SSLS143 - Pin addition to cantilever with offset heart

Summary:

The objective of this test is to validate the calculation of the options `EPSI_ELGA` and `DEGE_ELNO` for the multifibre beams of Euler-Bernoulli `POU_D_EM` , including if the reference axis is not confused with the locus of elastic centres.

The case test also validates the calculation of the elastic matrix (`RIGI_MECA`) when the axis of reference is not confused with the locus of elastic centres (offsetting).

1 Problem of reference

1.1 Geometry

A beam comforts length $L=1\,m$ (see Figure 1).

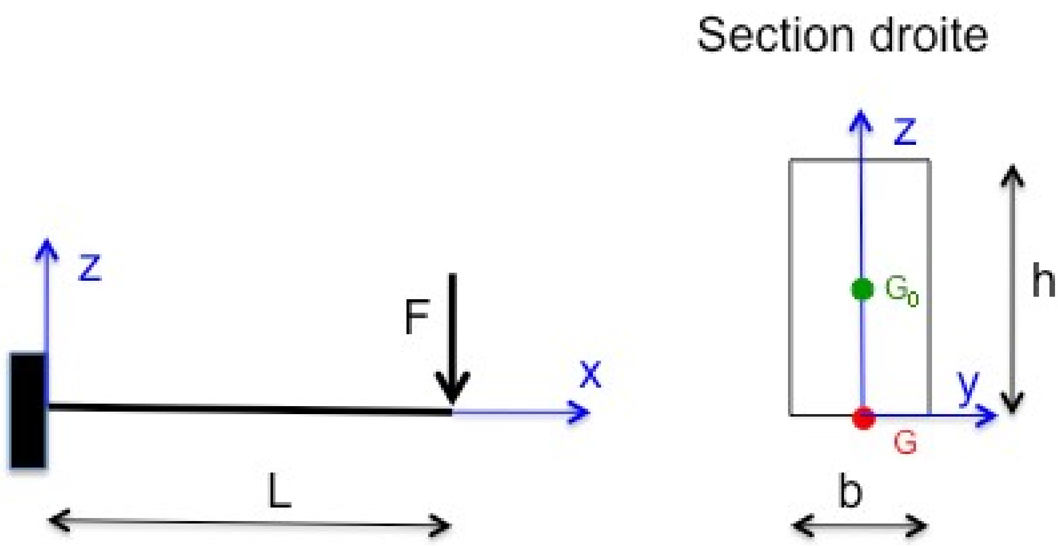

Figure 1: Beam comforts

1.2 Properties of material

The cross-section is rectangular $b\times h=0.4\times 1\,m^2$, and homogeneous with a material of modulus Young $E=3\times 10^{10}\,Pa$.

1.3 Boundary conditions and loadings

The beam is embedded at an end and is charged by a force $F=10^6\,N$ at its other end (see Figure 1)

2 Reference solution

2.1 Analytical expressions

On the basis of the embedded end, the expression of the bending moment is: $M(x)=F(x-L)$

The arrow at the end charged with the beam is $f=\frac{FL^3}{EI_{G_0}}$ where I_{G_0} is the quadratic moment calculated with the barycentre of the section G_0 : $I_{G_0}=\int_S (y-y_{G_0})^2 dS$.

Curve in a point located at a distance x embedding is $\chi_s(x)=-\frac{M(x)}{EI_{G_0}}$. Because of the offsetting of the reference axis, the lengthening of the beam (on the level of this axis) is worth:

$\epsilon_s(x)=-\frac{A_G}{S}\chi_s(x)$ where S is the surface of the section and A_G static moment of the section compared to an axis passing by G : $A_G=\int_S z dS$.

