SSLV134 - Circular crack in infinite medium

Summary

This test allows, after obtaining the field of displacement by MECA_STATIQUE, the calculation of the rate of refund of energy room for a circular crack plunged in a presumedly infinite medium.

For the first modeling, only a half space defined by the plan of the crack is represented. The bottom of crack is then a closed curve (a circle) and is defined as such in DEFI_FOND_FISS. The rate of refund local and total is compared to the analytical solution of reference.

Seven following modelings make it possible to calculate the stress intensity factors $K1$ and $K3$, in $3D$ and axisymmetric, calculated by POST_K1_K2_K3 and/or CALC_G.

- Modeling A tests G for a grid $3D$ with the closed crack,
- Modeling B tests $K1$ for a grid $3D$,
- Modeling C tests $K1$ for an axisymmetric grid,
- Modeling D tests the combination of $K1$ and $K3$ for a grid $3D$.
- Modeling G tests $K1$ for a grid $3D$ with a bottom of crack defined by two groups of nodes coïncidents.
- Modeling H tests $K1$ for a crack nonwith a grid (method X-FEM)
- Modeling I tests $K1$ for an axisymmetric crack nonwith a grid (method X-FEM)
- Modeling J tests G for a grid $3D$ with the crack closed for the incompressible elements,
- Modeling K tests $K1$ for an axisymmetric grid for the incompressible elements,
- Modeling L tests G for a grid $3D$ with the closed crack (method X-FEM).

Modelings E and F make it possible to validate the calculation of the bilinear form of G on the same problem. Lastly, modeling F makes it possible to validate the options G_MAX and CALC_K_MAX.

1 Problem of reference

1.1 Geometry

The crack is circular (penny shaped ace) of ray a , in the plan Oxy . So that the medium is regarded as infinite, the sizes characteristic of the solid mass are about 5 times higher than the ray a .

1.2 Material properties

Young modulus: $E=2.10^5\,MPa$

Poisson's ratio: $\nu=0.3$

Density: $\rho=7850\,kg/m^3$

1.3 Boundary conditions and loadings

Lower face : uniform constraint of traction $\sigma_z=1.\,MPa$

Higher face : uniform constraint of traction $\sigma_z=1.\,MPa$

According to modeling, one also has boundary conditions of symmetry and blocking of the movements of rigid body.

In the modeling D where only the quarter of the parallelepiped is represented, one uses boundary conditions of antisymetry for the loading of torsion: they amount imposing worthless tangential displacements on a face. The loading of torsion is introduced in the form of a tangential surface force (shearing distributed) applied to the lips of the crack.

- Upper lip: $F_X=-\tau\frac{Y}{a}$ and $F_Y=+\tau\frac{X}{a}$
- Lower lip: $F_X=+\tau\frac{Y}{a}$ and $F_Y=-\tau\frac{X}{a}$

2 Reference solution

2.1 Method of calculating used for the reference solution

For a circular crack of ray a in an infinite medium, subjected to a uniform traction σ according to the normal with the plan of the lips, the rate of refund of energy room $G(s)$ is independent of the curvilinear X-coordinate s and is worth [bib1]:

$$G(s)=\frac{(1-\nu^2)}{\pi E}4\sigma^2 a$$

then the coefficient of intensity of constraint $K1$ is given by the formula of Irwin:

$$G(s)=\frac{(1-\nu^2)}{E}K_1^2 \text{ that is to say } K_I=\frac{2\sigma\sqrt{a}}{\sqrt{\pi}}$$

If this crack is subjected to a shearing distributed on the lips: $\sigma_{\theta z}=\tau\frac{r}{a}$

(what is equivalent to a torsion ad infinitum), then one is in pure mode 3 and the stress intensity factor corresponding is worth:

$K_3=\frac{4\tau\sqrt{a}}{3\sqrt{\pi}}$ thus by the formula of Irwin $G(s)=\frac{(1+\nu)}{E}K_3^2$

In the presence of the two combined modes, one will have:

$$G(s)=\frac{(1-\nu^2)}{E}K_1^2+\frac{(1+\nu)}{E}K_3^2$$

The theta-method connects the rates of refund of energy total and local by the following variational equation:

$$G_{réf}(\boldsymbol{\theta})=\int_\Gamma G(s)\boldsymbol{\theta}\cdot\boldsymbol{m}(s)ds$$

where $\boldsymbol{m}(s)$ is the normal at the bottom of crack Γ and θ is the field speed of a virtual propagation of the crack.

If one chooses for θ the normal unit field at the bottom of crack, one obtains, since $G(s)$ is constant on all the bottom of crack:

$$G_{réf}(\boldsymbol{\theta})=G(s)\cdot 2\Pi a$$

2.2 Results of reference

Digital application (case with loading of traction only):

