SSLV100 - Hollow roll in plane deformations

Summary:

This test makes it possible to validate the elements of plane deformation on the following features:

- pressure distributed,
- matrix of rigidity,
- imposed displacements:
  - by ddl,
  - by face of element.

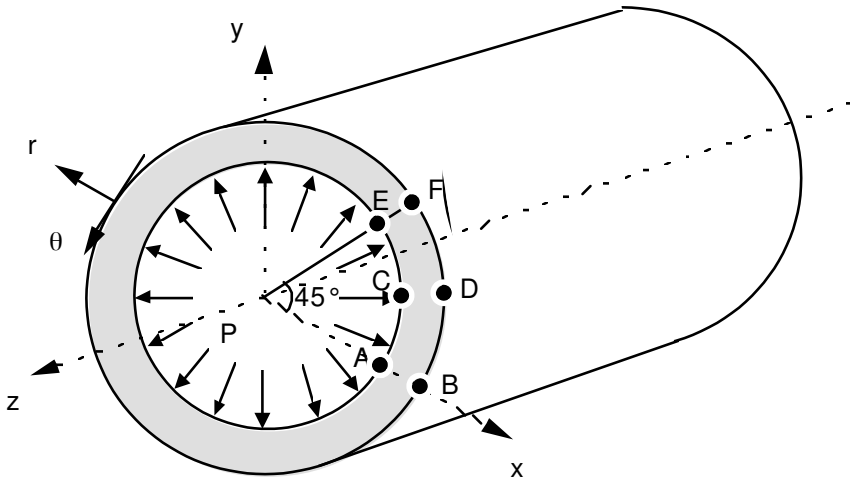
It understands 4 modelings.

The 3 first correspond to elements of the different type (linear and quadratic).

The last validates the displacements imposed by face (blocking of the normal component).

1 Problem of reference

1.1 Geometry



Rayon interne a = 0.1 m
Rayon externe b = 0.2 m

Coordinates of the points:

	A	B	C	D	E	F
x	0.100	0.200	$0.1\cos(22.5)$	$0.2\cos(22.5)$	$1/\sqrt{2}$	$\sqrt{2}$
y	0.	0.	$0.1\sin(22.5)$	$0.2\sin(22.5)$	$1/\sqrt{2}$	$\sqrt{2}$
z	0	0.	0.	0.	0.	0.

1.2 Properties of materials

$E = 2\,10^5\ Mpa$

$\nu = 0.3$

1.3 Boundary conditions and loadings

Internal pressure: $P = 60.\ MPa$

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical

$$\begin{aligned}
\sigma_{zz} &= 2\nu P\frac{a^2}{b^2-a^2}\\
\sigma_{rr} &= P\frac{a^2}{b^2-a^2}\left[1-\frac{b^2}{r^2}\right]\\
\sigma_{\theta\theta} &= P\frac{a^2}{b^2-a^2}\left[1+\frac{b^2}{r^2}\right]\\
\sigma_{r\theta} &= 0\\
u_r &= \frac{P}{E}\frac{a^2}{b^2-a^2}(1+\nu)\left[(1-2\nu)+\frac{b^2}{r^2}\right]r
\end{aligned}$$

One obtains:

For $r=0.1$ $u_r=5{,}72\,10^{-5}$ For $r=0.2$ $u_r=3{,}64\,10^{-5}$

$\sigma_{rr}=-60.$ $\sigma_{rr}=0.$

$\sigma_{\theta\theta}=100.$ $\sigma_{\theta\theta}=40.$

$\sigma_{zz}=12.$ $\sigma_{zz}=12.$

$\sigma_{r\theta}=0.$ $\sigma_{r\theta}=0.$

Passage in the system of Cartesian axes:

$$\begin{aligned}
\sigma_{xx} &= \sigma_{rr}\cos^2\theta+\sigma_{\theta\theta}\sin^2\theta-2\sigma_{r\theta}\sin\theta\cos\theta\\
\sigma_{yy} &= \sigma_{rr}\sin^2\theta+\sigma_{\theta\theta}\cos^2\theta+2\sigma_{r\theta}\sin\theta\cos\theta\\
\sigma_{xy} &= \sigma_{rr}\sin\theta\cos\theta-\sigma_{\theta\theta}\sin\theta\cos\theta-2\sigma_{r\theta}(\cos^2\theta-\sin^2\theta)
\end{aligned}$$

with:

- $\theta=0°$ at the points A and B ,
- $\theta=22.5°$ at the points C and D ,
- $\theta=45°$ at the points E and F .

