SSLV131 - Orthotropism in an unspecified reference mark

Summary

This case test validates modelings relating to linear elasticity which implement orthotropic materials whose properties are known in a reference mark defined by the user different from the total reference mark.

1 Problem of reference

1.1 Geometry

The total reference mark is the reference mark (A, X, Y, Z). In this reference mark the coordinates of the nodes are:

$A(0., 0., 0.)$
$B(3., 1., 0.)$
$C(2., 3., 0.)$
$D(3., 1., -1)$

The behavior of the tetrahedron will be studied $ABCD$ whose material properties are defined in a local reference mark (A, x, y, z) obtained by rotation of the total reference mark according to the nautical angles $(\alpha=30°, \beta=20°, \gamma=10°)$.

1.2 Properties of material

The materials used are orthotropic and isotropic transverse. In order to validate the orthotropic deformations of thermal origin, one does also a thermomechanical calculation.

One adopts the convention of terminology used in Code_Aster. That is to say suffixes L, T and N mean Longitudinal, Transverse and Normal.

The units are not specified.

$$E_L = 11000,\ E_T = 5000,\ E_N = 8000,$$
$$\nu_{LT} = 0.396,\ \nu_{LN} = 0.15,\ \nu_{TN} = 0.11$$
$$G_{LT} = 10500,\ G_{LN} = 7000,\ G_{TN} = 13000$$
$$\alpha_L = 10^{-3}, \alpha_T = 1.5\,10^{-3}, \alpha_N = 2\,10^{-3}$$

(It is known that $\nu_{LT} = \frac{E_L}{E_T}\nu_{TL}, \nu_{LN} = \frac{E_L}{E_N}\nu_{NL},\ \nu_{TN} = \frac{E_T}{E_N}\nu_{NT}$,

that is to say $\nu_{TL} = 0.18, \nu_{NL} = 0.1091, \nu_{NT} = 0.176$

For the transverse isotropy, one keeps the same values while knowing as:

$$E_T = E_L = 11000, \nu_{TN} = \nu_{LN} = 1.15 \text{ et } G_{LT} = \frac{E_L}{2(1+\nu_{LT})}$$

It is pointed out that these coefficients are defined in a local reference mark (A, L, T, N) turned with the nautical angles $(30°, 20°, 10°)$ compared to the total reference mark.

1.3 Boundary conditions and loadings

The boundary conditions are of Dirichlet type. One makes the assumption of a linear field of displacement in x and y so that the field of deformation is constant.

Conditions of Dirichlet	$dX=2x+3y+4z$ $dY=3x+5y+6z$ $dZ=4x+6y+7z$
Thermal conditions	Temperature imposed on all the structure of 100

One will thus impose:

- for the node A $dX=0, dY=0, dZ=0$
- for the node B $dX=9, dY=14, dZ=18$
- for the node C $dX=13, dY=21, dZ=26$
- for the node D $dX=5, dy=8, dZ=11$

2 Reference solution

2.1 Method of calculating

Calculation is analytical.
One used the formal calculation programme Mathématica to carry it out.

It is known that the field of displacement is:

$$dX=2x+3y+4z$$
$$dY=3x+5y+6z$$
$$dZ=4x+6y+7z$$

The field of deformations ε_G in the total reference mark is thus constant and equal to:

$$\varepsilon_G=\begin{vmatrix}2 & 3 & 4\\ 3 & 5 & 6\\ 4 & 6 & 7\end{vmatrix}$$

That is to say P the matrix of passage allowing to make pass a vector of the total reference mark (A, X, Y, Z) with the local reference mark (A, L, N, T).

That is to say $\varepsilon_{\mathbf{L}}$ the tensor of deformation in the local reference mark. One a: $\varepsilon_L=P\cdot\varepsilon_G\cdot P^T$

The tensor of Hooke H_L is known in the local reference mark, that is to say σ_L the tensor of the constraints in this reference mark. One a:

$$\sigma_L=H_L.\varepsilon_L$$

The tensor is obtained σ_G constraints in the total reference mark by: $\sigma_G=P^T.\sigma_L.P$

If a field of temperature is applied, the equations above are modified as follows:

The field of deformations ε_G in the total reference mark is always the same one:

$$\varepsilon_G = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 5 & 6 \\ 4 & 6 & 7 \end{vmatrix}$$

