SSLV04 - Hollow roll in plane constraints

Summary:

This test is drawn from Guide VPCS (test SSLV04/89) and has as an aim a hollow roll charged in internal pressure.

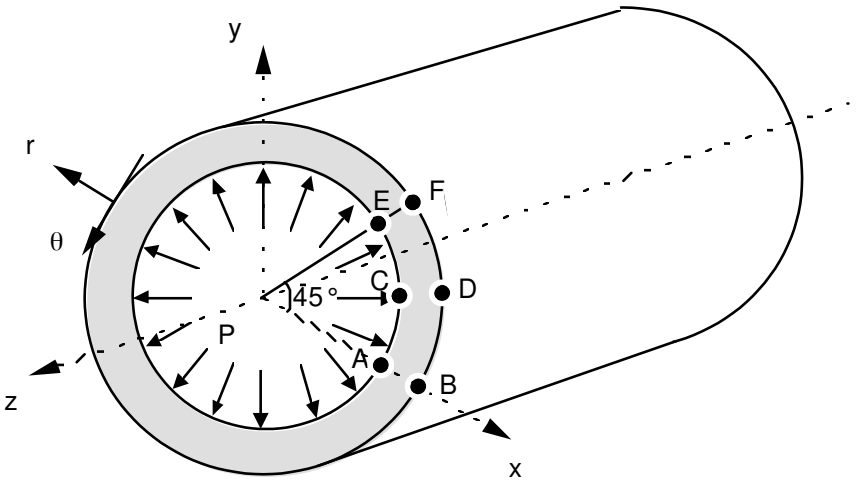
With this three-dimensional problem is dealt with various modelings:
- in 3D : 9 modelings (pentahedral, hexahedrons, tetrahedrons and pyramids, degrees 1 and 2),
- in 2D plane constraints: 4 modelings (triangles and quadrangles degrees 1 and 2, quadrangles with 9 nodes),
- in 2D axisymmetric: 3 modelings (triangles and quadrangles degrees 1 and 2, quadrangles with 9 nodes).

The features tested are:
- pressure distributed,
- basic effect (with fixed or variable pressure),
- imposed displacements,
- matrices of rigidity,
- strains and stresses with the nodes,
- nodal reactions (modeling K).

There are 16 modelings.

1 Problem of reference

1.1 Geometry



Rayon interne a = 0.1 m
Rayon externe b = 0.2 m

Coordinates of the points:

	A	B	C	D	E	F
x	0,100	0,200	$0.1\cos(22.5)$	$0.2\cos(22.5)$	$\sqrt{2}/2$	$\sqrt{2}$
y	0.	0.	$0.1\sin(22.5)$	$0.2\sin(22.5)$	$\sqrt{2}/2$	$\sqrt{2}$
z	0	0.	0.	0.	0.	0.

1.2 Material properties

The Young modulus of material is equal to $E=2.10^5\ MPa$.
The Poisson's ratio is equal to $\nu=0.3$.

1.3 Boundary conditions and loadings

Internal pressure:
$P=60\ MPa$

Pressure interns variable (modeling P only):
P vary linearly $60\ MPa$ with $t=1.\text{s}$ with $120\ MPa$ with $t=2.\text{s}$

2 Reference solution

2.1 Method of calculating used for the reference solution

In constraint planes (cylinder on free board at the ends)

$$\begin{aligned}
\sigma_{zz} &= 0 \\
\sigma_{rr} &= P\frac{a^2}{b^2-a^2}\left[1-\frac{b^2}{r^2}\right] \\
\sigma_{\theta\theta} &= P\frac{a^2}{b^2-a^2}\left[1+\frac{b^2}{r^2}\right] \\
\sigma_{r\theta} &= 0 \\
u_r &= \frac{P}{E}\frac{a^2}{b^2-a^2}\left[(1-\nu)+(1+\nu)\frac{b^2}{r^2}\right]r \\
\varepsilon_{rr} &= \frac{P}{E}\frac{a^2}{b^2-a^2}\left[(1-\nu)-(1+\nu)\frac{b^2}{r^2}\right] \\
\varepsilon_{\theta\theta} &= \frac{u_r}{r}
\end{aligned}$$

One obtains:

for $r=0.1$: $u_r=59.10^{-6}$, $\sigma_{rr}=-60.$, $\sigma_{\theta\theta}=100.$, $\sigma_{zz}=\sigma_{r\theta}=0.$, $\varepsilon_{rr}=-45.10^{-5}$, $\varepsilon_{\theta\theta}=59.10^{-5}$; for $r=0.2$: $u_r=40.10^{-6}$, $\sigma_{rr}=0.$, $\sigma_{\theta\theta}=40.$, $\sigma_{zz}=\sigma_{r\theta}=0.$, $\varepsilon_{rr}=-6.10^{-5}$, $\varepsilon_{\theta\theta}=2.10^{-4}$

Passage in the system of Cartesian axes:

$$\begin{aligned}
\sigma_{xx} &= \sigma_{rr}\cos^2\theta+\sigma_{\theta\theta}\sin^2\theta-2\sigma_{r\theta}\sin\theta\cos\theta \\
\sigma_{yy} &= \sigma_{rr}\sin^2\theta+\sigma_{\theta\theta}\cos^2\theta+2\sigma_{r\theta}\sin\theta\cos\theta \\
\sigma_{xy} &= \sigma_{rr}\sin\theta\cos\theta-\sigma_{\theta\theta}\sin\theta\cos\theta-2\sigma_{r\theta}\left(\cos^2\theta-\sin^2\theta\right)
\end{aligned}$$

with:

- $\theta=0°$ at the points A and B,
- $\theta=22.5°$ at the points C and D,
- $\theta=45°$ at the points E and F.

2.2 Results of reference

Displacements (u,v) and constraints $\left(\sigma_{xx},\sigma_{yy},\sigma_{zz},\sigma_{xy}\right)$ at the points A,B,C,D,E,F.

2.3 Bibliographical references

1 Guide VPCS. SSLV04/89

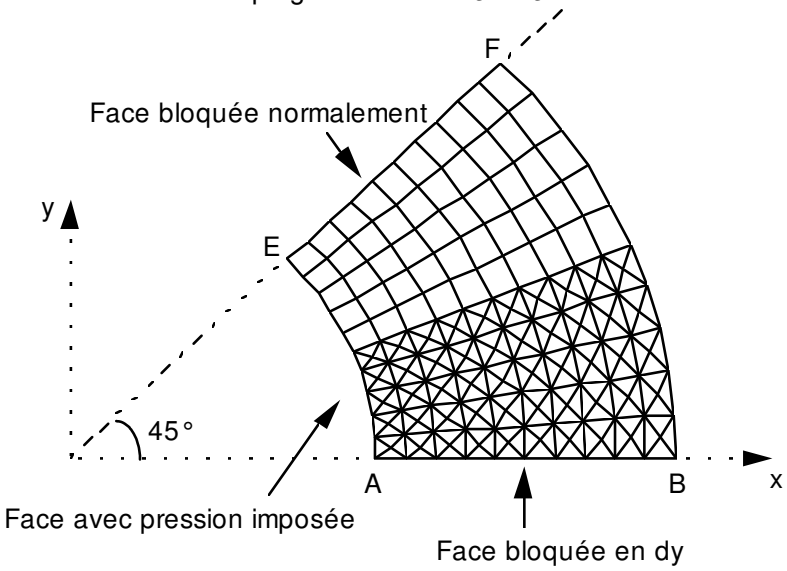
2 Y.C. FUNG. Foundations of solid mechanics. Prentice-hall, Inc. Englewood Cliffs. NJ. 1965 p. 243 to 245.

3 J. COURBON. Resistance of the materials p 649

3 Modeling A

3.1 Characteristics of modeling

Elements 3D (PENTA6 and HEXA8).
Grid obtained by extrusion starting from a grid $2D$ resembling the grid below (30 elements in the radial direction with déraffinement progressive and $15+15$ elements in the circumferential direction).



Along the axis Z : 1 layer of elements
Total thickness: 0.01m

Limiting conditions:

node F : $u_z=0$
face AB blocked in dy
face EF blocked normally
pressure on the face AE $p=60.$

Names of the nodes: $A=N993$ $B=N1443$ $C=N1$
$D=N31$ $E=N496$ $F=N495$

3.2 Characteristics of the grid

Many nodes: 1922
Many meshs and types: 900 PENTA6, 450 HEXA8 and 90 QUAD4 (faces internal skin).

3.3 Sizes tested and results

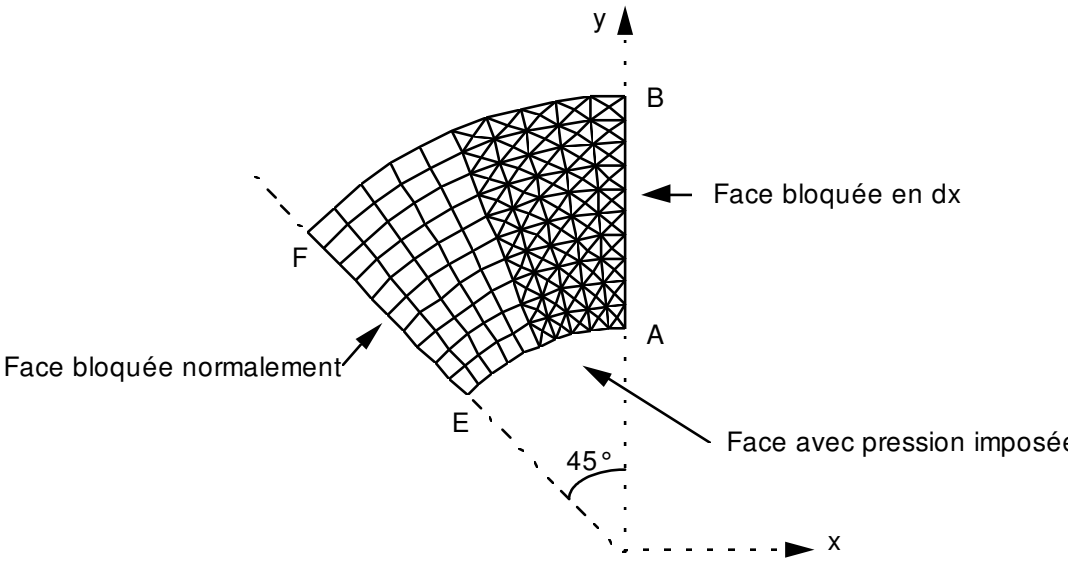
Localization	Size	Reference	*Aster*	% difference
A	u	5.9 10^{-5}	5.8950 10^{-5}	0.08
	v	0.	*eps*	-
	σ_{xx}	– 60.	– 59.2225	1.30
	σ_{yy}	100.	100.4159	0.42
	σ_{zz}	0.	0.3093	-
	σ_{xy}	0.	– 1.0442	-
	ε_{xx}	– 4.5 10^{-4}	– 4,472 10^{-4}	0.62
	ε_{yy}	5.9 10^{-4}	5,904 10^{-4}	0.08
	ε_{xy}	0.	– 6,788 10^{-5}	-
B	u	4 10^{-5}	3.9959 10^{-5}	0.10
	v	0.	*eps*	-
	σ_{xx}	0.	– 1.7246	-
	σ_{yy}	40.	39.2451	1.89
	σ_{zz}	0.	– 0.3761	-
	σ_{xy}	0.	– 0.2659	-
	ε_{xx}	– 0.6 10^{-4}	– 6,692 10^{-5}	11.54
	ε_{yy}	2. 10^{-4}	1,994 10^{-4}	0.31
	ε_{xy}	0.	– 1,728 10^{-6}	-
E	u	4.17193 10^{-5}	4.1708 10^{-5}	0.03
	v	4.17193 10^{-5}	4.1708 10^{-5}	0.03
	σ_{xx}	20.	19.0824	4.59
	σ_{yy}	20.	21.1394	5.70
	σ_{zz}	0.	0.0870	-
	σ_{xy}	– 80.	– 79.8831	0.15
	ε_{xx}	0.7 10^{-4}	0,636 10^{-4}	9.18
	ε_{yy}	0.7 10^{-4}	0,769 10^{-4}	9.92
	ε_{xy}	– 5.2 10^{-4}	– 5,192 10^{-4}	0.15
F	u	2.82843 10^{-5}	2.8302 10^{-5}	0.06
	v	2.82843 10^{-5}	2.8302 10^{-5}	0.06
	σ_{xx}	20.	18.9528	5.24
	σ_{yy}	20.	19.9104	0.45
	σ_{zz}	0.	0.1198	-
	σ_{xy}	– 20.	– 20.1809	0.90
	ε_{xx}	0.7 10^{-4}	0,647 10^{-4}	7.54
	ε_{yy}	0.7 10^{-4}	0,709 10^{-4}	1.35
	ε_{xy}	– 1.3 10^{-4}	– 1,312 10^{-4}	0.90