2.2 Results of reference

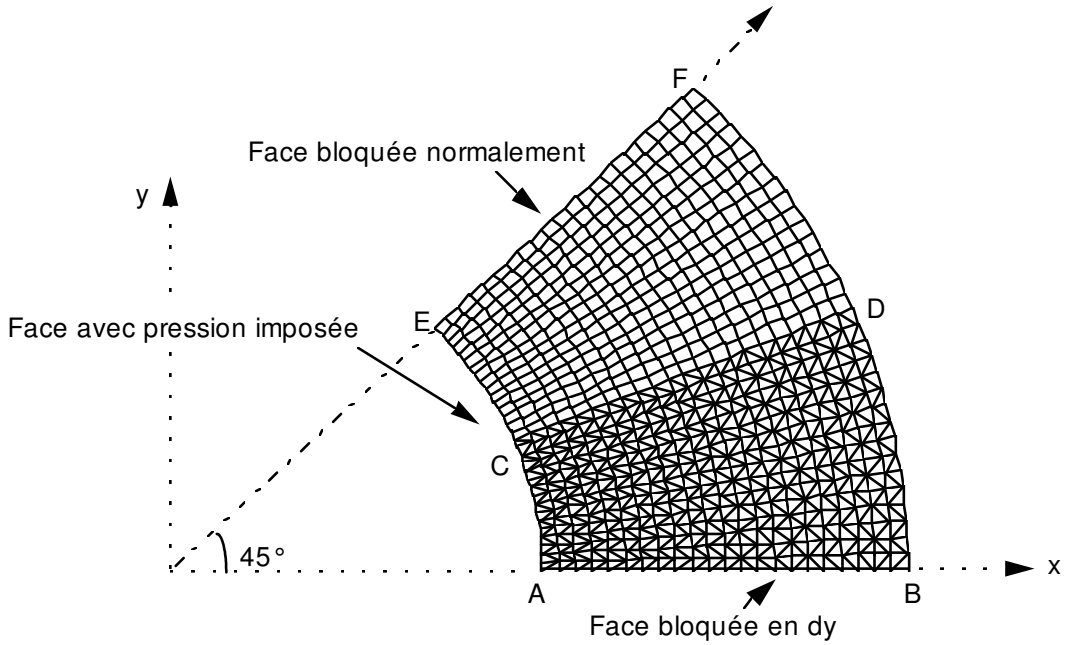
Displacements (u,v) and constraints $(\sigma_{xx},\sigma_{yy},\sigma_{zz},\sigma_{xy})$ at the points A,B,C,D,E,F .

2.3 Bibliographical references

[1] Y.C. FUNG. Fundations of solid mechanics. Prentice-hall, Inc. Englewood Cliffs. NJ. 1965 p. 243 to 245.

3 Modeling A

3.1 Characteristics of modeling: D-plan (QUAD4 + TRIA3)



Limiting conditions:

side AB	DDL_IMPO = (GROUP_NO = bordAB DY = 0.)
side EF	FACE_IMPO = (GROUP_MA = faceEF DNOR = 0.)
pressure on the face AE	PRES_REP = (GROUP_MA = faceAE CLOSE = 60.)

Names of the nodes: $A = N23$ $B = N1$ $C = N391$
$D = N369$ $E = N451$ $F = 751$