For the loading considered and $a=2m$, one obtains then:

$$G(s)=11.586 J/m^2$$
$$G_{réf}=145.60 J/m$$
$$K1=1.5958\text{E}6 J/m^2$$

For modeling G (3 different funds of crack) with the same loading, one obtains:

for $a=2m$

$$G(s)=10.586\,J/m^2$$
$$K1=1.5958\text{E}6\,J/m^2$$

for $a=1.88m$

$$G(s)=10.891\,J/m^2$$
$$K1=1.5472\text{E}6\,J/m^2$$

for $a=1.76\,m$

$$G(s)=10.196\,J/m^2$$
$$K1=1.4969\text{E}6\,J/m^2$$

Digital application (case with loading of torsion only):

$$G(s)=7.3565\,J/m^2$$
$$G_{réf}=92.44\,J/m$$
$$K1=1.0638\text{E}6\,J/m^2$$

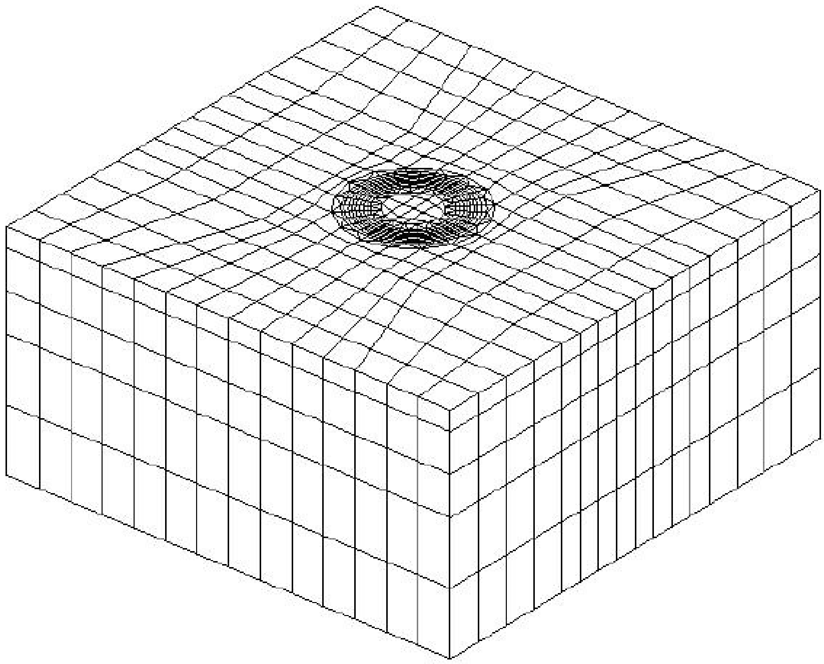
2.3 References bibliographical

1) Solution of Sneddon (1946) in G.C. SIH: Handbook of stress-intensity factors Institute of Fracture and Solid Mechanics - Lehigh University Bethlehem, Pennsylvania

3 Modeling A

The bottom of crack is closed. One calculates G.

3.1 Characteristics of modeling



The interest of this modeling is to represent the entirety of the bottom of crack which is a closed curve, without benefitting from symmetries of the problem.

Only the loading of traction is taken into account.

3.2 Characteristics of the grid

Many nodes: 11114
Number of meshs and type: 2432 PENTA15

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges.

3.3 Notice

The keyword is used SYME in the operator CALC_G to multiply automatically by two the rate of refund of energy calculated on only one lip of the crack, when the operand FOND_FISS is absent. When FOND_FISS is present, information on symmetry is recovered directly in the concept fond_fiss created via DEFI_FOND_FISS.

The principle is the same one for POST_K1_K2_K3. The indication of symmetry induced the calculation of the factors of intensity of the constraints and the rate of refund of energy G_IRWIN by interpolation of displacements of a single lip of the crack. The displacement of the nodes mediums of the edges of elements touching the bottom of crack to the quarter of these edges would make it possible to improve the precision of calculation.

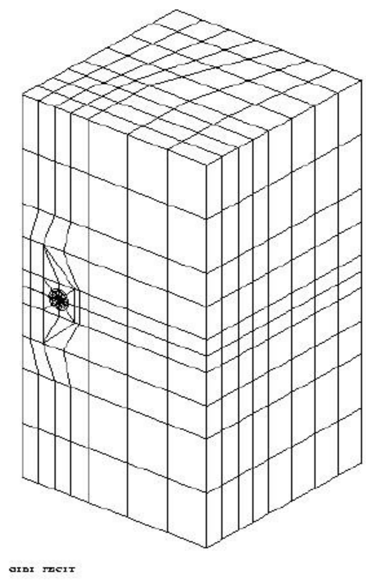
3.4 Sizes tested and results

Identification	Reference	*Type of reference*	$\%$ tolerance
G total	145.6	ANALYTICAL	1.2
G local Node $N403$ - G Lagrange	11,586	ANALYTICAL	3,0
G local Node $N2862$ - G Lagrange	11,586	ANALYTICAL	2,0
G local Node $N375$ - G Lagrange	11,586	ANALYTICAL	3,0
G local Node $N292$ - G Lagrange	11,586	ANALYTICAL	2.4
$max(G\,local)$ - G Lagrange	11,59	ANALYTICAL	2.5
$min(G\,local)$ - G Lagrange	11,59	ANALYTICAL	2,0
G local Node $N403$ - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G local Node $N2862$ - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G local Node $N375$ - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G local not 1 LAGRANGE / NB_POINT_FOND=33	11,586	ANALYTICAL	1,0
G local not 13 LAGRANGE / NB_POINT_FOND=33	11,586	ANALYTICAL	1,0
G local not 21 LAGRANGE / NB_POINT_FOND=33	11,586	ANALYTICAL	1,0
G local not 33 LAGRANGE / NB_POINT_FOND=33	11,586	ANALYTICAL	1,0
G local Node $N292$ - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G (POST_K1_K2_K3) - Node $N403$	11,586	ANALYTICAL	5,0
G (POST_K1_K2_K3) - Node $N2862$	11,586	ANALYTICAL	10,0
G (POST_K1_K2_K3) - Node $N375$	11,586	ANALYTICAL	5,0
G (POST_K1_K2_K3) - Node $N292$	11,586	ANALYTICAL	5,0