Deformation of a point of coordinates (x, y, z) is: $\epsilon=\epsilon_s(x)-\chi_s(x)z$, and the constraint at the same point is worth: $\sigma=E\epsilon$

2.2 Calculation of the characteristics of the cross-section

In order to eliminate uncertainty from the approximate digital calculation of the characteristics geometrical of cross-section (low number of fibres), the values used in the reference solution are calculated as in digital calculation:

$$S=\sum_{fibres} S_i \qquad A_G=\sum_{fibres} z_i S_i \qquad I_G=\sum_{fibres} z_i^2 S_i \qquad I_{G_0}=\sum_{fibres} (z_i-z_{G_0})^2 S_i$$

where z_i is the ordinate of the center of fibre i and S_i the surface of this fibre.

3 Modeling A

3.1 Characteristics of modeling

A modeling is used POU_D_EM.

the beam is modelled with 1 finite element and the section is discretized with 8 fibres. The reference axis east chooses voluntarily different from the barycentre, excentré of h/2 to the bottom, with the keyword COOR_AXE_POUTRE of DEFI_GEOM_FIBRE.

The section "is cut out" in 8 fibres (Figure 2). The coordinates of the centers of fibres and their surfaces are given in table 1.

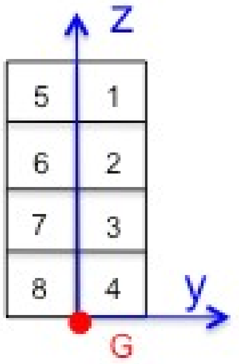

Figure 2: Section division in 8 fibres

Fibres	y_i	z_i	S_i	Fibres	y_i	z_i	S_i
1	0.1	0,875	0.05	5	-0.1	0,875	0.05
2	0.1	0,625	0.05	6	-0.1	0,625	0.05
3	0.1	0,375	0.05	7	-0.1	0,375	0.05
4	0.1	0,125	0.05	8	-0.1	0,125	0.05

Table 1: Characteristics of fibres

3.2 Characteristics of the grid

The grid contains 1 elements of the type SEG2.

3.3 Sizes tested and results

The digital calculation of the sizes of the cross-section gives:
$S=0.4\,m^2$ $A_G=0.2\,m^3$ $I_G=0.13125\,m^4$ and $I_{G_0}=0.03125\,m^4$

The arrow at the end charged with the beam is: $f=0.00035555\,m$

On the level of embedding $x=0$ one has the following values:
$M=-10^{-6}\,Nm$ $\chi_s=0.00106666\,m^{-1}$ $\epsilon_s=-0.00053333\,m^{-1}$

On the level of the first point of Gauss $x=(1-\frac{1}{\sqrt{3}})/2=0.21132486540518503\,m$ one has the following values: $M=-788675.13\,Nm$ $\chi_s=0.00084125\,m^{-1}$ $\epsilon_s=-0.0004206\,m^{-1}$

for the fibre n°1 ($z=0.875\,m$): $\epsilon=0.00031547$ and $\sigma=9464101.6\,Pa$ and for the fibre n°4 with $z=0.125\,m$ $\epsilon=-0.00031547$ and $\sigma=-9464101.6\,Pa$

Arrow at the end of the beam (DEPL):

Not	Component	**Value of reference**	**Tolerance**
SUPPORT	DZ	−3.5555555555555E-4	1.E-6

Deformations generalized with embedding (DEGE_ELNO):

Mesh	Node	Component	**Value of reference**	**Tolerance**
M1	N1	KY	1.066666666666667E-3	1.E-6
M1	N1	EPXX	−5.333333333333333E-3	1.E-6

Strains and stresses in fibres (EPSI_ELGA and SIEF_ELGA):

Mesh	Not	Under-point	Component	**Value of reference**	**Tolerance**
M1	1	1	EPXX	3.15470053837926E-4	1.E-6
M1	1	1	SIXX	9.46410161513778E6	1.E-6
M1	1	4	EPXX	−3.15470053837926E-4	1.E-6
M1	1	4	SIXX	−9.46410161513778E6	1.E-6

4 Summary of the results

Subject coarsely using the approximate characteristics of the cross-section with a grid for the calculation of the analytical values of reference, the arrow, the constraints and the deformations numerically calculated are identical to these values of reference.