SSLV117 – Validation of modeling second gradient of dilation in 3D

Summary:

This test makes it possible to validate modeling second gradieNT [R5.04.03] in 3D starting from analytical solutions. It is about an elastic ball of unit ray subjected to an imposed radial displacement and an internal pressure.

1 Problem of reference

1.1 Geometry

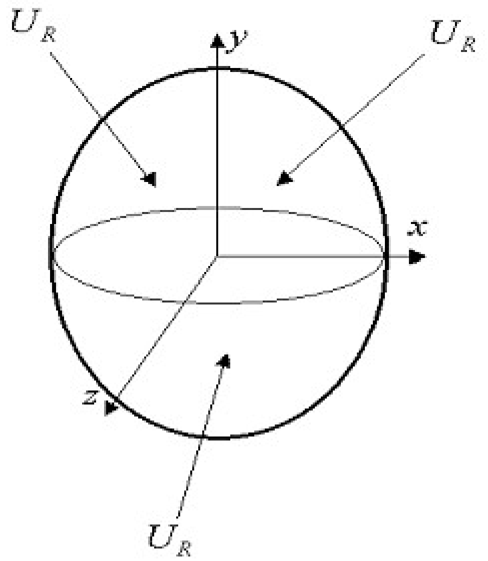

Figure 1.1-a

The geometry of the structure is a unit ball of ray for the analytical solution.

1.2 Properties of materials

In the context of the mediums of the second gradient, it is necessary to define properties materials for the parts attached to the first and second gradients of the field of displacement. The properties of material do not have any physical relevance but were established to simplify the analytical applications.

- Young modulus: $\mathrm{E}=1\,\mathrm{Pa}$;
- Poisson's ratio: $\nu=0$;
- Microscopic module of rigidity: $a_1=0.25\,\mathrm{Pa.m}^2$.
- Parameters material of the mediums of the second gradient: $a_2=a_3=a_4=a_5=0\,\mathrm{Pa.m}^2$

1.3 Boundary conditions and loadings

For digital reasons of simplifications, one does not consider that a eighth of the ball by symmetry:

- $DX=0$ on GROUP_MA 'SYME_X'
- $DY=0$ on GROUP_MA 'SYME_Y'
- $DZ=0$ on GROUP_MA 'SYME_Z'

For digital reasons of simplifications, one does not model the ball but the cube registers this ball for the application of the boundary conditions. One applies thus in the reference mark $X=(x, y, z)$:

- $U_R(X)=\left(\frac{r^4}{7}-\frac{1}{3}\right)X$ for the components DX , DY and DZ GROUP_MA ' BORD_EXT 'where r is the distance between the origin of the ball and the point of coordinate X .
- $G_R(X)=\left(r^4-1\right)$ for the components GONF GROUP_MA ' BORD_EXT '.

One applies finally the following internal forces in the reference mark $X=(x,y,z)$:
$F_I(X)=4(10-r^2)X$ for the components FX , FY and FZ GROUP_MA ' BORD_EXT '

2 Reference solution

2.1 Sizes and results of reference

The analytical solution is the following one:
$\nabla\chi(X)=4r^2X$. where $\nabla\chi(X)$ are the components DGONFX1, DGONFX2, DGONFX3 field SIEF_ELGA.

$\varepsilon_{xx}(X)=\left(\frac{r^4}{7}-\frac{1}{3}\right)+\frac{4r^2}{7}x*x$. where r indicate the distance between the origin of the ball and the point of coordinate $X=(x,y,z)$ (component EPXX field EPSI_ELGA).

$\varepsilon_{yy}(X)=\left(\frac{r^4}{7}-\frac{1}{3}\right)+\frac{4r^2}{7}y*y$. (component EPYY field EPSI_ELGA).

$\varepsilon_{zz}(X)=\left(\frac{r^4}{7}-\frac{1}{3}\right)+\frac{4r^2}{7}z*z$. (component EPZZ field EPSI_ELGA).

$\varepsilon_{xy}(X)=\frac{4r^2}{7}x*y$. (component EPXY field EPSI_ELGA).

$\varepsilon_{xz}(X)=\frac{4r^2}{7}x*z$. (component EPXZ field EPSI_ELGA).

$\varepsilon_{yz}(X)=\frac{4r^2}{7}y*z$. (component EPYZ field EPSI_ELGA).

2.2 Uncertainties on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling

The characteristics are identical to the reference solution. Modeling relating to the first gradient of the field of displacement is 3D and that bearing on the second gradient of dilation is 3D_DIL with the choice of interpolation P2-P1-P1.

3.2 Characteristics of the grid

The group of meshs $\mathrm{CUBE\_REG}$ is obtained by duplication of group of meshs named CUBE, whose objective is to accommodate modeling second gradient of dilation for the regularization.

Many nodes	9009
Number of TRIA6	768
Number of TETRA10	12288
Number of group of meshs	6

3.3 Sizes tested and results

The sizes tested relate to the standards of the errors $\theta(\nabla X)$ and $\theta(\varepsilon)$ according to the relations:

$$\theta(\nabla\chi)=\sqrt{\int_{boule}\left(\nabla\chi^{analytique}(X)-\nabla\chi^{numérique}(X)\right)}$$

$$\theta(\varepsilon)=\sqrt{\int_{boule}\left(\varepsilon^{analytique}(X)-\varepsilon^{numérique}(X)\right)}$$

The reference is regarded here as ' NON_REGRESSION ' as from the moment when the test relates to the standards of the errors $\theta(\nabla X)$ and $\theta(\varepsilon)$ and not on the components ∇X and ε .

Value tested	Moment	Reference	Criterion	Aster	Tolerance
$\theta(\varepsilon)$	1.0	'NON_REGRESSION'	RELATIVE	0.0038977	0.1%
$\theta(\nabla X)$	1.0	'NON_REGRESSION'	RELATIVE	0,06	0.1%

4 Summary of the results

This test makes it possible to validate the good performance of modeling second gradient of dilation in 3D by comparison with an analytical solution.