4 Modeling B

Calculation with POST_K1_K2_K3 in 3D

4.1 Characteristics of modeling



This modeling makes it possible to test the calculation of $K1$ using POST_K1_K2_K3 (method of extrapolation of displacements on the lips of the crack). The parameter ABSC_CURV_MAXI of the operator is calculated in POST_K1_K2_K3 so as to retain 5 nodes on the segment of extrapolation ($dmax = 0{,}35$).

Only the loading of traction is taken into account.

4.2 Characteristics of the grid

Many nodes: 6536
Number of meshs and type: 432 PENTA15 and 987 HEXA20

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

4.3 Notice

One represents only the quarter of the complete three-dimensional block and thus the quarter of the crack. Thus, it is necessary to divide the theoretical value of reference of the total rate of refund by 4: $G_{glob} = 145.60/4 = 36.40\ J/m$

4.4 Sizes tested and results

4.4.1 Results of **CALC_G**

Identification	Reference (analytical)	% tolerance
G room Node 49	11,59	3.0
G room Node 1710	11,59	2.0
G room Node 77	11,59	3.0
G total	36,4	1.2

4.4.2 Results of `POST_K1_K2_K3`

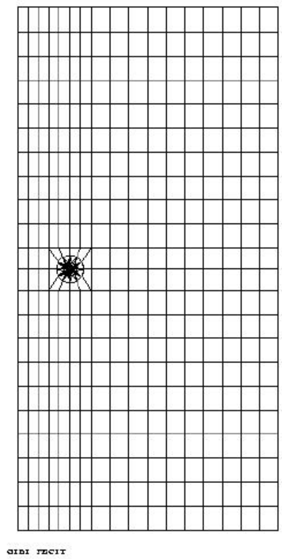
Identification	Reference (analytical)	% tolerance
G Node 49	11,586	2
G Node 77	11,586	2
G Node 1710	11,586	2
$K1$ Node 49	1,60E+006	1
$K1$ Node 77	1,60E+006	1
$K1$ Node 1710	1,60E+006	1

Other purely data-processing tests of the order `POST_K1_K2_K3` are also carried out.

5 Modeling C

Calculation with POST_K1_K2_K3 into axisymmetric.

5.1 Characteristics of modeling



This modeling makes it possible to test the calculation of $K1$ using POST_K1_K2_K3 (method of extrapolation of displacements on the lips of the crack) into axisymmetric.

Only the loading of traction is retained in this modeling.

Since one is in axisymmetric modeling, the relation between the rates of refund of energy total and local is [R7.02.01]:

$$G_{réf}(\theta)=G(s)\cdot a \text{ that is to say here } G_{réf}=23.17\,J/m$$

5.2 Characteristics of the grid

Many nodes: 1477
Number of meshs and type: 402 QUAD8 and 60 TRIA6

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

5.3 Sizes tested and results

Identification	Method	Reference	Type of reference	% tolerance
G	CALC_G	23,2	ANALYTICAL	1,8
$K1$	POST_K1_K2_K3	1,60E+006	ANALYTICAL	3
G	POST_K1_K2_K3	11,6	ANALYTICAL	1,8

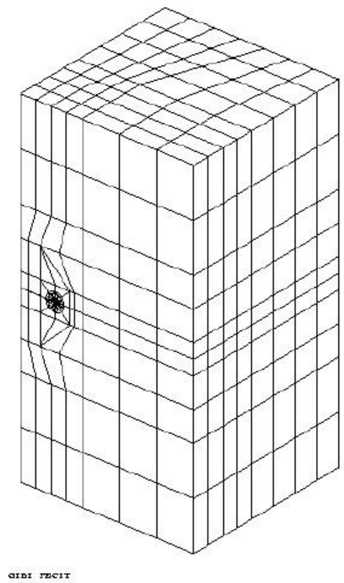
5.4 Notice

An order of STAT_NON_LINE is added to validate the modifications brought on xpesro.f . The got results this case test will be compared with those obtained from case test sslv134i for $\mathrm{X\text{-}FEM}$ into axisymmetric. In addition of the simple traction applied to the edges high and low of the plate, a rotation of $150\,trs/min$ around the symmetrical axis is applied.
With the option CALC_G, $G=21{,}4$ and $K_1=219\mathrm{E}{+}05$.

6 Modeling D

Calculation with POST_K1_K2_K3 in 3D for modes 1 and 3.

6.1 Characteristics of modeling



The boundary conditions following are successively applied:

- traction: as for modeling B;
- torsion.

This modeling makes it possible to test the calculation of $K1$ and $K3$ compounds using POST_K1_K2_K3 (method of extrapolation of displacements on the lips of the crack).