3.2 Characteristics of the grid

Many nodes: 759

Many meshs and types: 704 TRIA3, 352 QUAD4

3.3 Sizes tested and results

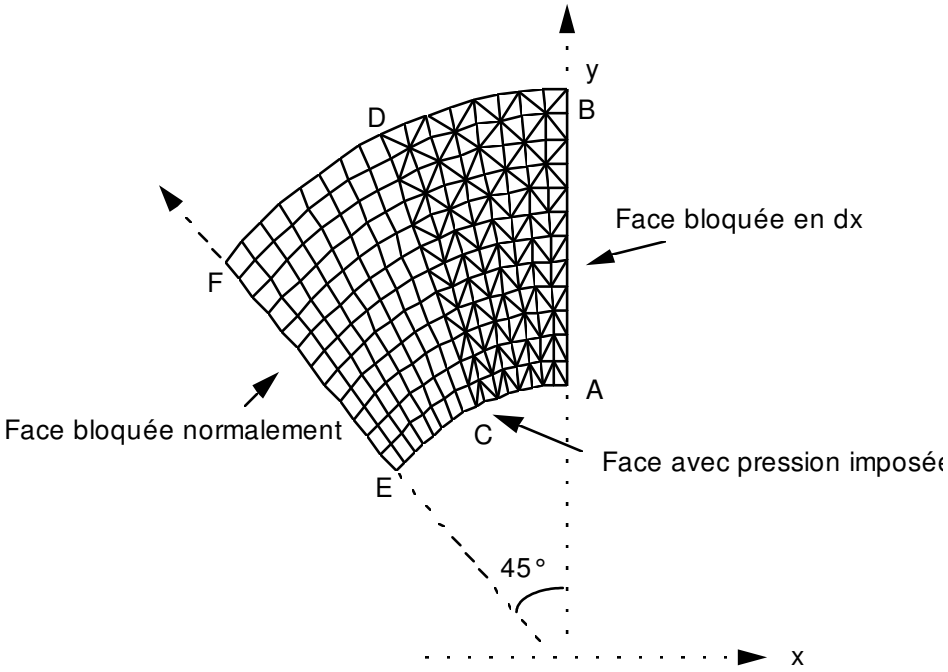
Place	Size	Reference	*Aster*	% difference	tolerance
With	u	5.72 10^{-5}	5.7155 10^{-5}	– 0.08	10^{-2}
	v	0.	epsilon	-	
	σ_{rr}	– 60.	– 56.3770	– 6.04	
	$\sigma_{\theta\theta}$	100.	96.3917	– 3.61	
	σ_{zz}	12.	12.0044	0.04	
	$\sigma_{r\theta}$	0.	– 0.9563	–	
C	u	5.28459 10^{-5}	5.2832 10^{-5}	– 0.03	10^{-2}
	v	2.18895 10^{-5}	2.1777 10^{-5}	– 0.51	
	σ_{rr}	– 36.56854	– 33.5312	– 8.31	
	$\sigma_{\theta\theta}$	76.56854	76.9335	0.48	
	σ_{zz}	12.	13.0207	8.51	
	$\sigma_{r\theta}$	– 56.56854	– 53.7445	– 4.99	
E	u	4.04465 10^{-5}	4.0400 10^{-5}	– 0.11	10^{-2}
	v	4.04465 10^{-5}	4.0400 10^{-5}	– 0.11	
	σ_{rr}	20.	23.4926	17.46	
	$\sigma_{\theta\theta}$	20.	25.4141	27.07	
	σ_{zz}	12.	14.6720	22.27	
	$\sigma_{r\theta}$	– 80.	– 78.3081	– 2.11	
B	u	3,640 10^{-5}	3.6405 10^{-5}	0.01	10^{-2}
	v	0.	epsilon	-	
	σ_{rr}	0.	– 0.4064	-	
	$\sigma_{\theta\theta}$	40.	39.8759	– 0.31	
	σ_{zz}	12.	11.8408	– 1.33	
	$\sigma_{r\theta}$	0.	– 0.4447	–	
D	u	3.36292 10^{-5}	3.3603 10^{-5}	– 0.08	10^{-2}
	v	1.39297 10^{-5}	1.3945 10^{-5}	0.11	
	σ_{rr}	5.85786	5.2229	– 10.84	
	$\sigma_{\theta\theta}$	34.14214	33.8961	– 0.72	
	σ_{zz}	12.	11.7357	– 2.20	
	$\sigma_{r\theta}$	– 14.14214	– 14.1755	0.24	
F	u	2.57387 10^{-5}	2.5710 10^{-5}	– 0.11	10^{-2}
	v	2.57387 10^{-5}	2.5710 10^{-5}	– 0.11	
	σ_{rr}	20.	19.1238	– 4.38	
	$\sigma_{\theta\theta}$	20.	19.6156	– 1.92	
	σ_{zz}	12.	11.6218	– 3.15	
	$\sigma_{r\theta}$	– 20.	– 20.1797	0.90	

3.4 Remarks

Increase in the error, when one passes from AB with CD then EF, is ascribable with the grid (density in elements QUAD4 lower than that in TRIA3).

