

## SSLV140 - Calculation of effective modules by a method Python

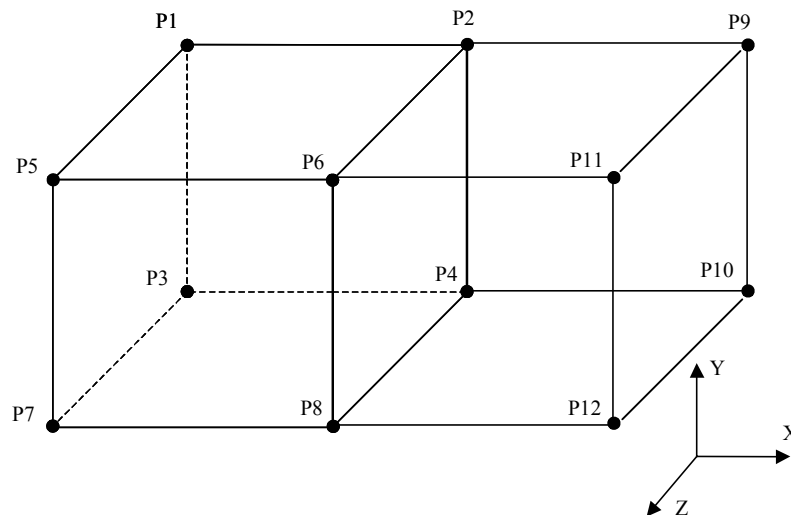
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### Summary:

One presents a test here having an analytical reference. The treated geometry is a set of two cubes having different elastic properties. The goal is to find the Young modulus of the mixture made up of these two cubes along two directions.

## 1 Problem of reference

### 1.1 Geometry



Following surfaces are defined:

- Face  $YZI$  : containing the nodes  $P1, P3, P5$  and  $P7$ .
- Face  $YZ2$  : containing the nodes  $P9, P10, P11$  and  $P12$ .
- Face  $XY1$  : containing the nodes  $P1, P2, P9, P3, P4$  and  $P10$ .
- Face  $XY2$  : containing the nodes  $P5, P6, P11, P7, P8$  and  $P12$ .
- Face  $XZ1$  : containing the nodes  $P3, P4, P10, P7, P8$  and  $P12$ .
- Face  $XZ2$  : containing the nodes  $P1, P2, P9, P5, P6$  and  $P11$ .

and following elements:

- Element  $M1$  : containing the nodes  $P1, P2, P3, P4, P5, P6, P7$  and  $P8$ .
- Element  $M2$  : containing the nodes  $P2, P9, P4, P10, P6, P11, P8$  and  $P12$ .

### 1.2 Material properties

Two materials are used:

- Material  $MAT1$  allotted to the element  $M1$  :  
Young modulus:  $E1 = 200000 \text{ MPa}$   
Poisson's ratio:  $\nu_1 = 0.3$
- Material  $MAT2$  allotted to the element  $M2$  :  
Young modulus:  $E2 = 100000 \text{ MPa}$   
Poisson's ratio:  $\nu_2 = 0.3$

## 1.3 Boundary conditions and loadings

### The first calculation:

It is a simple calculation of traction according to the direction  $X$  :

- A linear elastic strain is imposed  $\varepsilon_{xx} = 1$  on surface  $YZZ$  .
- Surface  $YZI$  does not move according to the direction  $X$  .

### The second calculation:

It is a simple calculation of traction according to the direction  $Y$  :

- A linear elastic strain is imposed  $\varepsilon_{yy} = 1$  on surface  $XZZ$  .
- Surface  $XZI$  does not move according to the direction  $Y$  .

## 2 Reference solution

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### 2.1 Method of calculating

According to the general theory of the homogenisation of composite materials [bib1], the effective Young moduli  $E_{xx}^{eff}$  and  $E_{yy}^{eff}$  according to the directions  $X$  and  $Y$  of a mixture having the form given above, are given by the following formulas:

$$\frac{1}{E_{xx}^{eff}} = \frac{f_1}{E_1} + \frac{f_2}{E_2}$$
$$E_{yy}^{eff} = f_1 E_1 + f_2 E_2$$

$f_1$  and  $f_2$  are the voluminal fractions of each material, in our case:

$$f_1 = f_2 = 0.5$$

### 2.2 Bibliographical references

- 1) Mr. BORNET, T. BRETHERAU and P. GILORMINI: Homogenisation in mechanics of materials (T1). Hermes Science Publications - 2001.

## 3 Modeling A

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### 3.1 Characteristics of the grid

Many nodes: 12.  
Modeling 3D : 2 quadratic elements of volume: HEXA8.

### 3.2 Features tested

Orders Python are inserted directly in the command file ASTER. These orders are used to write functions of postprocessing on the fields of results, like the averages, the trace of a tensor of deformations or constraints,... etc the fields of results are recovered by the order EXTR\_COMP.

### 3.3 Values tested

#### The first calculation:

The Young modulus following the direction  $X$  in this case is the average of the constraints  $\sigma_{xx}$  :

$$E_{xx}^{eff} = \langle \sigma_{xx} \rangle$$

#### The second calculation:

The Young modulus following the direction  $Y$  in this case is the average of the constraints  $\sigma_{yy}$  :

$$E_{yy}^{eff} = \langle \sigma_{yy} \rangle$$

Identification	Reference	Aster	% difference
$\langle \sigma_{xx} \rangle$	133333	134134	1.00
$\langle \sigma_{yy} \rangle$	150000	150000	0.00

## 4 Summary of the results

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The got results are in perfect agreement with the reference solution.