Localization	Size	Reference	Tolerance
Mesh M1380, not A			
SIRO_ELEM	SIG_NX	60.0	2.0%
Normal constraint with the face of the element	SIG_NY	0.0	1.0
	SIG_NZ	0.0	0.01
	SIG_N	-60.0	2.0%
SIRO_ELEM	SIG_TX	0.0	0.01
Tangential constraint in the plan of the element	SIG_TY	0.0	1.0%
	SIG_TZ	0.0	1.0
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the constraint	SIG_T1Y	0.0	0.01
tangential in the plan of the element	SIG_T1Z	0.0	1.0
	SIG_T1	0.0	1.0
SIRO_ELEM	SIG_T2X	0.0	2.0
Second value of the constraint	SIG_T2Y	-100.0	1.0%
tangential in the plan of the element	SIG_T2Z	0.0	0.01
	SIG_T2	100.0	1.0%
Mesh M1351, not E			
SIRO_ELEM	SIG_NX	42,426	1.5%
Normal constraint with the face of the element	SIG_NY	42,426	2.0%
	SIG_NZ	0.0	0.01
	SIG_N	-60.0	1.0%
SIRO_ELEM	SIG_TX	0.0	1.0
Tangential constraint in the plan of the element	SIG_TY	0.0	1.0
	SIG_TZ	0.0	0.2
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the constraint	SIG_T1Y	0.0	0.01
tangential in the plan of the element	SIG_T1Z	0.0	0.1
	SIG_T1	0.0	0.2
SIRO_ELEM	SIG_T2X	70,711	1.5%
Second value of the constraint	SIG_T2Y	-70,711	1.5%
tangential in the plan of the element	SIG_T2Z	0.0	0.01
	SIG_T2	100.0	1.0%

4 Modeling B

4.1 Characteristics of modeling

Elements 3D (PENTA15 and HEXA20).
Grid obtained by extrusion starting from the grid $2D$ below (modeling F)



Along the axis Z : 2 layers of elements
Total thickness: $0.01\mathrm{m}$

Limiting conditions:

node $F = u_z = 0$
face AB blocked in dx
face EF blocked normally
pressure on the face AE $p = 60.$

Names of the nodes: $A = NO2$ $B = NO361$ $C = NO121$
$D = NO584$ $E = NO155$ $F = NO503$

4.2 Characteristics of the grid

Many nodes: 2115
Many meshs and types: 400 PENTA15, 100 HEXA20 40 QUAD8 (faces skin interns)

4.3 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
A	u	0.	eps	-
	v	$5.9\ 10^{-5}$	$5.8944\ 10^{-5}$	– 0.09
	σ_{xx}	100.	99.6056	– 0.39
	σ_{yy}	– 60.	– 59.4473	– 0.92
	σ_{zz}	0.	0.0196	-
	σ_{xy}	0.	0.2481	-
	ε_{xx}	$5.9\ 10^{-4}$	$5.87\ 10^{-4}$	– 0.48
	ε_{yy}	$-4.5\ 10^{-4}$	$-4.47\ 10^{-4}$	– 0.74
	ε_{xy}	0.	$1.61\ 10^{-6}$	-
B	u	0.	eps	-
	v	$4\ 10^{-5}$	$3.9974\ 10^{-5}$	– 0.07
	σ_{xx}	40.	39.9711	0.07
	σ_{yy}	0.	0.0781	-
	σ_{zz}	0.	$5.7992\ 10^{-3}$	-
	σ_{xy}	0.	– 0.0182	-
	ε_{xx}	$2.\ 10^{-4}$	$1,997\ 10^{-4}$	– 0.13
	ε_{yy}	$-0.6.\ 10^{-4}$	$-0,596\ 10^{-4}$	– 0.67
	ε_{xy}	0.	-1.1810^{-7}	-
E	u	$4.17193\ 10^{-5}$	$4.1680\ 10^{-5}$	– 0.09
	v	$4.17193\ 10^{-5}$	$4.1680\ 10^{-5}$	– 0.09
	σ_{xx}	20.	20.0515	0.26
	σ_{yy}	20.	20.0264	0.13
	σ_{zz}	0.	– 0.0155	-
	σ_{xy}	80.	79.7918	– 0.26
	ε_{xx}	$0.7\ 10^{-4}$	$0,702\ 10^{-4}$	0.34
	ε_{yy}	$0.7\ 10^{-4}$	$0,701\ 10^{-4}$	0.11
	ε_{xy}	$5.2\ 10^{-4}$	$5.19\ 10^{-4}$	– 0.26
F	u	$-2.82843\ 10^{-5}$	$-2.82656\ 10^{-5}$	– 0.07
	v	$2.82843\ 10^{-5}$	$2.82656\ 10^{-5}$	– 0.07
	σ_{xx}	20.	20.0099	0.05
	σ_{yy}	20.	19.9980	– 0.01
	σ_{zz}	0.	$-3.90\ 10^{-4}$	-
	σ_{xy}	20.	20.0122	0.06
	ε_{xx}	$0.7\ 10^{-4}$	$0.7005\ 10^{-4}$	0.08
	ε_{yy}	$0.7\ 10^{-4}$	$0.6997\ 10^{-4}$	– 0.03
	ε_{xy}	$-1.3\ 10^{-4}$	$-1.3008\ 10^{-4}$	0.06

Localization	Size	Reference	Tolerance
Mesh MA751, not A			
SIRO_ELEM	SIG_NX	0.0	3.
Normal constraint with the face of the element	SIG_NY	60.0	0.5%
	SIG_NZ	0.0	0.01
SIRO_ELEM	SIG_TX	0.0	0.1
Tangential constraint in the plan of the element	SIG_TY	0.0	0.1
	SIG_TZ	0.0	0.1
SIRO_ELEM	SIG_T1X	0.0	0.01
First value constraint	SIG_T1Y	0.0	0.01
tangential in the plan of the element	SIG_T1Z	0.0	0.05
SIRO_ELEM	SIG_T2X	100.0	0.15%
Second value of the constraint	SIG_T2Y	0.0	4.5
tangential in the plan of the element	SIG_T2Z	0.0	0.20
Mesh MA769, not E			
SIRO_ELEM	SIG_NX	-42,426	6.0%
Normal constraint with the face of the element	SIG_NY	42,426	4.0%
	SIG_NZ	0.0	0.01
SIRO_ELEM	SIG_TX	0.0	0.01
Tangential constraint in the plan of the element	SIG_TY	0.0	0.01
	SIG_TZ	0.0	0.03
SIRO_ELEM	SIG_T1X	0.0	0.01
First value constraint	SIG_T1Y	0.0	0.01
tangential in the plan of the element	SIG_T1Z	0.0	0.01
SIRO_ELEM	SIG_T2X	70,711	3.5%
Second value of the constraint	SIG_T2Y	70,711	4.5%
tangential in the plan of the element	SIG_T2Z	0.0	0.1

5 Modeling C

5.1 Characteristics of modeling

Elements 3D (TETRA4).
AB is on the axis OX

Cutting:	21 equidistant nodes on the segments AB , CD and EF 21 equidistant nodes on the arcs ACE and BDF
Along the axis Z :	1 layer of elements
Total thickness:	$0.01\mathrm{m}$

Limiting conditions:

node F : $u_z=0$
face AB blocked in dy
face EF blocked normally
pressure on the face AE $p=60.$

Names of the nodes:	$A=N165$	$B=N4$	$C=N209$
	$D=N82$	$E=N244$	$F=N1068$

5.2 Characteristics of the grid

Many nodes: 1115
Many meshs and types: 3724 TETRA4 and 1760 TRIA3 (faces skin interns)

5.3 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
A	u	$5.9\ 10^{-5}$	$5.8901\ 10^{-5}$	– 0.17
	v	0.	eps	-
	σ_{xx}	– 60.	– 57.2290	– 4.62
	σ_{yy}	100.	97.8711	– 2.13
	σ_{zz}	0.	0.0568	-
	σ_{xy}	0.	– 2.6589	-
	ε_{xx}	$-4.5\ 10^{-4}$	$-4.33\ 10^{-4}$	– 3.77
	ε_{yy}	$5.9\ 10^{-4}$	$5.75\ 10^{-4}$	– 2.52
	ε_{xy}	0.	$-1.73\ 10^{-5}$	-
B	u	$4\ 10^{-5}$	$3.9878\ 10^{-5}$	– 0.30
	v	0.	eps	-
	σ_{xx}	0.	– 1.5296	-
	σ_{yy}	40.	40.9839	2.46
	σ_{zz}	0.	– 0.1006	-
	σ_{xy}	0.	– 0.8513	-
	ε_{xx}	$-0.6\ 10^{-4}$	$-6{,}897\ 10^{-4}$	14.95
	ε_{yy}	$2.\ 10^{-4}$	$2{,}074\ 10^{-4}$	3.68
	ε_{xy}	0.	$-5{,}534\ 10^{-5}$	-
E	u	$4.17193\ 10^{-5}$	$4.1655\ 10^{-5}$	– 0.15
	v	$4.17193\ 10^{-5}$	$4.1655\ 10^{-5}$	– 0.15
	σ_{xx}	20.	17.9096	– 10.45
	σ_{yy}	20.	21.8929	9.46
	σ_{zz}	0.	– 0.3679	-
	σ_{xy}	– 80.	– 77.6897	– 2.89
	ε_{xx}	$0.7\ 10^{-4}$	$0{,}573\ 10^{-4}$	– 18.20
	ε_{yy}	$0.7\ 10^{-4}$	$0{,}832\ 10^{-4}$	18.79
	ε_{xy}	$-5.2\ 10^{-4}$	$-5{,}050\ 10^{-4}$	– 2.89
F	u	$2.82843\ 10^{-5}$	$2.8251\ 10^{-5}$	– 0.12
	v	$2.82843\ 10^{-5}$	$2.8251\ 10^{-5}$	– 0.12
	σ_{xx}	20.	18.4444	– 7.78
	σ_{yy}	20.	19.8876	– 0.56
	σ_{zz}	0.	– 0.3910	-
	σ_{xy}	– 20.	– 20.1631	0.81
	ε_{xx}	$0.7\ 10^{-4}$	$0{,}630\ 10^{-4}$	– 10.05
	ε_{yy}	$0.7\ 10^{-4}$	$0{,}723\ 10^{-4}$	3.35
	ε_{xy}	$-1.3\ 10^{-4}$	$-1{,}311\ 10^{-4}$	0.81

