SSLV304 – Cylinder under variable external pressure

Summary:

The objective of this CAS-test is to validate the application of a pressure on an axisymmetric structure, starting from a decomposition in Fourier series of the load (modeling AXIS_FOURIER).

The pressure applied is function of the three coordinates of space (r, θ, z).

1 Problem of reference

1.1 Geometry

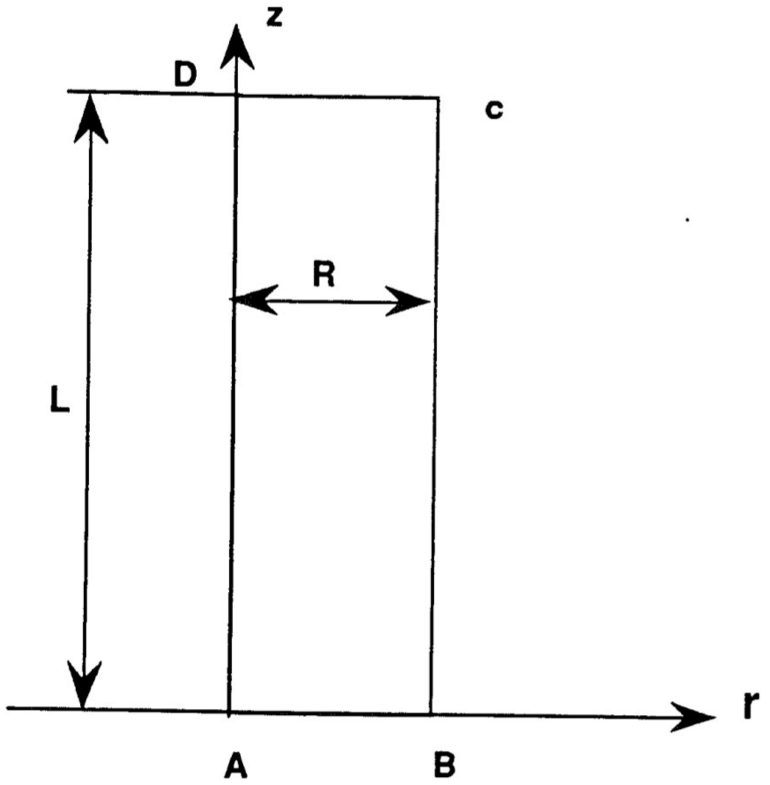

Figure 1.1 Geometry of the problem and system of loading

Length: $L=0.24\,m$
Ray: $R=0.006\,m$

1.2 Properties of material

Young modulus	$E=2.1\times10^{11}\,Pa$
Poisson's ratio	$\nu=0.3$

1.3 Boundary conditions and loadings

Imposed displacement:

Embedding on the side AB	$DX=0$, $DY=0$, $DZ=0$

Imposed loading: $p_0=10\,000\,Pa$

External pressure on the side BC	Radial component: $-p_0\frac{R}{L}\sin(\theta)$ (normal with the circumference)

	Axial component: $p_0 \frac{z}{L} \sin(\theta)$ (according to z)
External pressure on the side DC	Radial component: $p_0 \sin(\theta)$ (according to r) Circumferential component: $p_0 \cos(\theta)$ (according to θ) Axial component: $\frac{1-\nu}{\nu} \frac{p_0 r}{L} \sin(\theta)$ (normal with the section)

2 Reference solution

2.1 Method of calculating used for the reference solution

The deformation due to direct compression is given by:

$$u_r=\frac{p_0}{\nu E L}\times\frac{1}{1-2\nu}\times\frac{z^2}{2}\sin(\theta)\ ;\ u_z=\frac{-p_0}{\nu E L}rz\sin(\theta)\ ;\ u_\theta=\frac{p_0}{\nu E L}\times\frac{1}{1-2\nu}\times\frac{z^2}{2}\cos(\theta)\ .$$

The stress field is worth:

$$\sigma_{rr}=-\frac{p_0 r\sin(\theta)}{L}\ ;\ \sigma_{zz}=-\frac{1-\nu}{\nu L}p_0 r\sin(\theta)\ ;\ \sigma_{\theta\theta}=-\frac{p_0 r\sin(\theta)}{L}\ ;$$

$$\sigma_{rz}=\frac{p_0 z\sin(\theta)}{L}\ ;\ \sigma_{r\theta}=0\ ;\ \sigma_{\theta z}=\frac{p_0 z\cos(\theta)}{L}\ .$$

2.2 Results of reference

Radial displacements (DX), vertical (DY) and ortho-radial (DZ) at the points C and D for an angle $\theta=45°$.
Constraints at the points A and B .

2.3 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling A

Modeling AXIS_FOURIER.

3.2 Characteristics of the grid

Many nodes: 455
Many meshs and types: 400 TRIA3, 160 QUAD4

3.3 Sizes tested and results

For $\theta=45$,

Size	Component	Localization	Type of reference	Value of reference	Tolerance
DEPL	DX	Not D ($N461$)	'ANALYTICAL'	1.75099×10^{-8} m	3.7 %
DEPL	DY	Not D ($N461$)	'ANALYTICAL'	0.0	9.E-3 Pa
DEPL	DZ	Not D ($N461$)	'ANALYTICAL'	1.75099×10^{-8}	3.7 %
DEPL	DX	Not C ($N460$)	'ANALYTICAL'	1.75099×10^{-8}	3.7 %
DEPL	DY	Not C ($N460$)	'ANALYTICAL'	-3.5019×10^{-10}	8.8 %
DEPL	DZ	Not C ($N460$)	'ANALYTICAL'	1.75099×10^{-8}	3.7 %
SIGM_ELNO	SIXX	Not B ($N5$)	'ANALYTICAL'	-176.77	1.8 %
SIGM_ELNO	SIYY	Not B ($N5$)	'ANALYTICAL'	-412.48	3.8 %
SIGM_ELNO	SIZZ	Not B ($N5$)	'ANALYTICAL'	-176.77	3.2 %
SIGM_ELNO	SIXY	Not B ($N5$)	'ANALYTICAL'	0.0	5.0 Pa
SIGM_ELNO	SIXX	Not A ($N1$)	'ANALYTICAL'	0.0	45.0 Pa
SIGM_ELNO	SIYY	Not A ($N1$)	'ANALYTICAL'	0.0	105.0 Pa
SIGM_ELNO	SIZZ	Not A ($N1$)	'ANALYTICAL'	0.0	45.0 Pa
SIGM_ELNO	SIXY	Not A ($N1$)	'ANALYTICAL'	0.0	30.0 Pa

3.4 Remarks

The test reveals the need, in modelings with linear elements, to use very fine grids to arrive to satisfactory results.

4 Summary of the results

The results are in concord with the analytical solution.