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

6.2 Characteristics of the grid

Many nodes: 6536
Number of meshs and type: 432 PENTA15 and 987 HEXA20

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

6.3 Notice

The two loading cases (traction and torsion) are taken into account. It is thus necessary to cumulate the values of G for the two loadings. Moreover, one represents only the quarter of the complete three-dimensional block and thus the quarter of the crack, it is thus necessary to divide the theoretical value of reference of the total rate of refund by 4.

Thus

$$G(s)=(11.586+7.356)=18.943\,J/m^2$$

$$G=(145.60+92.44)/4=59,511\ J/m$$

Only traction contributes to $K1$, only torsion contributes to $K3$.

6.4 Sizes tested and results

Identification	Method	Localization	Reference	Type of reference	% tolerance
G	CALC_G Legendre	Node 49	18,94	ANALYTICAL	3,0
G	CALC_G Legendre	Node 1710	18,94	ANALYTICAL	2,0
G	CALC_G Legendre	Node 77	18,94	ANALYTICAL	3,0
G	CALC_G option CALC_G_GLOB	-	59,51	ANALYTICAL	1,2
K1	POST_K1_K1_K3	Node 49	$1.596\ 10^{6}$	ANALYTICAL	1,0
K1	POST_K1_K1_K3	Node 1710	$1.596\ 10^{6}$	ANALYTICAL	1,0
K1	POST_K1_K1_K3	Node 77	$1.596\ 10^{6}$	ANALYTICAL	2,0
K3	POST_K1_K1_K3	Node 49	$1.064\ 10^{6}$	ANALYTICAL	2,0
K3	POST_K1_K1_K3	Node 1710	$1.064\ 10^{6}$	ANALYTICAL	2,0
K3	POST_K1_K1_K3	Node 77	$1.064\ 10^{6}$	ANALYTICAL	1,0
G	POST_K1_K1_K3	Node 49	18,94	ANALYTICAL	2,0
G	POST_K1_K1_K3	Node 1710	18,94	ANALYTICAL	2,0
G	POST_K1_K1_K3	Node 77	18,94	ANALYTICAL	2,0

7 Modeling E

Calculation of the bilinear form of G .

7.1 Characteristics of modeling

The grid is identical to that of preceding calculations, but only the eighth of the block is retained (quadrant $Oxyz$)

Loading 1:	Idem modeling B.
Loading 2:	$Face\ x=10.$: uniform constraint of traction $\sigma_z=1$, $Face\ z=10.$: uniform constraint of traction $\sigma_x=1$ (shearing).
Loading 3:	Loading 1 + Loading 2.
Loading 4:	Loading 2 – Loading 1.

Four calculations are static are carried out producing displacements respectively $u, v, u+v$, and $v-u$.

7.2 Characteristics of the grid

Many nodes:	2774
Number of meshs and type:	392 HEXA20 and 216 PENTA15

7.3 Notice

The grid represents only the eighth of the complete three-dimensional block. On the other hand, the operand is used SYME in DEFI_FOND_FISS, which amounts representing a quarter of the crack. Thus, it is necessary to divide the theoretical value of reference of the total rate of refund by 4.

The bilinear form $g(u,v)$ check the following properties:

$$g(u,u)=G(u)$$

$$g(u,v)=\frac{G(u+v)-G(u-v)}{4}$$

from where

$$g(u-v,u+v)=\frac{G(2\mathrm{u})-G(-2\mathrm{v})}{4}=G(u)-G(v)$$

7.4 Sizes tested and results

Identification	Reference	*Type of reference*	% difference
G total: $G(u)$	36.4	AUTRE_ASTER	0,41%
G bilinear: $g(u,u)$	36.4	AUTRE_ASTER	0,41%
G total: $G(v)$	13,72	AUTRE_ASTER	0,10%
G bilinear: $g(v,v)-form.1$	13,72	AUTRE_ASTER	0,10%
G total: $G(u+v)$	95,04	AUTRE_ASTER	0,10%
G bilinear: $g(u+v,u+v)-form.1$	95,04	AUTRE_ASTER	0,10%

G total: $G(u-v)$	5,48	AUTRE_ASTER	0,10%
G bilinear: $g(u-v,u-v)-form.1$	5,48	AUTRE_ASTER	0,10%
G bilinear: $g(v,u)-form.2$	22,4	AUTRE_ASTER	0,10%
G bilinear: $g(u-v,u+v)-form.3$	22,82	AUTRE_ASTER	0,10%

One indicates by u displacement corresponding to loading 1, and v displacement corresponding to loading 2. Loadings 3 and 4 correspondent with displacements $(u+v)$ and $(v-u)$.

8 Modeling F

Calculation of the bilinear form of G room.

8.1 Characteristics of modeling

The grid is identical to that of modeling E.

Loading 1:	Idem modeling B.
Loading 2:	$Face\ x=10.$: uniform constraint of traction $\sigma_z=1$, $Face\ z=10.$: uniform constraint of traction $\sigma_x=1$ (shearing).
Loading 3:	Loading 1 + Loading 2.
Loading 4:	Loading 2 – Loading 1.

Four static calculations are carried out producing displacements respectively $u, v, u+v$, and $v - u$.

8.2 Characteristics of the grid

Many nodes:	2774
Number of meshs and type:	392 HEXA20 and 216 PENTA15

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges.