Localization	Size	Reference	Tolerance
Mesh M5444, not A			
SIRO_ELEM	SIG_NX	60.0	8.0%
Normal constraint with the face of the element	SIG_NY	0.0	1.5
	SIG_NZ	0.0	0.1
SIRO_ELEM	SIG_TX	0.0	0.1
Tangential constraint in the plan of the element	SIG_TY	0.0	1.0
	SIG_TZ	0.0	1.0
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the constraint	SIG_T1Y	0.0	0.01
tangential in the plan of the element	SIG_T1Z	0.0	1.5
SIRO_ELEM	SIG_T2X	0.0	2.5
Second value of the constraint	SIG_T2Y	-100.0	2.0%
tangential in the plan of the element	SIG_T2Z	0.0	0.20
Mesh M5404, not E			
SIRO_ELEM	SIG_NX	42,426	6.0%
Normal constraint with the face of the element	SIG_NY	42,426	9.0%
	SIG_NZ	0.0	1.00
SIRO_ELEM	SIG_TX	0.0	1.0
Tangential constraint in the plan of the element	SIG_TY	0.0	1.0
	SIG_TZ	0.0	1.0
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the constraint	SIG_T1Y	0.0	0.01
tangential in the plan of the element	SIG_T1Z	0.0	1.50
SIRO_ELEM	SIG_T2X	-70,711	2.0%
Second value of the constraint	SIG_T2Y	70,711	4.0%
tangential in the plan of the element	SIG_T2Z	0.0	0.10

6 Modeling D

6.1 Characteristics of modeling

Element 3D (TETRA10).

AB is on the axis OX

Cutting:	11 equidistant nodes on the segments AB , CD and EF 11 equidistant nodes on the arcs ACE and BDF
Along the axis Z :	1 layer of elements
Total thickness:	$0.01\mathrm{m}$

Limiting conditions:

node F : $u_z=0$
face AB blocked in dy
face EF blocked normally
pressure on the face AE $p=60.$

Names of the nodes:	$A=N184$	$B=N4$	$C=N207$
	$D=N50$	$E=N22$	$F=N726$

6.2 Characteristics of the grid

Many nodes: 1395
Many meshs and types: 652 TETRA10 and 480 TRIA6 (faces skin interns)

6.3 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
A	u	$5.9\ 10^{-5}$	$5.8974\ 10^{-5}$	– 0.04
	v	0.	*eps*	-
	σ_{xx}	– 60.	– 60.3816	0.64
	σ_{yy}	100.	99.1907	– 0.81
	σ_{zz}	0.	– 0.9707	-
	σ_{xy}	0.	– 0.2979	-
	ε_{xx}	$-4.5\ 10^{-4}$	$-4.49\ 10^{-4}$	– 0.17
	ε_{yy}	$5.9\ 10^{-4}$	$5.88\ 10^{-4}$	– 0.34
	ε_{xy}	0.	$-1.94\ 10^{-6}$	-
B	u	$4\ 10^{-5}$	$3.9989\ 10^{-5}$	– 0.03
	v	0.	*eps*	-
	σ_{xx}	0.	0.0388	-
	σ_{yy}	40.	40.0725	0.18
	σ_{zz}	0.	– 0.0046	-
	σ_{xy}	0.	0.1634	-
	ε_{xx}	$-0.6\ 10^{-4}$	$-0,599\ 10^{-4}$	– 0.15
	ε_{yy}	$2.\ 10^{-4}$	$2,003\ 10^{-4}$	0.16
	ε_{xy}	0.	$1,062\ 10^{-6}$	-
E	u	$4.17193\ 10^{-5}$	$4.17021\ 10^{-5}$	0.04
	v	$4.17193\ 10^{-5}$	$4.17021\ 10^{-5}$	0.04
	σ_{xx}	20.	19.1178	– 4.41
	σ_{yy}	20.	19.6399	– 1.80
	σ_{zz}	0.	– 1.0206	-
	σ_{xy}	– 80.	– 79.7804	– 0.27
	ε_{xx}	$0.7\ 10^{-4}$	$0,677\ 10^{-4}$	– 3.34
	ε_{yy}	$0.7\ 10^{-4}$	$0,711\ 10^{-4}$	1.50
	ε_{xy}	$-5.2\ 10^{-4}$	$-5,186\ 10^{-4}$	– 0.27
F	u	$2.82843\ 10^{-5}$	$2.82718\ 10^{-5}$	– 0.04
	v	$2.82843\ 10^{-5}$	$2.82718\ 10^{-5}$	– 0.04
	σ_{xx}	20.	20.1903	0.95
	σ_{yy}	20.	19.9023	– 0.49
	σ_{zz}	0.	– 0.0016	-
	σ_{xy}	– 20.	– 20.0570	0.28
	ε_{xx}	$0.7\ 10^{-4}$	$0,711\ 10^{-4}$	1.57
	ε_{yy}	$0.7\ 10^{-4}$	$0,692\ 10^{-4}$	– 1.10
	ε_{xy}	$-1.3\ 10^{-4}$	$-1,304\ 10^{-4}$	0.28

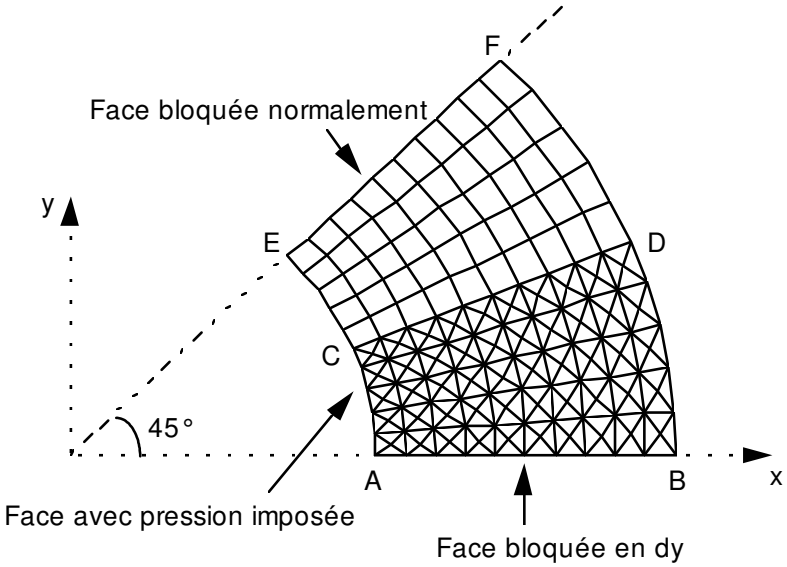
Localization	Size	Reference	Tolerance
Mesh M1111, not A			
SIRO_ELEM	SIG_NX	60.0	1.0%
Normal constraint with the face of the element	SIG_NY	0.0	3.0
	SIG_NZ	0.0	1.0
SIRO_ELEM	SIG_TX	0.0	1.0
Tangential constraint in the plan of the element	SIG_TY	0.0	2.5
	SIG_TZ	0.0	1.0
SIRO_ELEM	SIG_T1X	0.0	0.01
First principal value of the constraint	SIG_T1Y	0.0	0.01
tangential in the plan of the element	SIG_T1Z	0.0	1.0
SIRO_ELEM	SIG_T2X	0.0	4.5
Second principal value of the constraint	SIG_T2Y	-100.0	1.0%
tangential in the plan of the element	SIG_T2Z	0.0	0.1
Mesh M1093, not E			
SIRO_ELEM	SIG_NX	42,426	14%
Normal constraint with the face of the element	SIG_NY	42,426	5.0%
	SIG_NZ	0.0	1.0
SIRO_ELEM	SIG_TX	0.0	2.0
Tangential constraint in the plan of the element	SIG_TY	0.0	2.0
	SIG_TZ	0.0	1.0
SIRO_ELEM	SIG_T1X	0.0	0.01
First principal value of the constraint	SIG_T1Y	0.0	0.01
tangential in the plan of the element	SIG_T1Z	0.0	1.5
SIRO_ELEM	SIG_T2X	-70,711	5.0%
Second principal value of the constraint	SIG_T2Y	70,711	4.0%
tangential in the plan of the element	SIG_T2Z	0.0	2.0

7 Modeling E

7.1 Characteristics of modeling

Elements C_PLAN (TRIA3 + QUAD4)

Grid $2D$ resembling the grid below (30 elements in the radial direction with déraffinement progressive and $15+15$ elements in the circumferential direction).



