Reference	Reference	Tolerance ($\%$)
uN (node $N1$)	ANALYTICAL	0.	
uM (node $N26$)	ANALYTICAL	– 0.0267	3.0
uA (node $N4$)	ANALYTICAL	– 0.0267	3.0
$uC1$ (node $N22$)	ANALYTICAL	– 0.0933	1.0
uC (node $N23$)	ANALYTICAL	– 0.0933	1.0
uD (node $N29$)	ANALYTICAL	– 0.18	1.0
M_z (node $N27$)	ANALYTICAL	– 10.	1.0
M_z (node $N29$)	ANALYTICAL	0.	1.0

5 Modeling C

5.1 Characteristics of modeling

Mixed modeling: `3D`, `DKT` and `COQUE_C_PLAN`



Point N --> N1
Point A --> N4
Point C --> N23
Point D --> N29

In $N29$: load n°1: force F_y , load n°2: couples C_z or rotation dr_z
Displacements DY and DZ on the section 0 worthless on average (order `LIAISON_DDL`).

Note:

As the width of the beam is $a=3\text{mm}$, the Young modulus is multiplied by 3 in material of `COQUE_C_PLAN`.

5.2 Characteristics of the grid

Many nodes: 29
Many meshs and types: 1 `HEXA20` , 4 `TRIA3` / `DKT` , 1 `SEG3`/ `COQUE_C_PLAN`

5.3 Sizes tested and results

Identification	Type of Reference	Reference	Tolerance (%)
Load 1: *force* F_y			
uN (node $N8$)	`ANALYTICAL`	0.	6.0e-5
uM (node $N26$)	`ANALYTICAL`	– 0.0267	5.70
uA (node $N4$)	`ANALYTICAL`	– 0.0267	5.70
$uC1$ (node $N22$)	`ANALYTICAL`	– 0.0933	2.10
uC (node $N23$)	`ANALYTICAL`	– 0.0933	2.00
uD (node $N29$)	`ANALYTICAL`	– 0.18	2.0
MyD (node $N28$)	`ANALYTICAL`	-5.	0.50
$\sigma_{xx}(x,h/2)$ (node $N28$)	`ANALYTICAL`	-30.0000	0.50
$\varepsilon_{xx}(x,h/2)$ (node $N28$)	`ANALYTICAL`	– 5.0 10^{-5}	0.50

Load 2: *couples* C_z			
uN (node $N1$)	ANALYTICAL	0.	3.0e-6
uM (node $N26$)	ANALYTICAL	– 0.0010	0.50
uA (node $N4$)	ANALYTICAL	– 0.0010	0.50
$uC1$ (node $N22$)	ANALYTICAL	– 0.0040	0.20
uC (node $N23$)	ANALYTICAL	– 0.0040	0.20
uD (node $N29$)	ANALYTICAL	– 0.0090	0.10
M_{yD} (node $N28$)	ANALYTICAL	1.0	0.50
$\sigma_{xx}(x,h/2)$ (node $N28$)	ANALYTICAL	6.0000	0.50
$\varepsilon_{xx}(x,h/2)$ (node $N28$)	ANALYTICAL	$1.0\ 10^{-5}$	0.50