4 Modeling B

4.1 Characteristics of modeling: D-plan (QUAD8 + TRIA6)



Limiting conditions:

side AB	DDL_IMPO = (GROUP_NO = bordAB DY = 0.)
side EF	FACE_IMPO = (GROUP_MA = faceEF DNOR = 0.)
pressure on AE	PRES_REP = (GROUP_MA = faceAE CLOSE = 60.)

Names of the nodes: $A=N2$ $B=N48$ $C=N401$ $D=N424$ $E=N606$ $F=N494$

4.2 Characteristics of the grid

Many nodes: 729

Many meshs and types: 192 TRIA6, 96 QUAD8

4.3 Sizes tested and results

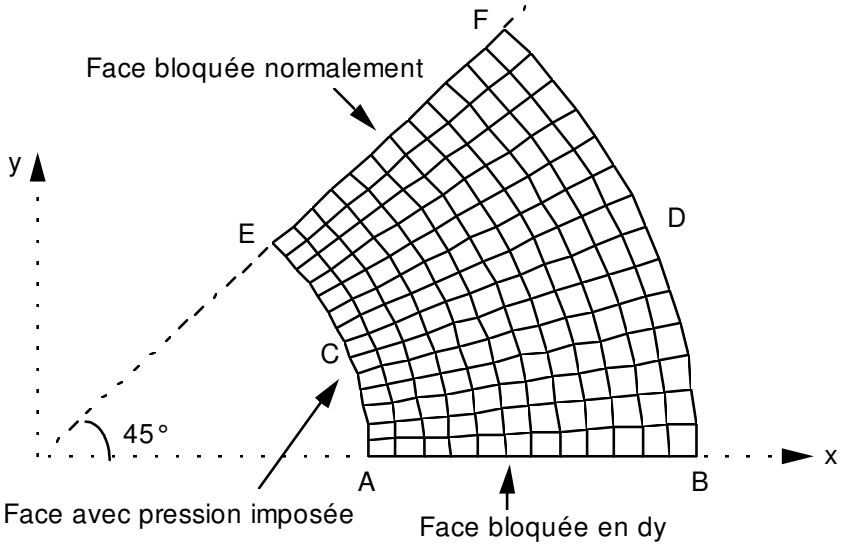
Place	Size	Reference	*Aster*	% difference	tolerance
A	u	0.	eps	-	10^{-2}
	v	$5.72\ 10^{-5}$	$5.7155\ 10^{-5}$	– 0.04	
	σ_{rr}	100.	99.7100	– 0.29	
	$\sigma_{\theta\theta}$	– 60.	– 59.7725	– 0.38	
	σ_{zz}	12.	11.9813	– 0.16	
	$\sigma_{r\theta}$	0.	0.2643	-	
C	u	$-2.18895\ 10^{-5}$	$-2.1881\ 10^{-5}$	– 0.04	10^{-2}
	v	$5.28459\ 10^{-5}$	$5.2826\ 10^{-5}$	– 0.04	
	σ_{rr}	76.56854	76.7005	0.17	
	$\sigma_{\theta\theta}$	– 36.56854	– 36.4500	– 0.32	
	σ_{zz}	12.	12.0751	0.63	
	$\sigma_{r\theta}$	– 56.56854	56.2844	– 0.50	
E	u	$-4.04465\ 10^{-5}$	$-4.0432\ 10^{-5}$	– 0.04	10^{-2}
	v	$4.04465\ 10^{-5}$	$4.0432\ 10^{-5}$	– 0.04	
	σ_{rr}	20.	20.0083	4.10^{-4}	
	$\sigma_{\theta\theta}$	20.	19.9988	-6.10^{-5}	
	σ_{zz}	12.	12.0021	2.10^{-4}	
	$\sigma_{r\theta}$	80.	79.8176	– 0.23	
B	u	0.	eps	-	10^{-2}
	v	$3,640\ 10^{-5}$	$3.6390\ 10^{-5}$	– 0.03	
	σ_{rr}	40.	39.9924	– 0.02	
	$\sigma_{\theta\theta}$	0.	– 0.001338	-	
	σ_{zz}	12.	11.9973	– 0.02	
	$\sigma_{r\theta}$	0.	– 0.04083	-	
D	u	$-1.39297\ 10^{-5}$	$-1.3926\ 10^{-5}$	– 0.03	10^{-2}
	v	$3.36292\ 10^{-5}$	$3.3619\ 10^{-5}$	– 0.03	
	σ_{rr}	34.14214	34.1361	– 0.02	
	$\sigma_{\theta\theta}$	5.85786	5.8948	0.63	
	σ_{zz}	12.	12.0093	0.08	
	$\sigma_{r\theta}$	14.14214	14.1596	0.12	
F	u	$-2.57387\ 10^{-5}$	$-2.5731\ 10^{-5}$	– 0.03	10^{-2}
	v	$2.57387\ 10^{-5}$	$2.5731\ 10^{-5}$	– 0.03	
	σ_{rr}	20.	20.0000	3.10^{-6}	
	$\sigma_{\theta\theta}$	20.	19.9996	-2.10^{-5}	
	σ_{zz}	12.	11.9999	-7.10^{-6}	
	$\sigma_{r\theta}$	20.	19.9975	-1.10^{-4}	