Limiting conditions:

side AB blocked in dy
side EF blocked normally
pressure on AE $p=60.$

Names of the nodes: $A=N1$ $B=N451$ $C=N496$
$D=N495$ $E=N990$ $F=N989$

7.2 Characteristics of the grid

Many nodes: 961
Many meshs and types: 900 TRIA3, 450 QUAD4

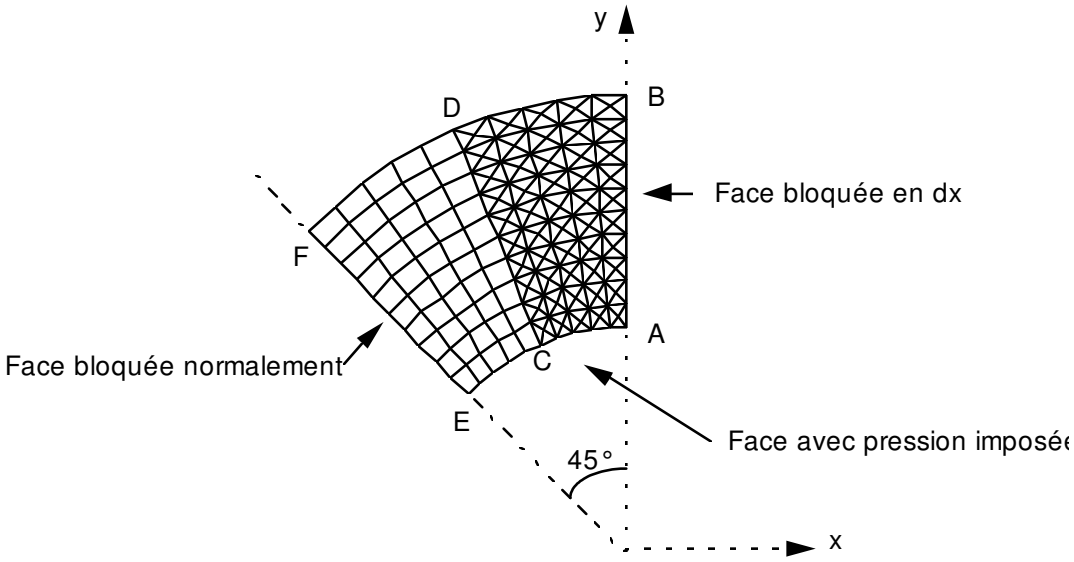
7.3 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
A	u	5.9 10^{-5}	5.8957 10^{-5}	– 0.07
	v	0.	eps	-
	σ_{xx}	– 60.	– 59.3645	– 1.06
	σ_{yy}	100.	100.2653	0.26
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	– 1.0472	-
	ε_{xx}	– 4.5 10^{-4}	– 4,472 10^{-4}	– 0.62
	ε_{yy}	5.9 10^{-4}	5,904 10^{-4}	0.06
	ε_{xy}	0.	– 6,807 10^{-6}	-
B	u	4. 10^{-5}	3.9965 10^{-5}	– 0.09
	v	0.	eps	-
	σ_{xx}	0.	– 1.4986	-
	σ_{yy}	40.	39.4415	– 1.40
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	– 0.2658	-
	ε_{xx}	– 0.6 10^{-4}	– 0,667 10^{-5}	11.09
	ε_{yy}	2. 10^{-4}	1,995 10^{-4}	– 0.27
	ε_{xy}	0.	– 1,728 10^{-6}	-
E	u	4.17193 10^{-5}	4.17101 10^{-5}	– 0.02
	v	4.17193 10^{-5}	4.17101 10^{-5}	– 0.02
	σ_{xx}	20.	19.0706	– 4.65
	σ_{yy}	20.	21.1354	5.68
	σ_{zz}	0.	0.	-
	σ_{xy}	– 80.	– 79.8720	– 0.16
	ε_{xx}	0.7 10^{-4}	0,636 10^{-4}	– 9.07
	ε_{yy}	0.7 10^{-4}	0,771 10^{-4}	10.10
	ε_{xy}	– 5.2 10^{-4}	– 5,192 10^{-4}	– 0.16
F	u	2.82843 10^{-5}	2.82996 10^{-5}	0.05
	v	2.82843 10^{-5}	2.82996 10^{-5}	0.05
	σ_{xx}	20.	18.9626	– 5.19
	σ_{yy}	20.	19.8483	– 0.76
	σ_{zz}	0.	0.	-
	σ_{xy}	– 20.	– 20.2466	1.23
	ε_{xx}	0.7 10^{-4}	0,650 10^{-4}	– 7.08
	ε_{yy}	0.7 10^{-4}	0,708 10^{-4}	1.14
	ε_{xy}	– 1.3 10^{-4}	– 1,316 10^{-4}	1.23

8 Modeling F

8.1 Characteristics of modeling

Elements C_plan (QUAD8 + TRIA6)



Limiting conditions:

side AB blocked in dx
side EF blocked normally
pressure on AE $p=60.$

Names of the nodes: $A=N2$ $B=N361$ $C=N121$
$D=N584$ $E=N155$ $F=N503$

8.2 Characteristics of the grid

Many nodes: 591
Many meshs and types: 200 TRIA6, 50 QUAD8

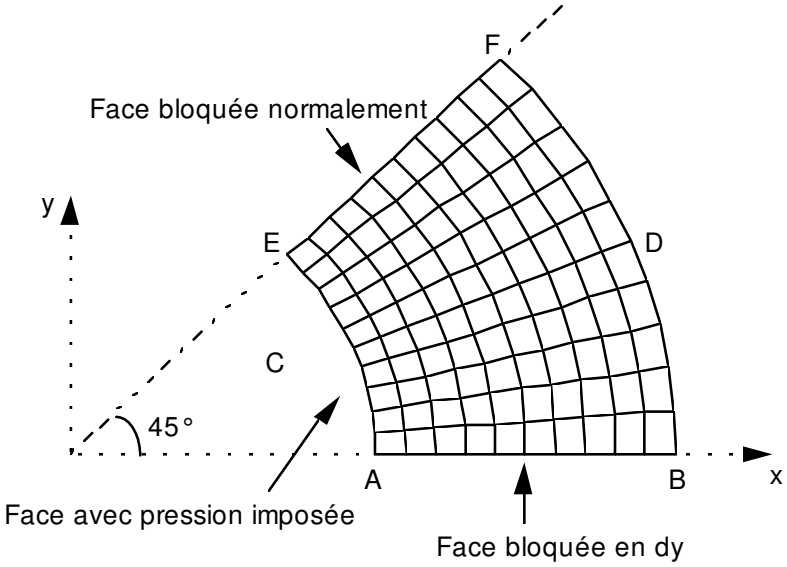
8.3 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
A	u	0.	0.	-
	v	$5.9\ 10^{-5}$	$5.8945\ 10^{-5}$	– 0.09
	σ_{xx}	100.	99.6095	– 0.39
	σ_{yy}	– 60.	– 59.4620	– 0.90
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	0.2441	-
	ε_{xx}	$5.9\ 10^{-4}$	$5,872\ 10^{-4}$	– 0.47
	ε_{yy}	$-4.5\ 10^{-4}$	$-4,467\ 10^{-4}$	– 0.73
	ε_{xy}	0.	$1,586\ 10^{-6}$	-
B	u	0.	eps	-
	v	$4\ 10^{-5}$	$3.9974\ 10^{-5}$	– 0.07
	σ_{xx}	40.	39.9774	– 0.06
	σ_{yy}	0.	0.0786	-
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	– 0.0181	-
	ε_{xx}	$2.\ 10^{-4}$	$1,998\ 10^{-4}$	– 0.11
	ε_{yy}	$-0.6\ 10^{-4}$	$-0,596\ 10^{-4}$	– 0.67
	ε_{xy}	0.	$-1,176\ 10^{-7}$	-
E	u	$-4.17193\ 10^{-5}$	$-4.16814\ 10^{-5}$	– 0.09
	v	$4.17193\ 10^{-5}$	$4.16814\ 10^{-5}$	– 0.09
	σ_{xx}	20.	20.0024	0.01
	σ_{yy}	20.	20.0045	0.02
	σ_{zz}	0.	0.	-
	σ_{xy}	80.	79.8164	– 0.23
	ε_{xx}	$0.7\ 10^{-4}$	$0.7001\ 10^{-4}$	0.01
	ε_{yy}	$0.7\ 10^{-4}$	$0.7002\ 10^{-4}$	0.03
	ε_{xy}	$5.2\ 10^{-4}$	$5,188\ 10^{-4}$	– 0.23
F	u	$-2.82843\ 10^{-5}$	$-2.82655\ 10^{-5}$	– 0.07
	v	$2.82843\ 10^{-5}$	$2.82655\ 10^{-5}$	– 0.07
	σ_{xx}	20.	20.0083	0.04
	σ_{yy}	20.	19.9915	– 0.04
	σ_{zz}	0.	0.	-
	σ_{xy}	20.	20.0138	0.07
	ε_{xx}	$0.7\ 10^{-4}$	$0.7005\ 10^{-4}$	0.08
	ε_{yy}	$0.7\ 10^{-4}$	$0.6995\ 10^{-4}$	– 0.08
	ε_{xy}	$-1.3\ 10^{-4}$	$1.3009\ 10^{-4}$	0.07

9 Modeling G

9.1 Characteristics of modeling

Modeling C_PLAN (QUAD9)



Limiting conditions:

side AB blocked in dy
side EF blocked normally
pressure on AE $p=60.$

Names of the nodes: $A=N1$ $B=N347$ $C=N21$
$D=N432$ $E=N39$ $F=N229$

9.2 Characteristics of the grid

Many nodes: 441
Many meshs and types: 100 QUAD9

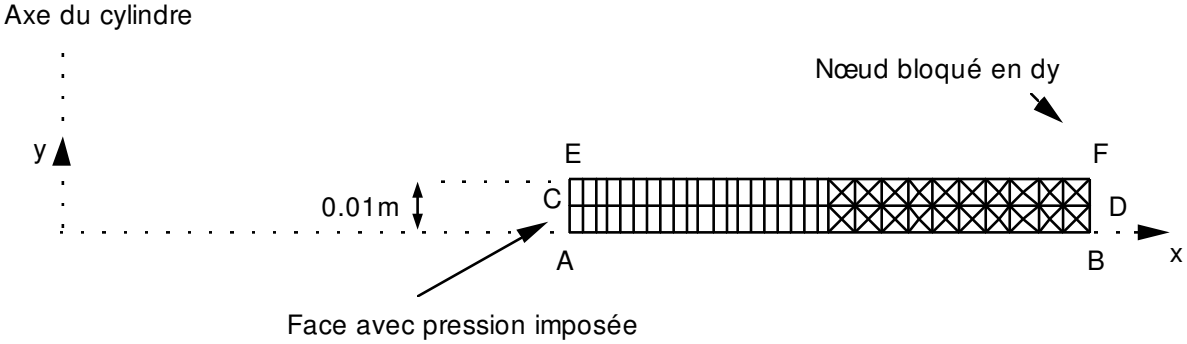
9.3 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
A	u	$5.9\ 10^{-5}$	$5.9000\ 10^{-5}$	0.00
	v	0.	eps	-
	σ_{xx}	– 60.	– 59.8354	– 0.27
	σ_{yy}	100.	99.8409	– 0.16
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	0.0283	-
	ε_{xx}	$-4.5\ 10^{-4}$	$-4{,}489\ 10^{-4}$	-0.24
	ε_{yy}	$5.9\ 10^{-4}$	$5{,}890\ 10^{-4}$	– 0.18
	ε_{xy}	0.	$-1{,}839\ 10^{-7}$	-
B	u	$4\ 10^{-5}$	$3.9999\ 10^{-5}$	– 0,001
	v	0.	eps	-
	σ_{xx}	0.	-0.0189	-
	σ_{yy}	40.	40.0182	0.05
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	$-3.6815\ 10^{-3}$	-
	ε_{xx}	$-0.6\ 10^{-4}$	$-0{,}601\ 10^{-4}$	0.20
	ε_{yy}	$2.\ 10^{-4}$	$2{,}001\ 10^{-4}$	0.06
	ε_{xy}	0.	$-2{,}393\ 10^{-8}$	-
E	u	$4.17193\ 10^{-5}$	$4.17195\ 10^{-5}$	0.00
	v	$4.17193\ 10^{-5}$	$4.17195\ 10^{-5}$	0.00
	σ_{xx}	20.	19.9745	– 0.13
	σ_{yy}	20.	20.0311	0.16
	σ_{zz}	0.	0.	-
	σ_{xy}	– 80.	– 79.8382	– 0.20
	ε_{xx}	$0.7\ 10^{-4}$	$0{,}698\ 10^{-4}$	– 0.25
	ε_{yy}	$0.7\ 10^{-4}$	$0{,}702\ 10^{-4}$	0.28
	ε_{xy}	$-5.2\ 10^{-4}$	$-5{,}189\ 10^{-4}$	– 0.20
F	u	$2.82843\ 10^{-5}$	$2.82839\ 10^{-5}$	– 0,001
	v	$2.82843\ 10^{-5}$	$2.82839\ 10^{-5}$	– 0,001
	σ_{xx}	20.	19.9960	– 0.02
	σ_{yy}	20.	20.0034	0.02
	σ_{zz}	0.	0.	-
	σ_{xy}	– 20.	– 20.0185	0.09
	ε_{xx}	$0.7\ 10^{-4}$	$0.6997\ 10^{-4}$	– 0.04
	ε_{yy}	$0.7\ 10^{-4}$	$0.7002\ 10^{-4}$	0.03
	ε_{xy}	$-1.3\ 10^{-4}$	$-1{,}301\ 10^{-4}$	0.09