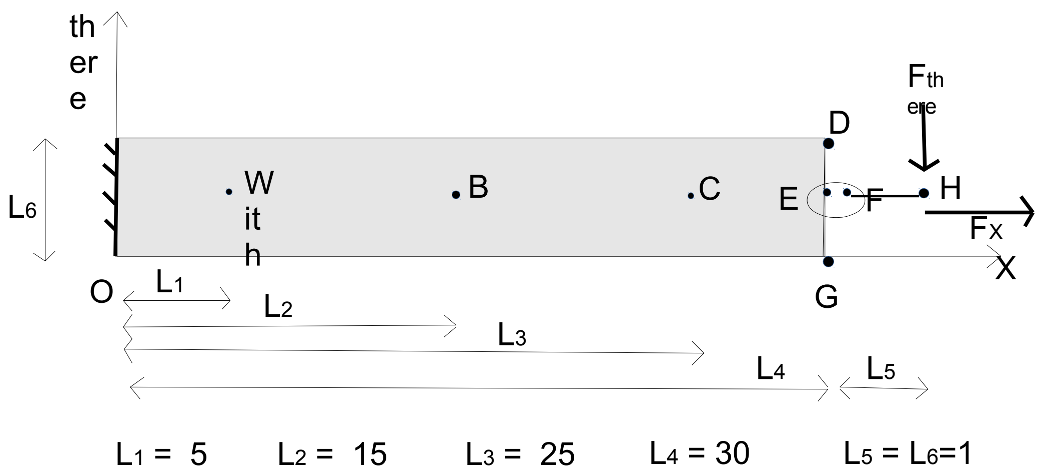
5.4 Remarks

The calculation of the efforts and the moment in the element COQUE_C_PLAN is carried out with the node medium $N28$ so that the interpolation is correct; knowing that the Young modulus is triple so that the product $E.I_z$ that is to say identical in all the model, the constraints are it too.

6 Modeling F

6.1 Characteristics of modeling

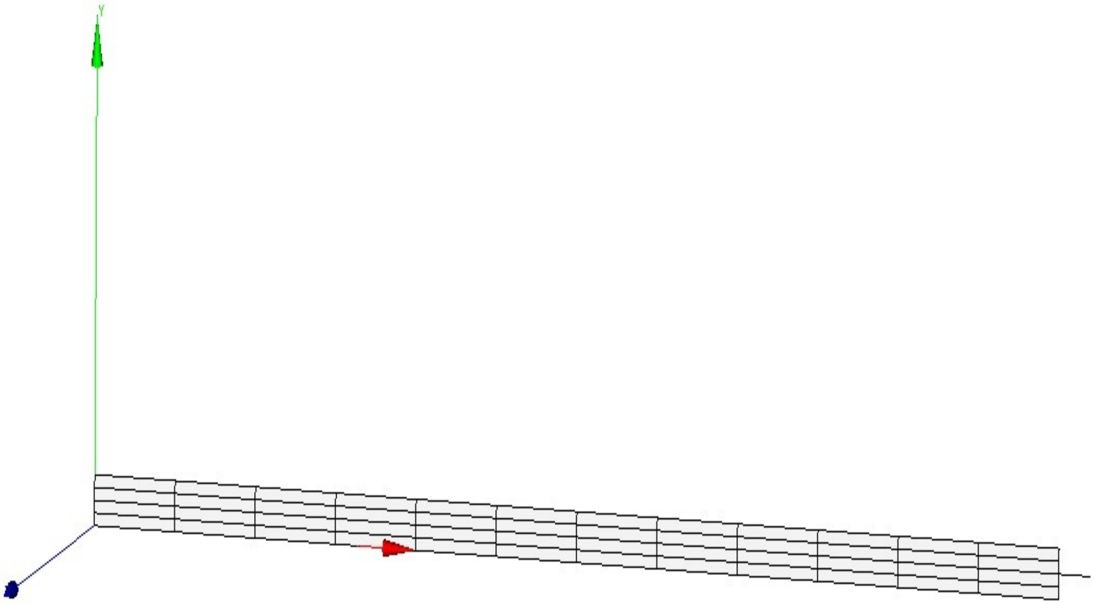
Mixed modeling: C_PLAN , 2D_DIS_TR



In H : load n°1: force F_x , load n°2: F_y
Displacements u_x and u_y worthless on the left edge of the 2D part.

6.2 Characteristics of the grid

Many nodes: 179
Many meshs and types : 8 SEG3, 1 SEG2, 48 QUAD8 (=12 X 4)