8.3 Notice

The grid represents only the eighth of the complete three-dimensional block. On the other hand, the operand is used SYME in DEFI_FOND_FISS, which amounts representing a quarter of the crack. Thus, it is necessary to divide the theoretical value of reference of the total rate of refund by 4.

The bilinear form $g(u,v)$ check the following properties:

$$g(u,u)=G(u)$$

$$g(u,v)=\frac{G(u+v)-G(u-v)}{4}$$

from where

$$g(u-v,u+v)=\frac{G(2\mathrm{u})-G(-2\mathrm{v})}{4}=G(u)-G(v)$$

Calculation with the options 'G_MAX' and 'CALC_K_MAX' illustrate the method to maximize G or $K1$ in the presence of signed and not signed loads. Loadings 1 and 3 definite like are signed here, loadings 2 and 4 being not signed.

The two options are characterized by the taking into account (CALC_K_MAX) or not (G_MAX) possible interpenetration of the lips of the crack; in these case test, the maximum approach obtained corresponds to the same combination of loads in both cases.

8.4 Sizes tested and results

The first two values tested are compared with the analytical reference solution. The other tests are of standard not-regression.

Node	Smoothing	R_inf	R_sup	Identification	Type of Reference	*Reference*	Tolerance (%)
N2667	Lag-Lag	0.1	1.0	G room: $G(u)$	AUTRE_ASTER	11,58	0,80%
N2667	Lag-Lag	0.1	1.0	G bilinear: $g(u,u)$	AUTRE_ASTER	11,58	0,80%
N2667	Lag-Lag	0.1	1.0	G_MAX : $g(u,u)$	NON_REGRESSION	30.26	0,60%
N2667	Leg-Leg	0.5	1.5	G room: $G(v)$	NON_REGRESSION	4.38	0,50%
N2667	Leg-Leg	0.5	1.5	G bilinear: $g(v,v)$	NON_REGRESSION	4.38	0,50%
N2667	Lag-Lag	0.5	1.5	G total: $G(u+v)$	NON_REGRESSION	30.45	0,50%

9 Modeling G

Calculation with `POST_K1_K2_K3` in 3D.

9.1 Characteristics of modeling

This modeling makes it possible to test calculations of $K1$ and G using `POST_K1_K2_K3` when the bottom of crack was defined either by only one lists nodes, but by two lists of nodes in the operator `DEFI_FOND_FISS`.

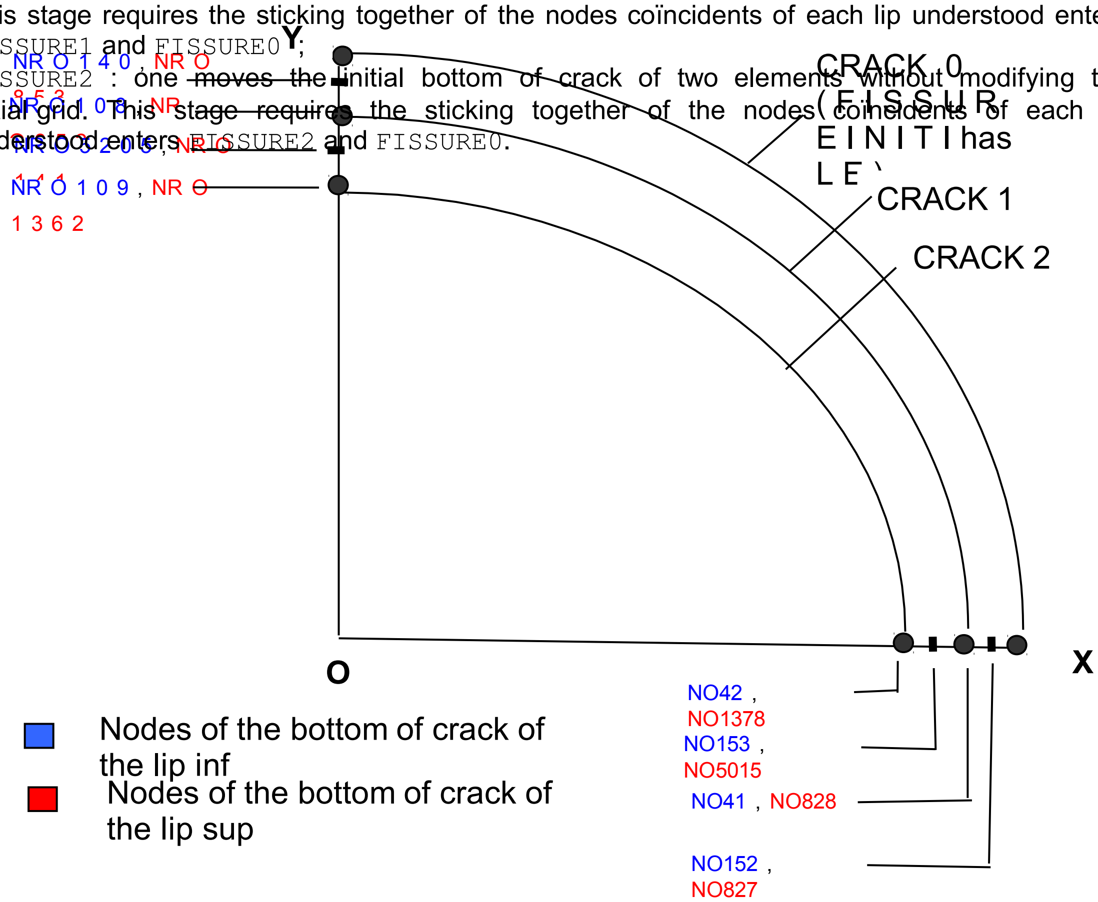
Note:

> *This kind of bottom of crack is obtained by the keywords `FOND_INF` and `FOND_SUP` of the operator `DEFI_FOND_FISS [U4.82.01]` . The two funds of crack must be geometrically confused.*

The grid is identical to that of modeling b: it contains a circular crack noted `FISSURE0` on the diagram below.