10 Modeling H

10.1 Characteristics of modeling

Elements AXIS (TRIA3 + QUAD4)



Limiting conditions:

node F blocked in dy
pressure on AE $p=60.$

Names of the nodes: $A=N111$ $B=N1$ $C=N112$
$D=N3$ $E=N113$ $F=N4$

10.2 Characteristics of the grid

Many nodes: 113
Many meshs and types: 40 QUAD4, 80 TRIA3

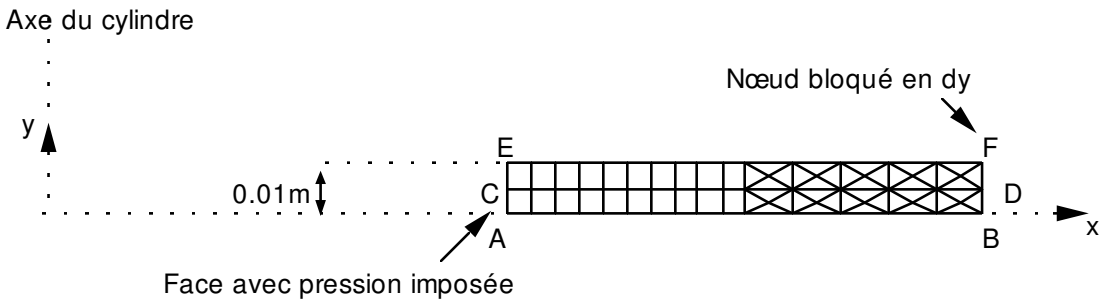
10.3 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
A	u	5.9 10^{-5}	5.8992 10^{-5}	– 0.01
	v	0.	-	-
	σ_{xx}	– 60.	– 56.6060	– 5.66
	σ_{yy}	0.	1.0383	-
	σ_{zz}	100.	101.2924	1.29
	σ_{xy}	0.	– 1.1635	-
	ε_{xx}	– 4.5 10^{-4}	– 4.36 10^{-4}	– 2.99
	ε_{yy}	0.	– 6.18 10^{-5}	-
	ε_{zz}	5.9 10^{-4}	5,898 10^{-4}	– 0.03
	ε_{xy}	0.	– 1.06 10^{-6}	-
B	u	4 10^{-5}	3.9997 10^{-5}	– 0.01
	v	0.	-	-
	σ_{xx}	0.	– 0.8951	-
	σ_{yy}	0.	– 0.4106	-
	σ_{zz}	40.	39.6001	– 1.00
	σ_{xy}	0.	– 0.1281	-
	ε_{xx}	– 0.6 10^{-4}	– 0,632 10^{-4}	5.43
	ε_{yy}	0.	– 6,011 10^{-5}	-
	ε_{zz}	2. 10^{-4}	1,999 10^{-4}	– 0.02
	ε_{xy}	0.	– 8,325 10^{-7}	-
E	u	5.9 10^{-5}	5.8992 10^{-5}	– 0.01
	v	0.	-	-
	σ_{xx}	– 60.	– 56.6060	– 5.66
	σ_{yy}	0.	1.0383	-
	σ_{zz}	100.	101.2924	1.29
	σ_{xy}	0.	1.1635	-
	ε_{xx}	– 4.5 10^{-4}	– 4,365 10^{-4}	– 2.99
	ε_{yy}	0.	– 6,184 10^{-5}	-
	ε_{zz}	5.9 10^{-4}	5,898 10^{-4}	– 0.03
	ε_{xy}	0.	1,063 10^{-6}	-
F	u	4 10^{-5}	3.9997 10^{-5}	– 0.01
	v	0.	-	-
	σ_{xx}	0.	– 0.4221	-
	σ_{yy}	0.	– 0.2280	-
	σ_{zz}	40.	39.8015	– 0.50
	σ_{xy}	0.	– 0.0020	-
	ε_{xx}	– 0.6 10^{-4}	– 0,615 10^{-4}	2.45
	ε_{yy}	0.	– 6,021 10^{-5}	-
	ε_{zz}	2. 10^{-4}	1.9998 10^{-4}	– 0.01
	ε_{xy}	0.	– 1,280 10^{-8}	-

11 Modeling I

11.1 Characteristics of modeling

Elements AXIS (TRIA6 + QUAD8)



Limiting conditions:

Node F blocked in dy
pressure on AE $p=60.$

Names of the nodes: $A=N8$ $B=N174$ $C=N5$
$D=N170$ $E=N3$ $F=N159$

11.2 Characteristics of the grid

Many nodes: 175
Many meshs and types: 20 QUAD8, 40 TRIA6

11.3 Sizes tested and results

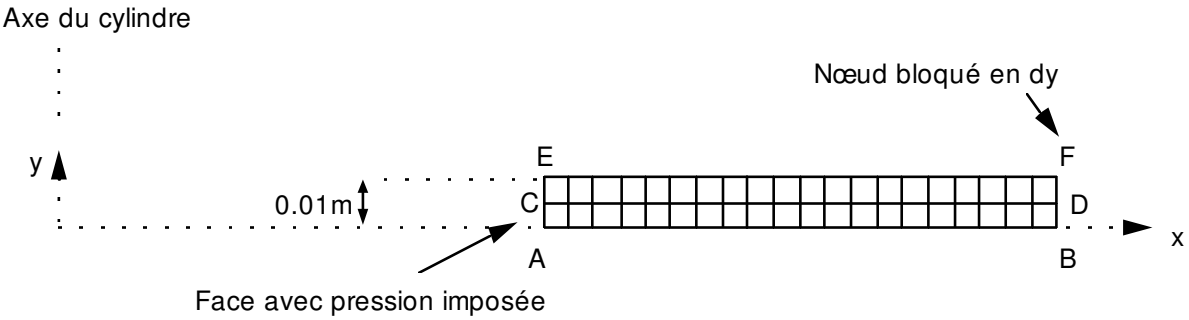
Localization	Size	Reference	*Aster*	% difference
A	u	$5.9\ 10^{-5}$	$5.9000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	– 60.	– 59.8976	– 0.17
	σ_{yy}	0.	– 0.0024	-
	σ_{zz}	100.	99.9089	– 0.09
	σ_{xy}	0.	– 0.0137	-
	ε_{xx}	$-4.5\ 10^{-4}$	$-4{,}493\ 10^{-4}$	– 0.14
	ε_{yy}	0.	$-6{,}003\ 10^{-5}$	-
	ε_{zz}	$5.9\ 10^{-4}$	$5{,}894\ 10^{-4}$	– 0.10
	ε_{xy}	0.	$-8{,}895\ 10^{-8}$	-
B	u	$4\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	0.	0.0308	-
	σ_{yy}	0.	– 0.0020	-
	σ_{zz}	40.	39.9738	– 0.07
	σ_{xy}	0.	0.0131	-
	ε_{xx}	$-0.6\ 10^{-4}$	$-0{,}598\ 10^{-4}$	– 0.33
	ε_{yy}	0.	$-6{,}002\ 10^{-5}$	-
	ε_{zz}	$2.\ 10^{-4}$	$1{,}998\ 10^{-4}$	– 0.09
	ε_{xy}	0.	$8{,}495\ 10^{-8}$	-
E	u	$5.9\ 10^{-5}$	$5.9000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	– 60.	– 59.8976	– 0.17
	σ_{yy}	0.	– 0.0024	-
	σ_{zz}	100.	99.9089	– 0.09
	σ_{xy}	0.	0.0137	-
	ε_{xx}	$-4.5\ 10^{-4}$	$-4{,}493\ 10^{-4}$	– 0.14
	ε_{yy}	0.	$-6{,}003\ 10^{-5}$	-
	ε_{zz}	$5.9\ 10^{-4}$	$5{,}894\ 10^{-4}$	– 0.10
	ε_{xy}	0.	$8{,}895\ 10^{-8}$	-
F	u	$4\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	0.	0.0308	-
	σ_{yy}	0.	– 0.0020	-
	σ_{zz}	40.	39.9738	– 0.07
	σ_{xy}	0.	– 0.0131	-
	ε_{xx}	$-0.6\ 10^{-4}$	$-0{,}598\ 10^{-4}$	– 0.33
	ε_{yy}	0.	$-6{,}002\ 10^{-5}$	-
	ε_{zz}	$2.\ 10^{-4}$	$1{,}998\ 10^{-4}$	– 0.09
	ε_{xy}	0.	$-8{,}495\ 10^{-8}$	-

There is compared to the end of this test a loading in rotation into axisymmetric pure with the same loading as a Fourier mode 0. One finds many identical results.