4.4 Remarks

Evolution of the error induced by the following grid AB , CD or EF , is clearly attenuated compared to modeling A.

5 Modeling C

5.1 Characteristics of modeling: D-plan (QUAD9)



Limiting conditions:

side AB	DDL_IMPO =	(GROUP_NO = bordAB	DY = 0.)
side EF	FACE_IMPO =	(GROUP_MA = faceEF	DNOR = 0.)
pressure on AE	PRES_REP =	(GROUP_MA = faceAE	CLOSE = 60.)

Names of the nodes: $A = N1$ $B = N47$ $C = N351$
$D = N374$ $E = N569$ $F = N423$

5.2 Characteristics of the grid

Many nodes: 725

Many meshs and types: 168 QUAD9

5.3 Sizes tested and results

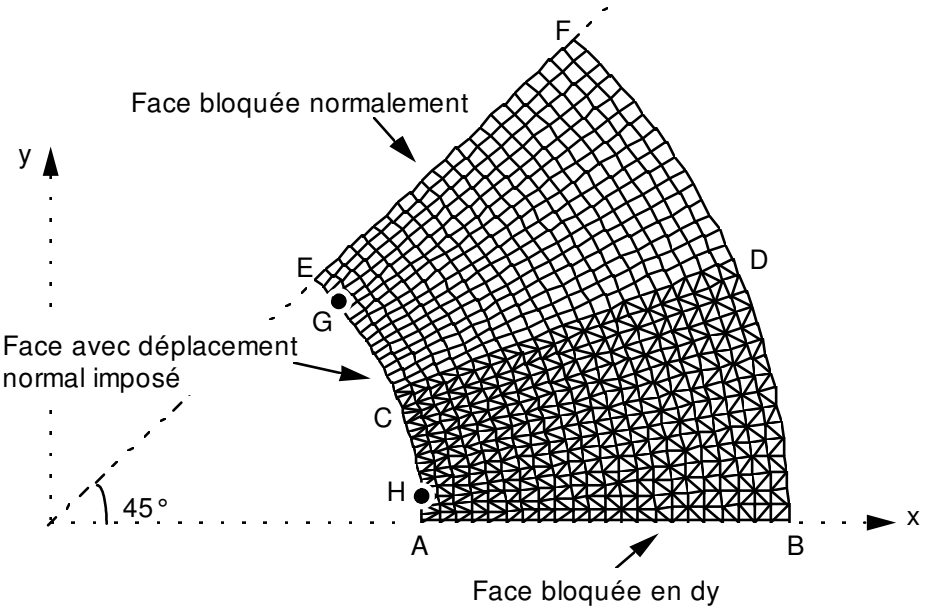
Place	Size	Reference	*Aster*	% difference	tolerance
A	u	5.72 10^{-5}	5.7173 10^{-5}	– 0.05	10^{-2}
	v	0.	epsilon	-	
	σ_{rr}	– 60.	– 56.8334	– 0.27	
	$\sigma_{\theta\theta}$	100.	99.84	– 0.16	
	σ_{zz}	12.	12.0023	0.02	
	$\sigma_{r\theta}$	0.	– 0.00272	-	
C	u	5.28459 10^{-5}	5.2821 10^{-5}	– 0.05	10^{-2}
	v	2.18895 10^{-5}	2.1879 10^{-5}	– 0.05	
	σ_{rr}	– 36.56854	– 36.45	0.32	
	$\sigma_{\theta\theta}$	76.56854	76.46	– 0.14	
	σ_{zz}	12.	12,003	0.02	
	$\sigma_{r\theta}$	– 56.56854	– 56.45	0.2	
E	u	4.04465 10^{-5}	4.0427 10^{-5}	– 0.05	10^{-2}
	v	4.04465 10^{-5}	4.0427 10^{-5}	– 0.05	
	σ_{rr}	20.	19,996	– 2. 10^{-4}	
	$\sigma_{\theta\theta}$	20.	20,011	5.5 10^{-4}	
	σ_{zz}	12.	12,002	2. 10^{-4}	
	$\sigma_{r\theta}$	– 80.	– 79,837	0.2	
B	u	3,640 10^{-5}	3.6386 10^{-5}	0.04	10^{-2}
	v	0.	epsilon	-	
	σ_{rr}	0.	– 2.7 10^{-3}	-	
	$\sigma_{\theta\theta}$	40.	40.0011	– 4. 10^{-4}	
	σ_{zz}	12.	11.9995	– 4. 10^{-4}	
	$\sigma_{r\theta}$	0.	– 4.8 10^{-4}	-	
D	u	3.36292 10^{-5}	3.3617 10^{-5}	– 0.04	10^{-2}
	v	1.39297 10^{-5}	1.3924 10^{-5}	– 0.04	
	σ_{rr}	5.85786	5.8557	– 0.03	
	$\sigma_{\theta\theta}$	34.14214	34,143	2.5 10^{-5}	
	σ_{zz}	12.	11.9996	3. 10^{-5}	
	$\sigma_{r\theta}$	– 14.14214	– 14.1435	– 1. 10^{-4}	
F	u	2.57387 10^{-5}	2.5729 10^{-5}	– 0.04	10^{-2}
	v	2.57387 10^{-5}	2.5729 10^{-5}	– 0.04	
	σ_{rr}	20.	19,999	– 3. 10^{-5}	
	$\sigma_{\theta\theta}$	20.	20.0002	1. 10^{-5}	
	σ_{zz}	12.	11.9999	– 9. 10^{-6}	
	$\sigma_{r\theta}$	– 20.	– 20.0025	– 0.01	