One carries out the calculation of the coefficients of intensity of constraints for the 3 following funds of crack:

- `FISSURE0` : crack of modeling B (`FISSURE0`) defined by only one entity;
- `FISSURE1` : one moves the initial bottom of crack of an element without modifying the initial grid. This stage requires the sticking together of the nodes coïncidents of each lip understood enters `FISSURE1` and `FISSURE0`;
- `FISSURE2` : one moves the initial bottom of crack of two elements without modifying the initial grid. This stage requires the sticking together of the nodes coïncidents of each lip understood enters `FISSURE2` and `FISSURE0`.



Note:

When one applies conditions of sticking together (`LIAISON_GROUP`) and of symmetry to the nodes of the lips , superabundant conditions of blockings are generated. To cure this problem, nodes of only one of the lips have at the same time conditions of symmetry and sticking together and for the other lip, the nodes have only conditions of sticking together.

Only the loading of traction is taken into account.

9.2 Characteristics of the grid

Many nodes: 5227
Number of meshs and type: 432 `PENTA15` and 784 `HEXA20`

Note:

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision (elements of Barsoum).
However, the presence of nodes to the quarter in the restuck part of the crack disturbs calculation significantly. It is thus recommended to duplicate the structure of data grid as many times as funds of crack thanks to the order `CREA_MAILLAGE` , and successively to move the nodes with the quarter on each grid. It is what is made in this case test.

9.3 Sizes tested and results

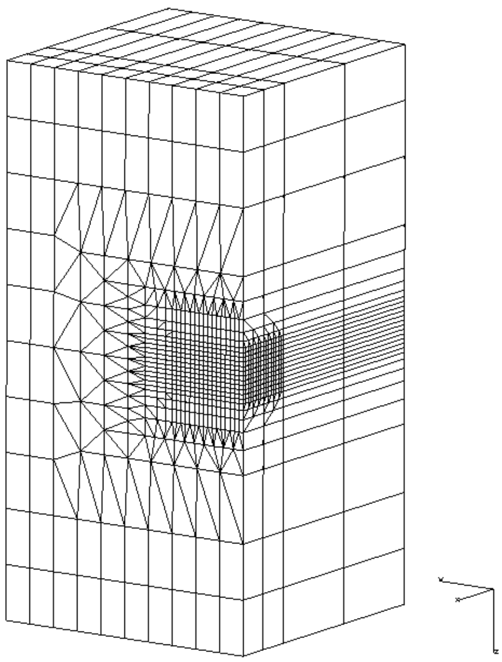
Results of `POST_K1_K2_K3`

Crack	Identification	Node	Reference	*Type of reference*	% tolerance
Fissure1	$K1$	Node 69	1,547E+06	ANALYTICAL	0,50
	G	Node 69	1,089E+01	ANALYTICAL	1,00
Fissure2	$K1$	Node 70	1,497E+06	ANALYTICAL	0,50
	G	Node 70	1,020E+01	ANALYTICAL	1,00

10 Modeling H

Method X-FEM.

10.1 Characteristics of modeling



This modeling makes it possible to test the calculation of $K1$ using POST_K1_K2_K3 and CALC_G (option CALC_K_G) on a crack nonwith a grid (method X-FEM).

Only the loading of traction is taken into account. Conditions of symmetry are imposed on the two side faces.

10.2 Characteristics of the grid

Many nodes: 6100
Number of meshs and type: 1500 PENTA6 and 4600 HEXA8 (linear grid)

10.3 Sizes tested and results

The values tested are the factors of intensity of the constraints $K1$ along the bottom of crack, calculated either by POST_K1_K2_K3 (method 3), that is to say by CALC_G. The test is carried out in 3 points of the bottom of crack: items 1 (first point), 10 and 24 (last point).

The average quadratic error corresponds to the following quantity:

$$\varepsilon=\sqrt{\frac{\int_{\Gamma}(K_I^{ref}-K_I^{Aster})^2\,ds}{\int_{\Gamma}(K_I^{ref})^2\,ds}}$$

Identification	Reference	*Type of reference*	% tolerance

POST_K1_K2_K3			
$K1$ - point 1	1.595e6	ANALYTICAL	12,00
$K1$ - point 10	1.595e6	ANALYTICAL	3,00
$K1$ - point 24	1.595e6	ANALYTICAL	18,00
Average quadratic error			9,90

CALC_G			
$K1$ - point 1	1.595e6	ANALYTICAL	7.00
$K1$ - point 10	1.595e6	ANALYTICAL	7,00
$K1$ - point 24	1.595e6	ANALYTICAL	10,00
Average quadratic error			5,3

Remarks

Precision of the results got on a crack nonwith a grid (method X-FEM) is slightly less less good than for a crack with a grid. Results got with the operator CALC_G remain however satisfactory.