12 Modeling J

12.1 Characteristics of modeling

Elements AXIS (QUAD9)



Limiting conditions:

node F blocked in dy
pressure on AE $p=60.$

Names of the nodes: $A=N196$ $B=N1$ $C=N200$
$D=N5$ $E=N202$ $F=N7$

12.2 Characteristics of the grid

Many nodes: 205
Many meshs and types: 40 QUAD9

12.3 Sizes tested and results

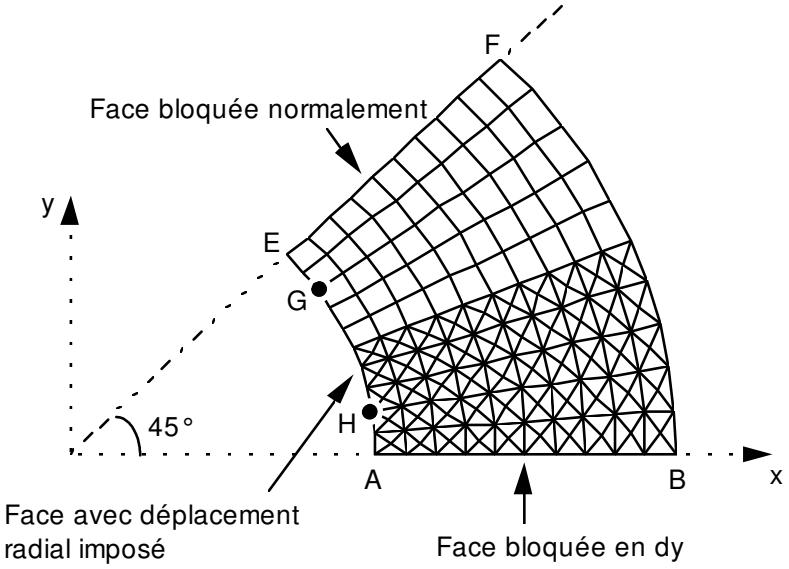
Localization	Size	Reference	*Aster*	% difference
A	u	$5.9\ 10^{-5}$	$5.9000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	– 60.	– 59.8997	– 0.17
	σ_{yy}	0.	– 0.0035	-
	σ_{zz}	100.	99.9080	– 0.09
	σ_{xy}	0.	– 0.0141	-
	ε_{xx}	$-4.5\ 10^{-4}$	$-4,494\ 10^{-4}$	– 0.14
	ε_{yy}	0.	$-6,003\ 10^{-5}$	-
	ε_{zz}	$5.9\ 10^{-4}$	$5,894\ 10^{-4}$	– 0.10
	ε_{xy}	0.	$-9,156\ 10^{-8}$	-
B	u	$4.\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	0.	0.0070	-
	σ_{yy}	0.	– 0.0001	-
	σ_{zz}	40.	39.9936	– 0.02
	σ_{xy}	0.	0.0010	-
	ε_{xx}	$-0.6\ 10^{-4}$	$-0.5996\ 10^{-4}$	– 0.07
	ε_{yy}	0.	$-6,000\ 10^{-5}$	-
	ε_{zz}	$2.\ 10^{-4}$	$1.9996\ 10^{-4}$	– 0.02
	ε_{xy}	0.	$6,748\ 10^{-9}$	-
E	u	$5.9\ 10^{-5}$	$5.9000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	– 60.	– 59.8997	– 0.17
	σ_{yy}	0.	– 0.0035	-
	σ_{zz}	100.	99.9080	– 0.09
	σ_{xy}	0.	0.0141	-
	ε_{xx}	$-4.5\ 10^{-4}$	$-4,494\ 10^{-4}$	– 0.14
	ε_{yy}	0.	$-6,003\ 10^{-5}$	-
	ε_{zz}	$5.9\ 10^{-4}$	$5,894\ 10^{-4}$	– 0.10
	ε_{xy}	0.	$-9,156\ 10^{-8}$	-
F	u	$4.\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	0.	0.0070	-
	σ_{yy}	0.	– 0.0001	-
	σ_{zz}	40.	39.9936	– 0.02
	σ_{xy}	0.	– 0.0010	-
	ε_{xx}	$-0.6\ 10^{-4}$	$-0.5996\ 10^{-4}$	– 0.07
	ε_{yy}	0.	$-6,000\ 10^{-5}$	-
	ε_{zz}	$2.\ 10^{-4}$	$1.9996\ 10^{-4}$	– 0.02
	ε_{xy}	0.	$-6,748\ 10^{-9}$	-

13 Modeling K

13.1 Characteristics of modeling

Elements 3D (PENTA6 and HEXA8)

Grid obtained by extrusion starting from the grid $2D$ below (modeling E)



Along the axis Z : 2 layers of elements
Total thickness: $0.01\mathrm{m}$

Limiting conditions:

node F : $u_z=0$
face AB blocked in dy
face EF blocked normally
face AE radial displacement imposed on $5.9E-5m$

Names of the nodes:	$A=No1$	$C=No36$	$D=No166$
plan $z=0.005$	$A2=No172$	$C2=No242$	$D2=No5025$
plan $z=0.01$	$A3=No173$	$C3=No243$	$D3=No503$

Names of the nodes:	$E=No41$	$H=No9$	$G=No38$
plan $z=0.005$	$E2=No252$	$H2=No188$	$G2=No246$
plan $z=0.01$	$E3=No253$	$H3=No189$	$G3=No247$

13.2 Characteristics of the grid

Many nodes: 513
Many meshs and types: 400 PENTA6, 100 HEXA8 40 QUAD4 (faces skin interns)

13.3 Remarks

The loading is here in imposed displacement, contrary to other modelings. The reactions are tested.

13.4 Sizes tested and results

Localization	Size	Reference	*Aster*	% difference
C	Fx	1.0884 E-3	1.0953 E-3	0.64
	Fy	4.5084 E-4	4.5836 E-4	1.67
C_2	Fx	2.1768 E-3	2.1571 E-3	– 0.91
	Fy	9.0170 E-4	9.1304 E-4	1.26
C_3	Fx	1.0884 E-3	1.0953 E-3	0.64
	Fy	4.5084 E-4	4.5836 E-4	1.67
H	Fx	1.1636 E-3	1.1709 E-3	0.63
	Fy	1.8429 E-4	1.8527 E-4	0.53
G	Fx	1.0045 E-3	1.0144 E-3	0.99
	Fy	6.1550 E-4	6.2117 E-4	0.92
H_2	Fx	2.3272 E-3	2.3173 E-3	– 0.43
	Fy	3.6858 E-4	3.6669 E-4	– 0.51
G_2	Fx	2.0090 E-3	1.9951 E-3	– 0.69
	Fy	1.2310 E-3	1.2214 E-3	– 0.78

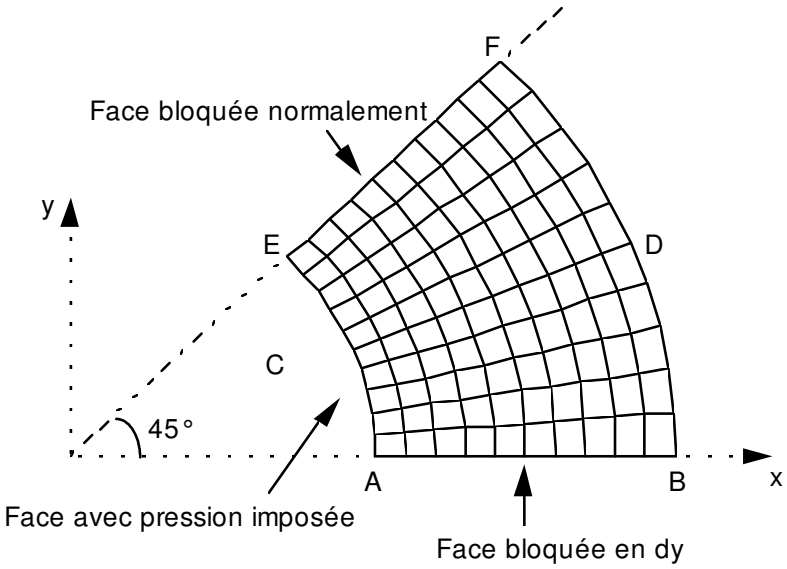
13.5 Remarks

One checks that the nodal forces of reactions are worthless in all the nodes, except on the nodes of surface AE and of surfaces EF and AB .

14 Modeling L

14.1 Characteristics of modeling

Elements 3D (PYRAM5)



Along the axis Z : each parallelepiped is cut out in 6 pyramids
Total thickness: 0.01m

Limiting conditions:

node F : $u_z=0$
face AB blocked in dy
face EF blocked normally
pressure on the face AE $p=60.$

Names of the nodes: $A=N267$ $B=N142$ $E=N29$ $F=N1$

14.2 Characteristics of the grid

Many nodes: 342
Many meshs and types: 600 PYRAM5 620 QUAD4 (faces skin interns)

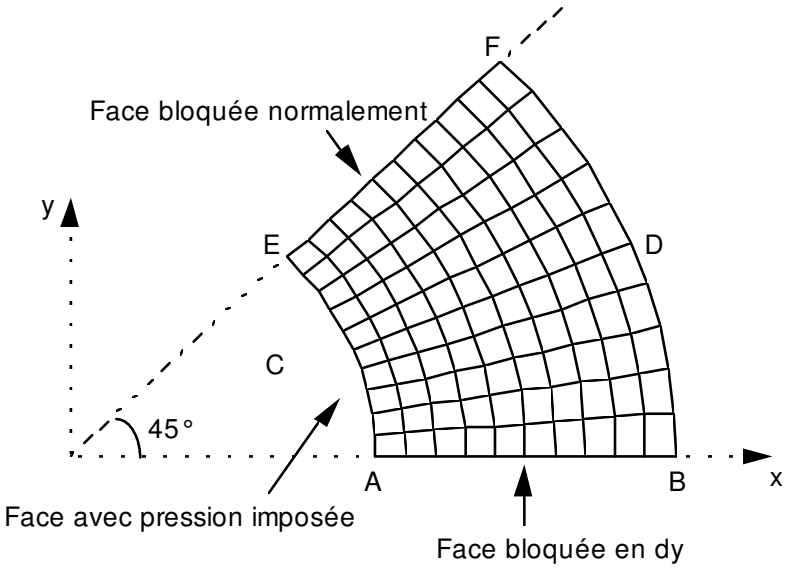
14.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
u - node (A)	5.9 10^{-5}	5.8873 10^{-5}	– 0.21
v - node (A)	0.	eps	-
σ_{xx} - node (A)	– 60.	– 52.9567	– 11.74
σ_{yy} - node (A)	100.	91.7830	– 8.22
σ_{zz} - node (A)	0.	– 1.1206	-
σ_{xy} - node (A)	0.	– 4.5121	-
ε_{xx} - node (A)	– 4.5 10^{-4}	– 4,008 10^{-4}	– 10.94
ε_{yy} - node (A)	5.9 10^{-4}	5,400 10^{-4}	– 8.47
ε_{xy} - node (A)	0.	– 2,933 10^{-5}	-
u - node (B)	4. 10^{-5}	3.9936 10^{-5}	– 0.16
v - node (B)	0.	eps	-
σ_{xx} - node (B)	0.	– 0.7670	-
σ_{yy} - node (B)	40.	39.5319	– 1.17
σ_{zz} - node (B)	0.	– 0.3115	-
σ_{xy} - node (B)	0.	– 1.5858	-
ε_{xx} - node (B)	– 0.6 10^{-4}	– 0,627 10^{-4}	4.44
ε_{yy} - node (B)	2. 10^{-4}	1,993 10^{-4}	– 0.36
ε_{xy} - node (B)	0.	– 1,031 10^{-5}	-
u - node (E)	4.17193 10^{-5}	4.16293 10^{-5}	– 0.21
v - node (E)	4.17193 10^{-5}	4.16293 10^{-5}	– 0.21
σ_{xx} - node (E)	20.	19.3586	– 3.21
σ_{yy} - node (E)	20.	31.5151	57.57
σ_{zz} - node (E)	0.	2.5686	-
σ_{xy} - node (E)	– 80.	– 77.2309	– 3.46
ε_{xx} - node (E)	0.7 10^{-4}	0,457 10^{-4}	– 34.76
ε_{yy} - node (E)	0.7 10^{-4}	1,247 10^{-4}	78.12
ε_{xy} - node (E)	– 5.2 10^{-4}	– 5,020 10^{-4}	– 3.46
u - node (F)	2.82843 10^{-5}	2.82393 10^{-5}	– 0.16
v - node (F)	2.82843 10^{-5}	2.82393 10^{-5}	– 0.16
σ_{xx} - node (F)	20.	18.9523	– 5.24
σ_{yy} - node (F)	20.	20.9510	4.75
σ_{zz} - node (F)	0.	0.0035	-
σ_{xy} - node (F)	– 20.	– 20.9897	4.95
ε_{xx} - node (F)	0.7 10^{-4}	0,633 10^{-4}	– 9.60
ε_{yy} - node (F)	0.7 10^{-4}	0,763 10^{-4}	8.96
ε_{xy} - node (F)	– 1.3 10^{-4}	– 1,364 10^{-4}	4.95