5.4 Remarks

Evolution of the error induced by the following grid AB , CD or EF , is clearly attenuated compared to modeling A.

6 Modeling D

6.1 Characteristics of modeling: D-plan (QUAD4 + TRIA3)



Limiting conditions:

side AB D DL_IMPO: (GROUP_NO = bordAB DY = 0.)

side EF FACE_IMPO: (GROUP_MA = faceEF DNOR = 0.)

on AE normal displacement imposed on $5.72E-5m$

FACE_IMPO: (GROUP_MA = faceAE DNOR = -5.72 E-5)

Names of the nodes: $A=N23$ $B=N1$ $C=N391$ $D=N369$ $E=N451$ $F=N751$ $H=N92$ $G=N447$

6.2 Characteristics of the grid

Many nodes: 759

Many meshs and types: 704 TRIA3, 352 QUAD4

6.3 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
C	F_x	0.1360	0.14069	3.45
	F_y	0,056	0.05827	4.06
H	F_x	0.14686	0.13608	7.34
	F_y	0.0108	0.0100	7.04
G	F_x	0.1138	0,114	0.19
	F_y	0,093	0.0936	0.61

6.4 Remarks

One checks that the nodal forces of reaction are worthless in all the nodes, except on the nodes of the line AE , EF and AB .

7 Summary of the results

D_plan

Summary of the errors max in %		modeling With	B	C
Displacements	WITH, B	0.08	0.04	0.05
	C, D	0.51	0.04	0.05
	E, F	0.11	0.04	0.05
Constraints σ_{xx}	WITH, B	6.04	0.29	0.27
	C, D	10.84	0.17	0.32
	E, F	17.46	4.10^{-4}	2.10^{-4}
Constraints σ_{yy}	WITH, B	3.61	0.38	0.16
	C, D	0.72	0.63	0.14
	E, F	27.07	2.10^{-5}	$5.5.10^{-4}$
Constraints σ_{zz}	WITH, B	1.33	0.16	0.02
	C, D	8.51	0.63	0.02
	E, F	22.27	2.10^{-4}	2.10^{-4}
Constraints σ_{xy}	WITH, B	-	-	-
	C, D	4.99	0.50	0.2
	E, F	2.11	0.23	0.2

These 3 modelings appreciably have the same number of nodes; the results got with elements of order 1 (modeling A in TRIA3 and QUAD4) are definitely less precise, in particular on the internal wall.