It is pointed out that the grid used is linear; the use of a finer grid makes it possible to improve the precision of the result, but to the detriment of the computing times.

The ray of enrichment around the bottom of crack (parameter RAYON_ENRI of DEFI_FISS_XFEM) a negligible influence on the results has of CALC_G and a weak influence on the results of POST_K1_K2_K3. For the operator CALC_G, smoothings of the type LAGRANGE do not allow to have easily useable results; a smoothing of the type LEGENDRE is thus to privilege.

11 Modeling I

Method X-FEM into axisymmetric.

11.1 Characteristics of modeling

This modeling makes it possible to test the calculation of $K1$ using POST_K1_K2_K3 and CALC_G (option CALC_K_G) on an axisymmetric crack nonwith a grid (method X-FEM).

Two types of loadings are considered. First is of simple traction applied to the edges high and Bace of the plate. Second is of simple traction applied to the edges high and low of the plate and a rotation of $150\,trs/min$ around the symmetrical axis.

11.2 Characteristics of the grid

Many nodes: 20301
Number of meshs and type: 20000 QUA4 and 600 SE2 (linear grid)

11.3 Sizes tested and results

One tests the values of G and K_I calculated by the order CALC_G option 'CALC_K_G' and by the order POST_K1_K2_K3, as well as the .valor of G calculated by the order CALC_G (option 'CALC_G'). Since modeling is axisymmetric, the relation between the rates of refund of energy total and local is [R7.02.01]: $G_{réf}(\theta)=G(s)\cdot a$, that is to say here $G_{réf}=23.17\,J/m$ for the value of G calculated with the option 'CALC_G'.

Loading 1: simple traction applied to the edges high and Bace of the plate

Identification	Type of reference	Value of reference	Tolerance ($\%$)
G (CALC_G option 'CALC_K_G')	'ANALYTICAL'	11.59	2.1%
$K1$ (CALC_G option 'CALC_K_G')	'ANALYTICAL'	1,60E+06	6.0%
G (CALC_G option 'CALC_K_G')	'AUTRE_ASTER'	11.78	0.4%
$K1$ (CALC_G option 'CALC_K_G')	'AUTRE_ASTER'	1,64E+06	2.0%
G (CALC_G option 'CALC_G')	'ANALYTICAL'	23.17	2.1%
G (POST_K1_K2_K3)	'ANALYTICAL'	11.59	6.0%
$K1$ (POST_K1_K2_K3)	'ANALYTICAL'	1,60E+06	6.0%

Loading 2: simple traction applied to the edges high and low of the plate and a rotation of 150 $trs/\ min$ around the symmetrical axis_

Identification	Type of reference	Value of reference	Tolerance ($\%$)
G (CALC_G option 'CALC_K_G')	'AUTRE_ASTER'	2136.52	0.3%
$K1$ (CALC_G option	'AUTRE_ASTER'	2,191E+07	2.5%

'CALC_K_G')			
G (CALC_G option 'CALC_G')	'AUTRE_ASTER'	4273.04	0.3%
G (POST_K1_K2_K3)	'AUTRE_ASTER'	2136.52	4.0%
$K1$ (POST_K1_K2_K3)	'AUTRE_ASTER'	2,191E+07	2.5%

11.4 Remarks

In this case test, the ratio a/W between the size of the crack a and the width W is $0,2$. The effects edges thus contribute to the difference between the digital solution for a finished edge and the reference solution for a continuous medium.

12 Modeling J

Fund of crack closed, calculation of G for the incompressible elements.

12.1 Characteristics of modeling

Identical to modeling A except the elements used are 3D_INCO_UPG and 3D_INCO_UP.

12.2 Characteristics of the grid

Identical to modeling A

12.3 Notice

The keyword is used SYME in the operator CALC_G to multiply automatically by two the rate of refund of energy calculated on only one lip of the crack, when the operand FOND_FISS is absent. When FOND_FISS is present, information on symmetry is recovered directly in the concept fond_fiss created via DEFI_FOND_FISS.

12.4 Sizes tested and results

Identification	Reference	Type of reference	% tolerance
G total	145,600	ANALYTICAL	2.0
G local Node $N403$ - G Lagrange	11,586	ANALYTICAL	4.0
G local Node $N2862$ - G Lagrange	11,586	ANALYTICAL	1.0
G local Node $N375$ - G Lagrange	11,586	ANALYTICAL	3.5
G local Node $N292$ - G Lagrange	11,586	ANALYTICAL	4.0
$max(G\,local)$ - G Lagrange	11,590	ANALYTICAL	5.0
$min(G\,local)$ - G Lagrange	11,586	ANALYTICAL	1.0
G local Node $N403$ - G Lagrange_no_no	11,586	ANALYTICAL	2.5
G local Node $N2862$ - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G local Node $N375$ - G Lagrange_no_no	11,586	ANALYTICAL	2.0
G local Node $N292$ - G Lagrange_no_no	11,586	ANALYTICAL	3.0
$max(G\,local)$ - G Lagrange_no_no	11,715	ANALYTICAL	2.0
$min(G\,local)$ - G Lagrange_no_no	11,575	ANALYTICAL	2.0

13 Modeling K

One tests K into axisymmetric for the incompressible elements

13.1 Characteristics of modeling

Identical to modeling C except the elements used are AXIS _INCO_UPG and AXIS _INCO_UP.

