

SSLV139 - Buckling of a circular plate subjected to a compressive force uniformly distributed on its contour

Summary:

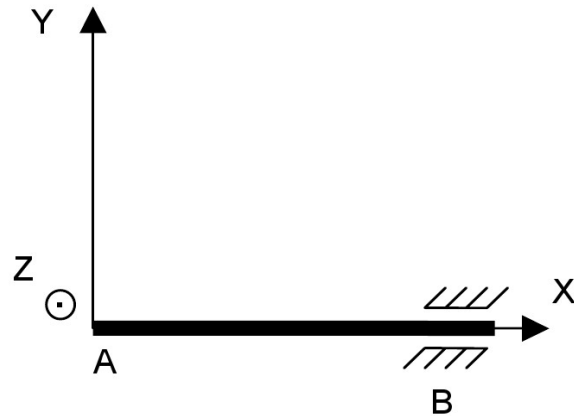
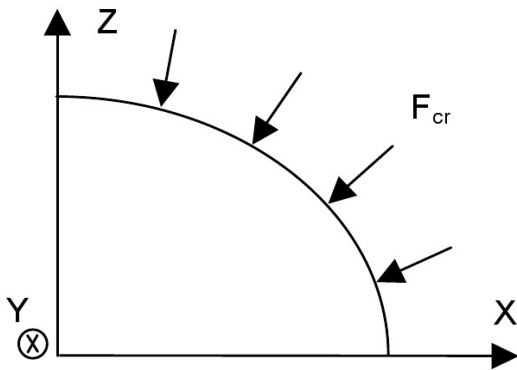
This test represents a calculation of stability of a circular plate subjected to a compressive force uniformly distributed on its contour. We determine the critical load leading to the elastic buckling of Euler as well as the associated modal deformation.

This test validates modeling `AXIS_FOURIER` for linear buckling with meshes `QUAD8` and `TRIA6` (circumferential mode equal to zero), and modeling `AXIS` with meshes `QUAD8`.

The critical load obtained is compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



- Thickness of the plate: $h=0.0005\text{ m}$
- Ray of the plate : $R=0.115\text{ m}$

1.2 Properties of material

The properties of material constituting the plate are:

$$E = 2.1 \cdot 10^{11} \text{ Pa} \quad \text{Young modulus}$$

$$\nu = 0.3 \quad \text{Poisson's ratio}$$

1.3 Boundary conditions and loadings

Boundary conditions:

- on the contour of the plate (not B) : following displacement $Y=0$ and rotation around $Z=0$

Loading: one applies a compressive force uniformly distributed F on the external contour of the plate.

1.4 Initial conditions

Without object.

2 Reference solution

2.1 Method of calculating

The value of the critical load is given in [bib1] by the following expression:

$$F_{cr} = \frac{14.68 D}{R^2}$$

with: D the bending stiffness of the plate (in $N.m$) defined by the following expression:

$$D = \frac{E h^3}{12(1-\nu^2)}$$

This critical load is associated with a circumferential mode equal to 0.

2.2 Sizes and results of reference

For the characteristics given, the critical load is worth:

$$F_{cr} = 2668.315 N/m$$

2.3 Uncertainties on the solution

Analytical solution

2.4 Bibliographical references

- 1) S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, second edition, DUNOD (1966)

3 Modeling A

3.1 Characteristics of modeling

Modeling AXIS_FOURIER (QUAD8): 3 degrees of freedom per node (DX, DY, DZ)

Limiting conditions:

- group of meshes AD : $DX=0, DZ=0$
- group of meshes BC : $DY=0, DZ=0$

Characteristics of the discretization

- Sides AB and CD : 460 elements
- Sides AD and BC : 4 elements

3.2 Characteristics of the grid

Many nodes: 6449

Many meshes: 1840 (QUAD8)

3.3 Sizes tested and results

Identification	Reference
Pressure criticizes ($n=0$)	$8.4935 \cdot 10^5 Pa$
Displacement DY with the node D	1

3.4 Remarks

- Critical pressure P_{cr} of reference, used in the command file, was obtained starting from the critical load referred in the paragraph [§2.2]:

$$P_{cr} = \frac{F_{cr}}{2\pi h} = 849350.94 N/m^2$$

- The standardisation of the clean mode for largest of the components of translation implies a value of reference equal to 1 for displacement DY with the node D .

4 Modeling B

4.1 Characteristics of modeling

Modeling AXIS (QUAD8): 2 degrees of freedom per node (DX, DY)

Limiting conditions:

- group of meshes AD : $DX=0$
- group of meshes BC : $DY=0$

Characteristics of the discretization

- Sides AB and CD : 460 elements
- Sides AD and BC : 4 elements

4.2 Characteristics of the grid

Many nodes: 6449
Many meshes: 1840 (QUAD8)

4.3 Sizes tested and results

Identification	Reference
Pressure criticizes ($n=0$)	$8.4935 \cdot 10^5 Pa$
Displacement DY with the node D	1

4.4 Remarks

- Critical pressure P_{cr} of reference, used in the command file, was obtained starting from the critical load referred in the paragraph [§2.2]:

$$P_{cr} = \frac{F_{cr}}{2\pi h} = 849350.94 N/m^2$$

- The standardisation of the clean mode for largest of the components of translation implies a value of reference equal to 1 for displacement DY with the node D .

5 Modeling C

5.1 Characteristics of modeling

Modeling AXIS_FOURIER (TRIA6): 3 degrees of freedom per node (DX , DY , DZ)

Limiting conditions:

- group of meshes AD : $DX=0$, $DZ=0$.
- group of meshes BC : $DY=0$, $DZ=0$.

Characteristic of the discretization

- Sides AB and CD : 690 elements
- Sides AD and BC : 6 elements

5.2 Characteristics of the grid

Many nodes: 17,964

Many meshes: 8,280 (TRIA6)

5.3 Sizes tested and results

Identification	Reference
Pressure criticizes ($n=0$)	$8.4935 \cdot 10^5 Pa$
Displacement DY with the node D	1

5.4 Remarks

- Critical pressure P_{cr} of reference, used in the command file, was obtained starting from the critical load referred in the paragraph [§2.2]:

$$P_{cr} = \frac{F_{cr}}{2\pi h} = 849350.94 N/m^2$$

- The standardisation of the clean mode for largest of the components of translation implies a value of reference equal to 1 for displacement DY with the node D .

6 Summary of the results

The got results are very satisfactory for the meshes QUAD8 independently of modeling used (AXIS or AXIS_FOURIER) : uncertainties on the critical pressure do not exceed 0.104%.

However, it will be noted that modeling AXIS_FOURIER is definitely less precise with meshes TRIA6 that with meshes QUAD8.

This test with licence to test and compare modelings AXIS and AXIS_FOURIER in linear buckling of Euler of a circular mean structure subjected to an external force of compression uniformly distributed on its contour.