15 Modeling M

15.1 Characteristics of modeling

Elements 3D (PYRAM13)



Along the axis Z : each parallelepiped is cut out in 6 pyramids
Total thickness: 0.01m

Limiting conditions:

node F : $u_z=0$
face AB blocked in dy
face EF blocked normally
pressure on the face AE $p=60.$

Names of the nodes: $A=N1403$ $B=N734$ $E=N152$ $F=N4$

15.2 Characteristics of the grid

Many nodes: 1703
Many meshs and types: 600 PYRAM13 620 QUAD8 (faces skin interns)

15.3 Sizes tested and results

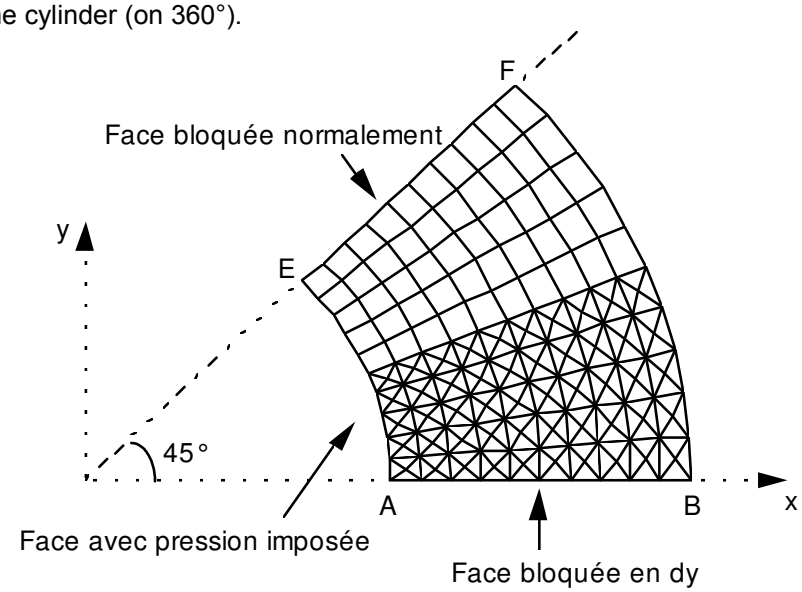
Identification	Reference	*Aster*	% difference
u - node (A)	5.9 10^{-5}	5.8999 10^{-5}	– 0,002
v - node (A)	0.	eps	-
σ_{xx} - node (A)	– 60.	– 59.9880	– 0.02
σ_{vv} - node (A)	100.	100.1277	0.13
σ_{zz} - node (A)	0.	0.0425	-
σ_{xv} - node (A)	0.	– 0.0913	-
ε_{xx} - node (A)	– 4.5 10^{-4}	– 4,502 10^{-4}	0.04
ε_{vv} - node (A)	5.9 10^{-4}	5,906 10^{-4}	0.09
ε_{xv} - node (A)	0.	– 5,934 10^{-7}	-
u - node (B)	4. 10^{-5}	4.0000 10^{-5}	0.00
v - node (B)	0.	eps	-
σ_{xx} - node (B)	0.	– 0.0276	-
σ_{vv} - node (B)	40.	40.0331	0.08
σ_{zz} - node (B)	0.	0.0024	-
σ_{xv} - node (B)	0.	0.0126	-
ε_{xx} - node (B)	– 0.6 10^{-4}	– 0,602 10^{-4}	0.32
ε_{vv} - node (B)	2. 10^{-4}	2,002 10^{-4}	0.10
ε_{xv} - node (B)	0.	8,177 10^{-8}	-
u - node (E)	4.17193 10^{-5}	4.17183 10^{-5}	– 0,002
v - node (E)	4.17193 10^{-5}	4.17183 10^{-5}	– 0,002
σ_{xx} - node (E)	20.	19.9787	-0.11
σ_{vv} - node (E)	20.	20.1612	0.81
σ_{zz} - node (E)	0.	0.0425	-
σ_{xv} - node (E)	– 80.	– 80.0580	0.07
ε_{xx} - node (E)	0.7 10^{-4}	0,696 10^{-4}	– 0.59
ε_{vv} - node (E)	0.7 10^{-4}	0,708 10^{-4}	1.11
ε_{xv} - node (E)	– 5.2 10^{-4}	– 5,204 10^{-4}	0.07
u - node (F)	2.82843 10^{-5}	2.82844 10^{-5}	0.00
v - node (F)	2.82843 10^{-5}	2.82844 10^{-5}	0.00
σ_{xx} - node (F)	20.	20.0224	0.11
σ_{vv} - node (F)	20.	19.9901	– 0.05
σ_{zz} - node (F)	0.	0.0031	-
σ_{xv} - node (F)	– 20.	– 19.9818	– 0.09
ε_{xx} - node (F)	0.7 10^{-4}	0,701 10^{-5}	0.17
ε_{vv} - node (F)	0.7 10^{-4}	0,699 10^{-4}	– 0.13
ε_{xv} - node (F)	– 1.3 10^{-5}	– 1,299 10^{-5}	– 0.09

16 Modeling NR

16.1 Characteristics of modeling

Elements 3D (PENTA15 and HEXA20)

Grid obtained by extrusion starting from a grid $2D$ resembling the grid below (8 elements in the radial direction, $4+4$ elements in the circumferential direction) and duplicated to have a complete section of the cylinder (on 360°).



Along the axis Z : 1 layer of elements
Total thickness: 0.01m

Limiting conditions:

face AB blocked in dy
face EF blocked normally

pressure on the face AE $p=60.$
basic effect on the sections $p=60.$

Names of the nodes: $A=N5349$ $B=N6092$ $C=N433$
$D=N441$ $E=N2180$ $F=N1632$

16.2 Characteristics of the grid

Many nodes: 8832
Many meshs and types: 1024 PENTA15, 512 HEXA20, 1176 QUAD8 and 2048 TRIA6.

16.3 Remarks

Contrary to preceding modelings, one takes into account here the basic effect applying to the sections at the ends of the cylinder.

16.4 Sizes tested and results

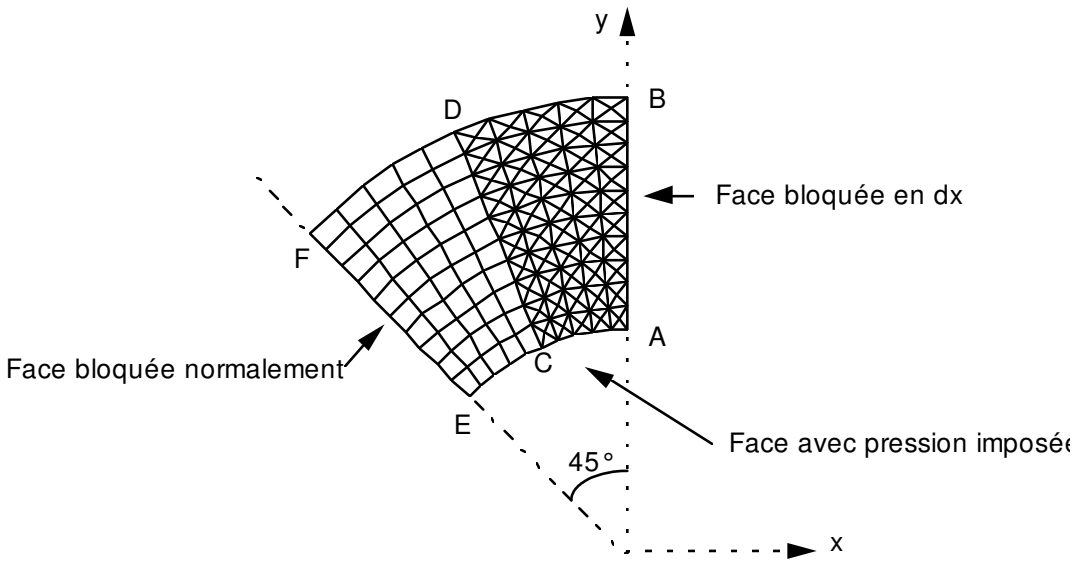
Identification	Reference	*Aster*	% difference
u - node (A)	5.6 10^{-5}	5.6000 10^{-5}	0.00
v - node (A)	0.	eps	-
σ_{xx} - node (A)	– 60.	-59.5972	– 0.67
σ_{yy} - node (A)	100.	99.5187	– 0.48
σ_{zz} - node (A)	20.	19.9584	– 0.21
σ_{xy} - node (A)	0.	eps	-
u - node (B)	3.4 10^{-5}	3.4000 10^{-5}	0.00
v - node (B)	0.	eps	-
σ_{xx} - node (B)	0.	2.0877 10^{-2}	-
σ_{yy} - node (B)	40.	39.9685	– 0.08
σ_{zz} - node (B)	20.	19.9946	– 0.03
σ_{xy} - node (B)	0.	eps	-
σ_{xx} - node (E)	20.	20.3287	1.64
σ_{yy} - node (E)	20.	20.3287	1.64
σ_{zz} - node (E)	20.	20.1739	0.87
σ_{xy} - node (E)	– 80.	-79.9775	– 0.03
σ_{xx} - node (F)	20.	20.0176	0.09
σ_{yy} - node (F)	20.	20.0176	0.09
σ_{zz} - node (F)	20.	20.0072	0.04
σ_{xy} - node (F)	– 20.	-20.0027	0.01

17 Modeling O

17.1 Characteristics of modeling

Elements C_PLAN (QUAD8 + TRIA6)

Grid $2D$ resembling the grid below (8 elements in the radial direction, $4+4$ elements in the circumferential direction) and duplicated to have a complete section of the cylinder (on 360°).