That is to say $\varepsilon_{\mathbf{L}}$ the tensor of deformation in the local reference mark. One a: $\varepsilon_{\mathbf{L}} = P.\varepsilon_G.P^T$
The tensor of the mechanical deformations in the local reference mark is thus worth:

$$\varepsilon_L^{mec} = \varepsilon_L - \varepsilon_L^{ther} \text{ with } \begin{cases} \varepsilon_{Lxx}^{ther} = \alpha_L (T - T_{ref}) \\ \varepsilon_{Lyy}^{ther} = \alpha_T (T - T_{ref}) \\ \varepsilon_{Lzz}^{ther} = \alpha_N (T - T_{ref}) \end{cases}$$, other components being worthless

The tensor of Hooke H_L is known in the local reference mark. That is to say σ_L the tensor of the constraints in this reference mark. One a:

$$\sigma_L = H_L . \varepsilon_L^{mec}$$

The tensor is obtained σ_G constraints in the total reference mark by: $\sigma_G = P^T . \sigma_L . P$

2.2 Results of reference

They are obtained by carrying out the operations described above with Mathematica.

2.3 Uncertainties on the solution

Uncertainty is worthless because the solution is analytical.

2.4 Bibliographical references

For the description of the matrices of Hooke for materials isotropic transverse and orthotropic for plane modelings 3D, constraints and plane deformations, the selected reference was:

'Matrix of Hooke for orthotropic materials'. Report interns applications in Mechanics n° 79 - 018 of Jean-Claude Masson CISI.

3 Modeling A

3.1 Characteristics of modeling

Modeling 3D is put in work. One tests materials isotropic transverse and orthotropic (with possibly taken into account of thermal deformations of origins)

Note:

- The transverse isotropy is not tested for the plane constraints because this case corresponds to the isotropy.
- For the axisymmetric case the stress field depends on the point of calculation.
- This point is selected at the point of integration of the triangle (i.e it is the centre of gravity of the triangle).
- It is pointed out that the orthotropism in an unspecified reference mark is not available for modeling as a Fourier because there is then coupling of all the components of the tensor of constraints:

Implementation the current makes it possible to use only the symmetrical components from which one can find the antisymmetric components but so that it is possible, it is not necessary that the slips induce tensile stresses.

3.2 Characteristics of the grid

There is an element tetrahedron with 4 nodes $ABCD$.

3.3 Values tested

Identification	**Reference**
Case of the transverse isotropy 3D	
name of the result: $Mest1$	
field of displacement	
$dy(c)$	21
field EPSI_ELGA	
$EPXY$	3
$EPXZ$	4
$EPYZ$	6
field SIEF_ELGA	
$SIXX$	43310.760
$SIYY$	72798.710
$SIZZ$	62459.356
$SIXY$	39567.891
$SIXZ$	31078.597
$SIYZ$	84049.301
field SIGM_ELNO	
$SIXX$	43310.760
field emel-elga Ep	1.19123 E6
Field emel-elno-elga Ep	1.19123 E6

Case of the orthotropism 3D	
name of the result: $Mest2$	
field of displacement	
$dy(c)$	21
field EPSI_ELGA	
$EPXY$	3
$EPXZ$	4
$EPYZ$	6
field SIEF_ELGA	
$SIXX$	601.8754
$SIYY$	80053.665
$SIZZ$	78596.607
$SIXY$	83948.263
$SIXZ$	17339.093
$SIYZ$	126571.71
field enel-elga Ep	$1.55286.10^{6}$
field enel-elno-elga Ep	$1.55286.10^{6}$
Case of the orthotropism with taking into account of the thermal deformations (order STAT_NON_LINE)	
name of the result: $Mest3$	
field of displacement	
$dy(c)$	21
Identification	**Reference**
field EPSI_ELGA	
$EPXY$	3
$EPXZ$	4
$EPYZ$	6
field SIEF_ELGA	
$SIXX$	1226.2014
$SIYY$	78597.064
$SIZZ$	76585.792
$SIXY$	83710.907
$SIXZ$	17255.703
$SIYZ$	126657.367
Case of the orthotropism with taking into account of the thermal deformations (order MECA_STATIQUE)	
name of the result: $Mest4$	
field of displacement	
$dy(c)$	21
field EPSI_ELGA	
$EPXY$	3
$EPXZ$	4
$EPYZ$	6
field SIEF_ELGA	

SIXX	1226.2014
SIYY	78597.064
SIZZ	76585.792
SIXY	83710.907
SIXZ	17255.703
SIYZ	126657.367

4 Summary of the results

Results provided by Mathématica and *Aster* are identical for all modelings usable with materials isotropic transverse and orthotropic.