6.3 Sizes tested and results

Identification	Type of Reference	Reference	Tolerance ($\%$)
Load 1: *force* F_x			
$u_x(A)$ (node $N13$)	ANALYTICAL	2,50	1.0e-5
$u_x(B)$ (node $N75$)	ANALYTICAL	7,50	1.0e-5
$u_x(C)$ (node $N131$)	ANALYTICAL	12,50	1.0e-5
$u_x(D)$ (node $N141$)	ANALYTICAL	15,00	1.0e-5
$u_x(E)$ (node $N133$)	ANALYTICAL	15,00	1.0e-5
$u_x(F)$ (node $N178$)	ANALYTICAL	15,00	1.0e-5
$u_x(G)$ (node $N125$)	ANALYTICAL	15,00	1.0e-5
$u_x(H)$ (node $N179$)	ANALYTICAL	15,25	1.0e-5
Load 2: *force* F_y			
$u_y(A)$ (node $N13$)	ANALYTICAL	2.125e-2	1.0e-2
$u_y(B)$ (node $N75$)	ANALYTICAL	-1.6875e-1	1.0e-2
$u_y(C)$ (node $N131$)	ANALYTICAL	-4,0625	1.0e-2
$u_y(D)$ (node $N141$)	ANALYTICAL	-0,54	1.0e-2
$u_y(E)$ (node $N133$)	ANALYTICAL	-0,54	1.0e-2
$u_y(F)$ (node $N178$)	ANALYTICAL	-0,54	1.0e-2
$u_y(G)$ (node $N125$)	ANALYTICAL	-0,54	1.0e-2
$u_y(H)$ (node $N179$)	ANALYTICAL	-0,54	1.0e-2

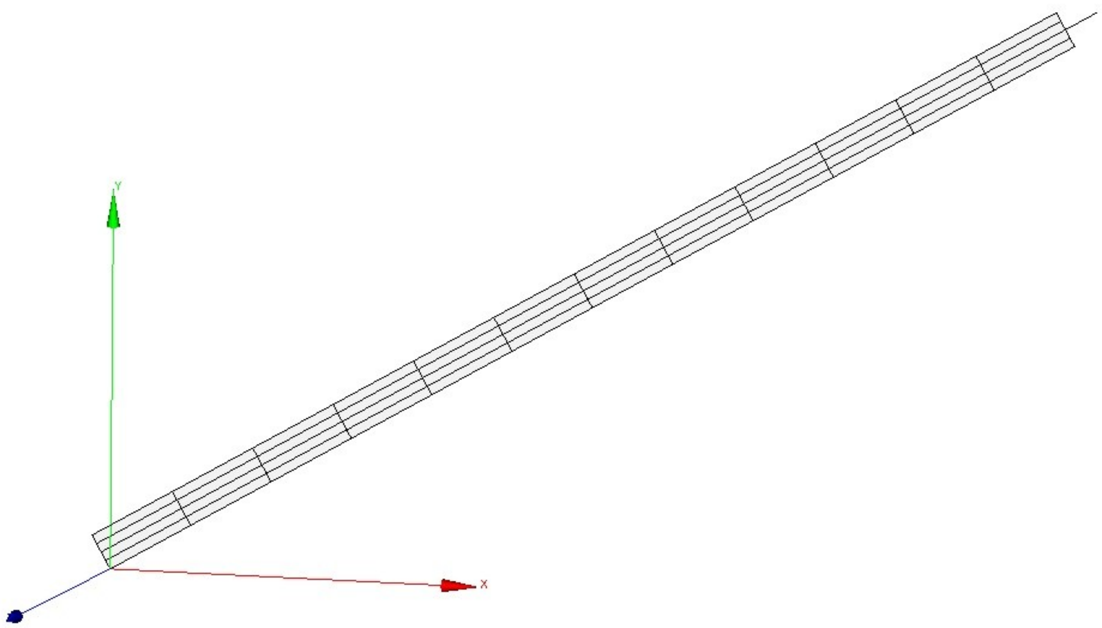
7 Modeling G

7.1 Characteristics of modeling

Modeling G = Modeling F (`C_PLAN` , `2D_DIS_TR)` + rotation of 30° in the beginning.