Limiting conditions:

side AB blocked in dx
side EF blocked normally
pressure on AE $p=60.$

Names of the nodes: $A=N249$ $B=N992$ $C=N1667$
$D=N1588$ $E=N3776$ $F=N3228$

17.2 Characteristics of the grid

Many nodes: 3840
Many meshs and types: 1026 TRIA6, 512 QUAD8

17.3 Sizes tested and results

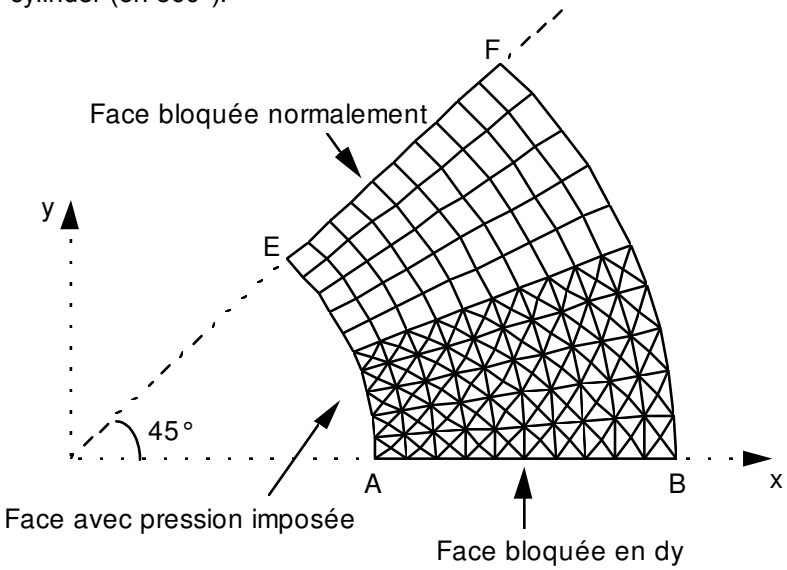
Identification	Reference	*Aster*	% difference
u - node (A)	$5.9\ 10^{-5}$	$5.8999\ 10^{-5}$	– 0,002
v - node (A)	0.	eps	-
σ_{xx} - node (A)	– 60.	-59.5340	– 0.78
σ_{yy} - node (A)	100.	99.5453	– 0.45
σ_{zz} - node (A)	0.	0.	-
σ_{xy} - node (A)	0.	eps	-
u - node (B)	$4.\ 10^{-5}$	$3.99996\ 10^{-5}$	0.00
v - node (B)	0.	0.	-
σ_{xx} - node (B)	0.	$2.6874\ 10^{-2}$	-
σ_{yy} - node (B)	40.	39.9716	– 0.07
σ_{zz} - node (B)	0.	0.	-
σ_{xy} - node (B)	0.	eps	-
u - node (E)	$4.17193\ 10^{-5}$	$4.17215\ 10^{-5}$	0,005
v - node (E)	$4.17193\ 10^{-5}$	$4.17215\ 10^{-5}$	0,005
σ_{xx} - node (E)	20.	20.2875	1.44
σ_{yy} - node (E)	20.	20.2875	1.44
σ_{zz} - node (E)	0.	0.	-
σ_{xy} - node (E)	– 80.	-79.9196	– 0.10
u - node (F)	$2.82843\ 10^{-5}$	$2.82841\ 10^{-5}$	– 0,001
v - node (F)	$2.82843\ 10^{-5}$	$2.82841\ 10^{-5}$	– 0,001
σ_{xx} - node (F)	20.	20.0167	0.08
σ_{yy} - node (F)	20.	20.0167	0.08
σ_{zz} - node (F)	0.	0.	-
σ_{xy} - node (F)	– 20.	-19.9993	– 0,004

18 Modeling P

18.1 Characteristics of modeling

Elements 3D (PENTA15 and HEXA20) – even grid that modeling NR

Grid obtained by extrusion starting from a grid $2\mathrm{D}$ resembling the grid below (8 elements in the radial direction, $4+4$ elements in the circumferential direction) and duplicated to have a complete section of the cylinder (on 360°).



Along the axis Z : 1 layer of elements
Total thickness: $0.01\mathrm{m}$

Limiting conditions:

face EF blocked normally
face AB blocked in dy
pressure on the face AE fp
basic effect on the sections fp

With fp : linear pressure function of being worth time $60.$ with $t=1\mathrm{s}$ and $120.$ with $t=2\mathrm{s}$

Names of the nodes: $A=N5349$ $B=N6092$ $C=N433$
$D=N441$ $E=N2180$ $F=N1632$

18.2 Characteristics of the grid

Many nodes: 8832
Many meshs and types: 1024 PENTA15, 512 HEXA20, 1176 QUAD8 and 2048 TRIA6.

18.3 Remarks

Contrary to modeling NR, one tests here a basic pressure and effect variables according to time. A linear variation of the pressure involves a linear variation of the constraints.

18.4 Sizes tested and results

Identification	Reference	*Aster*	% difference
U-node (A) with t=1. S	$5.6\ 10^{-5}$	$5.6000\ 10^{-5}$	0.00
v-node (A) with t=1. S	0.	eps	-
σ_{xx} - node (A) with t=1. S	– 60.	-59.5414	– 0.76
σ_{yy} - node (A) with t=1. S	100.	99.6183	– 0.38
σ_{zz} - node (A) with t=1. S	20.	19.9976	– 0.01
σ_{xy} - node (A) with t=1. S	0.	eps	-
σ_{xx} - node (A) with t=2. S	– 120.	-119.0829	– 0.76
σ_{yy} - node (A) with t=2. S	200.	199.2366	– 0.38
σ_{zz} - node (A) with t=2. S	40.	39.9952	– 0.01
σ_{xy} - node (A) with t=2. S	0.	eps	-
U-node (B) with t=1. S	$3.4\ 10^{-5}$	$3.4000\ 10^{-5}$	0.00
v-node (B) with t=1. S	0.	eps	-
σ_{xx} - node (B) with t=1. S	0.	$2.6761\ 10^{-2}$	-
σ_{yy} - node (B) with t=1. S	40.	39.9740	– 0.06
σ_{zz} - node (B) with t=1. S	20.	19.9973	– 0.01
σ_{xy} - node (B) with t=1. S	0.	eps	-
σ_{xx} - node (B) with t=2. S	0.	$5.3523\ 10^{-2}$	-
σ_{yy} - node (B) with t=2. S	80.	79.9480	– 0.06
σ_{zz} - node (B) with t=2. S	40.	39.9946	– 0.01
σ_{xy} - node (B) with t=2. S	0.	eps	-
σ_{xx} - node (E) with t=1. S	20.	20.3287	1.64
σ_{yy} - node (E) with t=1. S	20.	20.3287	1.64
σ_{zz} - node (E) with t=1. S	20.	20.1739	0.87
σ_{xy} - node (E) with t=1. S	– 80.	-79.9775	– 0.03
σ_{xx} - node (E) with t=2. S	40.	40.6575	1.64
σ_{yy} - node (E) with t=2. S	40.	40.6575	1.64
σ_{zz} - node (E) with t=2. S	40.	40.3479	0.87
σ_{xy} - node (E) with t=2. S	– 160.	-159.9550	– 0.03
σ_{xx} - node (F) with t=1. S	20.	20.0176	0.09
σ_{yy} - node (F) with t=1. S	20.	20.0176	0.09
σ_{zz} - node (F) with t=1. S	20.	20.0072	0.04
σ_{xy} - node (F) with t=1. S	– 20.	-20.0027	0.01
σ_{xx} - node (F) with t=2. S	40.	40.0351	0.09
σ_{yy} - node (F) with t=2. S	40.	40.0351	0.09
σ_{zz} - node (F) with t=2. S	40.	40.0144	0.04
σ_{xy} - node (F) with t=2. S	– 40.	-40.0054	0.01

19 Summary of the results

Summary of the errors max in %

3D	Localization	MOD A	MOD B	MOD C	MOD D	MOD L	MOD M	MOD NR	MOD P
elem		pe6, h8	pe15, h20	te4	te10	py5	py13	pe15, h20	pe15, h20
geom		45°	45°	45°	45°	45°	45°	360°	360°
Nb No		1922	2115	1115	1395	342	1703	8832	8832
Dépl.	WITH, E	0.08	0.09	0.17	0.04	0.21	0.00	0.00	0.00
	B, F	0.10	0.07	0.30	0.04	0.16	0.00	0.00	0.00
σ_{xx}	WITH, E	4.59	0.39	10.45	4.41	11.74	0.11	1.64	1.64
	B, F	5.24	0.07	7.78	0.95	5.24	0.11	0.09	0.09
σ_{vv}	WITH, E	5.70	0.92	9.46	1.80	57.57	0.81	1.64	1.64
	B, F	1.89	0.01	2.46	0.49	4.75	0.08	0.09	0.09
σ_{zz}	WITH, E	Good	Good	Good	Good	Good	Good	0.87	0.87
	B, F	Good	Good	Good	Good	Good	Good	0.04	0.04
σ_{xv}	WITH, E	0.15	0.26	2.89	0.27	3.46	0.07	0.03	0.03
	B, F	0.90	0.06	0.81	0.28	4.95	0.09	0.01	0.01

C_PLAN	Localization	MOD E	MOD F	MOD G	MOD O
Type of elements		tria3, quad4	tria6, quad8	quad9	tria6, quad8
Modelled geometry		45°	45°	45°	360°
Many nodes		961	591	441	384
Displacements	WITH, E	0.07	0.09	0.00	0.01
	B, F	0.09	0.07	0.00	0.00
Constraints σ_{xx}	WITH, E	4.65	0.39	0.27	1.44
	B, F	5.19	0.06	0.02	0.08
Constraints σ_{yy}	WITH, E	5.68	0.90	0.16	1.44
	B, F	1.40	0.04	0.05	0.08
Constraints σ_{zz}	WITH, E	Good	Good	Good	Good
	B, F	Good	Good	Good	Good
Constraints σ_{xy}	WITH, E	0.16	0.23	0.20	0.10
	B, F	1.23	0.07	0.09	0.00

AXIS	Localization	MOD H	MOD I	MOD J
Type of elements		tria3, quad4	tria6, quad8	quad9
Many nodes		113	175	205
Displacements	WITH, E	0.01	0.00	0.00
	B, F	0.01	0.00	0.00
Constraints σ_{xx}	WITH, E	5.66	0.17	0.17
	B, F	Good	Good	Good
Constraints σ_{yy}	WITH, E	Good	Good	Good
	B, F	Good	Good	Good
Constraints σ_{zz}	WITH, E	1.29	0.09	0.09
	B, F	1.00	0.07	0.02
Constraints σ_{xy}	WITH, E	Good	Good	Good
	B, F	Good	Good	Good

- The results are more precise with elements of order 2.
- The problem is adapted more to an axisymmetric modeling. The results are better.
- The grids remain insufficient for the elements 3D of order 1: constraints and deformations of modelings A, C, E and L (especially for modeling L in PYRAM5).
- The pyramids give results similar to the other elements 3D, to grid are equivalent.

- Modelings NR and P with basic effect and pressure constant or variables give of good results.