13.2 Characteristics of the grid

Identical to modeling C

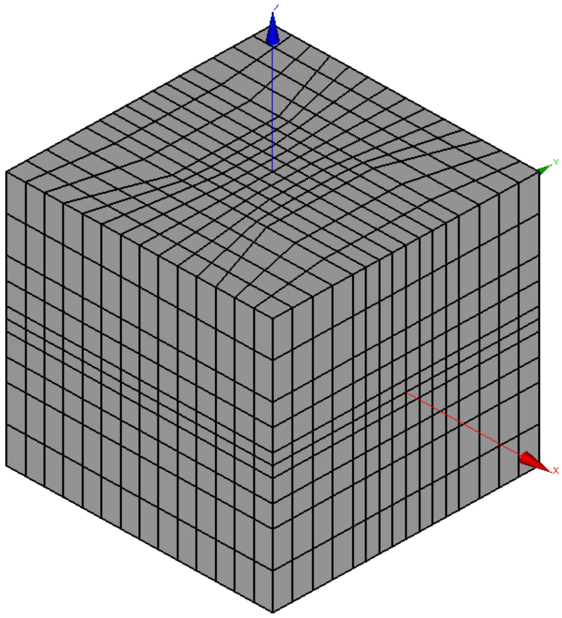
13.3 Sizes tested and results

Identification	Method	Reference	Type of reference	% tolerance
G	CALC_G	23,2	ANALYTICAL	2
G	CALC_K_G	23,2	ANALYTICAL	2
$K1$	CALC_K_G	1.595769E6	ANALYTICAL	3.5
$K1$	CALC_K_G	1.643E+06	ANALYTICAL	3

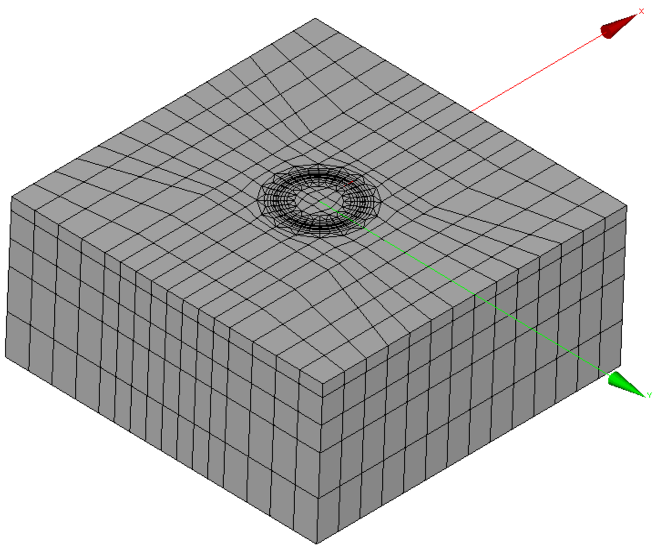
14 Modeling L

The bottom of crack is closed. One calculates G.

14.1 Characteristics of modeling



The interest of this modeling is to set out again of modeling A and to symmetrize the grid.



Thus, we find the discretization of the crown of modeling A.

Only the loading of traction is taken into account.

14.2 Characteristics of the grid

Many nodes: 18571
Number of meshs and type: 3136 HEXA20 and 1728 PENTA15

14.3 Sizes tested and results

Identification	Reference	*Type of reference*	% tolerance
G local not 3 LAGRANGE / NB_POINT_FOND=16	11,586	ANALYTICAL	5, 0
G local not 8 LAGRANGE / NB_POINT_FOND=16	11,586	ANALYTICAL	1, 0
G local not 11 LAGRANGE / NB_POINT_FOND=16	11,586	ANALYTICAL	4.0
G local not 15 LAGRANGE / NB_POINT_FOND=16	11,586	ANALYTICAL	2, 0

15 Summaries of the results

The conclusions of this case test are the following ones:

- The definition and the calculation of G room on closed funds of crack is validated. One checks in particular the independence of G room with respect to the angle for an axisymmetric crack and a loading. One notes a variation of less 2% on the whole of the bottom of crack by the methods 'LAGRANGE' and 'LAGRANGE_NO_NO'.

- The order POST_K1_K2_K3, which makes it possible to calculate the stress intensity factors by exploiting the jump of displacements on the lips of the crack, is also validated. This method, less precise than CALC_G, allows to obtain here (with a suitable grid: nodes mediums of the edges touching the bottom of crack moved with the quarter of these edges) of the values of $K1$ and $K3$ with less 2% reference.

  Three methods of interpolation are used and give close results. Method 3 is interesting because it provides a single value of the stress intensity factors and not a maximum value and a minimal value.

  The use of POST_K1_K2_K3 to study a crack by relaxation of nodes is tested and gives satisfactory results.

- One validates the calculation of the bilinear form of G and options G_MAX and CALC_K_MAX.

- Method X-FEM allows to evaluate the factors of intensity of the constraints K on a grid not fissured with an error lower than 10% .
- One validates calculations for elements 3D_INCO_UPG, 3D_INCO_UP, AXIS_INCO_UPG and AXIS_INCO_UP