7.2 Characteristics of the grid

Many nodes: 179
Many meshs and types : `8 SEG3, 1 SEG2, 48 QUAD8 (=12 X 4)`



7.3 Sizes tested and results

Identification	Type of Reference	Reference	Tolerance (%)
Load 1: *force* F_x			
$u_x(A)$ (node $N13$)	ANALYTICAL	2,165064	1.0e-6
$u_x(B)$ (node $N75$)	ANALYTICAL	6,495191	1.0e-6
$u_x(C)$ (node $N131$)	ANALYTICAL	10,825318	1.0e-6
$u_x(D)$ (node $N141$)	ANALYTICAL	12,990381	1.0e-6
$u_x(E)$ (node $N133$)	ANALYTICAL	12,990381	1.0e-6
$u_x(F)$ (node $N178$)	ANALYTICAL	12,990381	1.0e-6
$u_x(G)$ (node $N125$)	ANALYTICAL	12,990381	1.0e-6

$u_x(H)$ (node $N179$)	ANALYTICAL	13,208874	1.0e-6
Load 2: *force* F_y			
$u_y(A)$ (node $N13$)	ANALYTICAL	-1.8403e-2	1.0e-2
$u_y(B)$ (node $N75$)	ANALYTICAL	-0,146142	1.0e-2
$u_y(C)$ (node $N131$)	ANALYTICAL	-0,351823	1.0e-2
$u_y(D)$ (node $N141$)	ANALYTICAL	-0,467654	1,5
$u_y(E)$ (node $N133$)	ANALYTICAL	-0,467654	1.0e-2
$u_y(F)$ (node $N178$)	ANALYTICAL	-0,467654	1.0e-2
$u_y(G)$ (node $N125$)	ANALYTICAL	-0,467654	1,5
$u_y(H)$ (node $N179$)	ANALYTICAL	-0,467654	1.0e-2

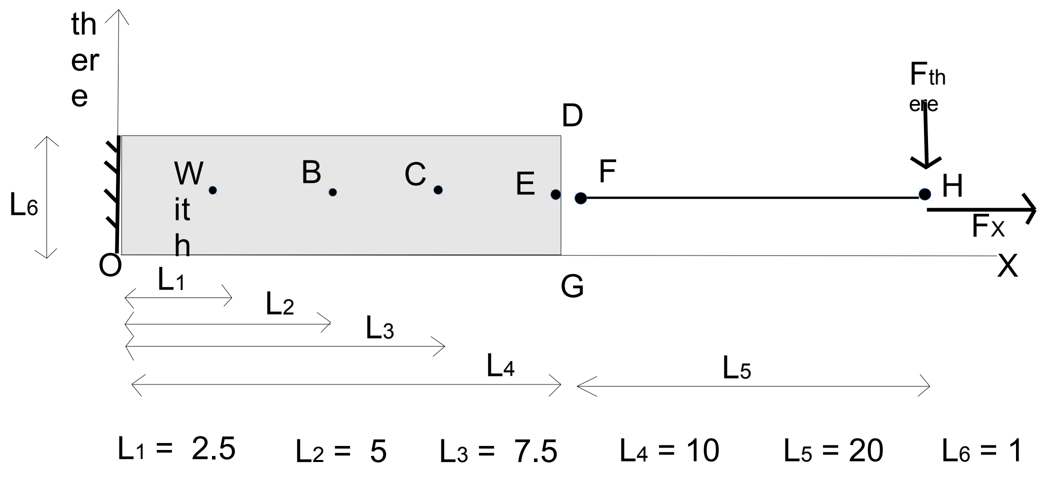
7.4 Remarks

The analytical values of displacement are those of modeling G multiplied by the cosine of 30 degrees.

8 Modeling H

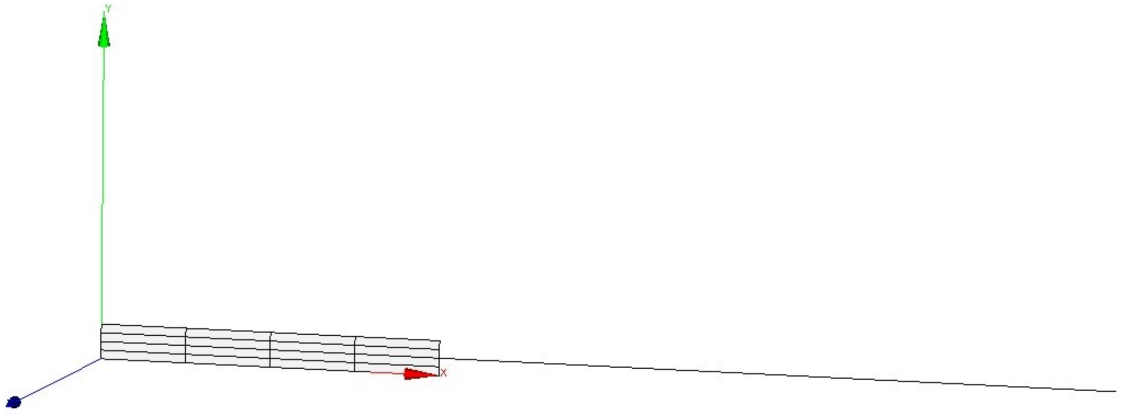
8.1 Characteristics of modeling

Mixed modeling: `C_PLAN` , `POU_D_E`



8.2 Characteristics of the grid

Many nodes: 67
Many meshs and types : `8 SEG3, 1 SEG2, 16 QUAD8 (=4 X 4)`



8.3 Sizes tested and results

Identification	Type of Reference	Reference	Tolerance ($\%$)
Load 1: *force* F_x			
$u_x(A)$ (node $N12$)	ANALYTICAL	1,25	1.0e-6
$u_x(B)$ (node $N13$)	ANALYTICAL	2,50	1.0e-6
$u_x(C)$ (node $N14$)	ANALYTICAL	3,75	1.0e-6
$u_x(D)$ (node $N25$)	ANALYTICAL	5,00	1.0e-6
$u_x(E)$ (node $N15$)	ANALYTICAL	5,00	1.0e-6
$u_x(F)$ (node $N66$)	ANALYTICAL	5,00	1.0e-6
$u_x(G)$ (node $N5$)	ANALYTICAL	5,00	1.0e-6
$u_x(H)$ (node $N67$)	ANALYTICAL	15,00	1.0e-6
Load 2: *force* F_y			
$u_y(A)$ (node $N12$)	ANALYTICAL	-5.46875e-3	1.0e-2
$u_y(B)$ (node $N13$)	ANALYTICAL	-2.125e-2	1.0e-2
$u_y(C)$ (node $N14$)	ANALYTICAL	-4.640625e-2	1.0e-2
$u_y(D)$ (node $N25$)	ANALYTICAL	-0,08	1.0e-2
$u_y(E)$ (node $N15$)	ANALYTICAL	-0,08	1.0e-2
$u_y(F)$ (node $N66$)	ANALYTICAL	-0,08	1.0e-2
$u_y(G)$ (node $N5$)	ANALYTICAL	-0,08	1.0e-2
$u_y(H)$ (node $N67$)	ANALYTICAL	-0,54	1.0e-3

9 Summary of the results

The studied cases are:

Modeling	Connections
A: 3D, DKT and POU_D_E	LIAISON_GROUP
B: 3D, DKT, POU_D_E and DIS_TR	LIAISON_GROUP; LIAISON_ELEM: 3D_POU, COQ_POU
C: 3D, DKT and COQUE_C_PLAN	LIAISON_GROUP
D: 3D, COQUE_3D, POU_D_E and DIS_TR	LIAISON_GROUP; LIAISON_ELEM: 3D_POU, COQ_POU
F: C_PLAN , 2D_DIS_TR	LIAISON_ELEM: 2D_POU
G: C_PLAN , 2D_DIS_TR	LIAISON_ELEM: 2D_POU
H: C_PLAN , POU_D_E	LIAISON_ELEM: 2D_POU

- The grid is very coarse in elements $3\mathrm{D}$ and plates. The test deserves a finer modeling, since the results are influenced by the way of describing the conditions of embedding in O. Modeling A conduit with an error of 14% to the maximum,
- however with a good taking into account of these conditions, the solution is definitely better (modeling B conduit with an error of 3% maximum).
- The comparisons of the constraints and efforts give good performances (modeling B). For the element of hull $1\mathrm{D}$, the results are very good.