SSLS137 - Prestressed concrete plate with a excentré cable in inflection

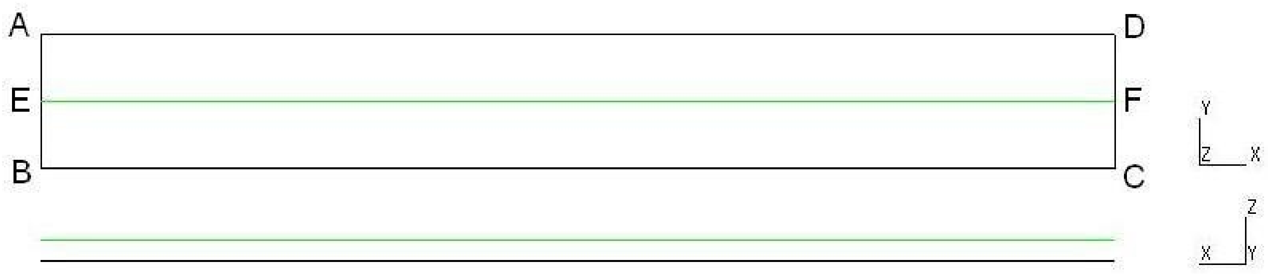
Summary:

The purpose of this test is to validate the macro-order CALC_PRECONT for the elements hulls. The reference solution is established starting from the theory of the beams. The first modeling is made with elements 3D and aim at validating the use of the theory of the beams like reference. Two modelings of hulls are then proposed (DKT and Q4GG) like two types of different meshs (TRIA3, QUAD4) for each modeling. Lastly, modeling F takes again modeling D by replacing the elastic law of behavior of the concrete by the nonlinear law of behavior GLRC_DAMAGE.

1 Description

1.1 Geometry

The model of study is a rectangular plate length $L=4\mathrm{m}$, of width $l=0.5\mathrm{m}$ and thickness $t=0.2\mathrm{m}$. A reinforcement of inflection made up of only one braces, crosses the plate to its half and is excentré of a distance $e_z=0.075\mathrm{m}$ median plane of the plate along the axis z . From a geometrical point of view, one does not know that the surface of the cross-section of the bar $A=0.00015\mathrm{m}^2$. The figure below illustrates the position of the cable for the model of study. The median plane of the plate is defined by the rectangle $ABCD$ and cables it by the segment EF .



1.2 Properties of materials

The properties of the concrete for the plate and steel for the cable are indexed in the table according to.

Material	Concrete	Steel
Young modulus	$4\times10^{10}\,Pa$	$1.93\times10^{11}\,Pa$
Poisson's ratio	0.0 *	0.0
Density	$2500\,kg/m^3$	$7850\,kg/m^3$
Elastic ultimate stress	n/a	$1.94\times10^{11}\,Pa$

*Il is justified thereafter that the model of study can be comparable to a beam and that one will take a Poisson's ratio for the concrete equal to 0.0 .

1.3 Boundary conditions and loadings

The side defined by segment AB of the plate is embedded, all the degrees of freedom on this side are blocked and the other edges are left free to allow him a configuration of type beam. The order of application of the loadings is defined in the following way.

1.3.1 Loading 1:

It corresponds to the setting in tension of the cable. One imposes a tension in the cable equalizes with $3.75\times10^5\,N$, and only the node F cable located at the free edge of the plate is active. The losses of tension by relieving of steel and retreat of anchoring are neglected.

1.3.2 Loading 2:

The second loading applies a pressure $P_0=10^5\,Pa$, carried by the axis $-z$, on all the plate.

1.4 Principal stages of the tests

The macro-order is used DEFI_CABLE_BP to obtain the relations kinematics between the plate and the cable as well as the loading related to the tension in the cable. Only the node F cable is considered active.

One launches then the macro-order CALC_PRECONT to carry out the setting into prestressed plate starting from the tension of the cable given.

The order STAT_NON_LINE is also used to apply the loading of pressure to the plate. One extracts then the maximum value from displacement on the free edge of the plate to the node D , this in order to compare them with the values of reference and thus to validate the setting in tension of the cables of prestressed for the elements hulls.

2 Reference solution

For each modeling, the objective is to find the good initial tension of prestressed cable after the application of CALC_PRECONT but also to determine the maximum displacement of the plate to the node D along the axis z after application of the pressure.

2.1 Results of reference

To determine maximum displacement, one uses the theory of the beams working in inflection. For a beam fixed at an end and free of the other under loading distributed, maximum displacement at the loose lead called marks with arrows, is given by:

$$f=-\frac{qL^4}{8\mathrm{EI}}$$

q : the force divided into N/m .

L : the length of the beam in m .

E : the Young modulus of the plate, therefore concrete in Pa .

$I=\frac{t^3 l}{12}$: quadratic moment of the beam compared to the axis there of m^4 .

<u>For the compressive force:</u>

Thanks to its offsetting, the cable takes part in the rigidity of the model:

$$(EI)_{eq}=E_b\frac{t^3 l}{12}+E_a a_x l\times e_z^2$$

$a_x=\frac{A}{dx}$: is the rate of reinforcement.

$$(EI)_{eq}=13.50\,MN.m^2$$

The arrow under the loading of pressure is calculated as follows:

$$f_p=\frac{-P_0 l\times L^4}{8(EI)_{eq}}$$

Then:

$$f_p=-0.118552\mathrm{m}$$

<u>For the setting in tension of the cable of prestressed:</u>

The cable put in tension applies a compressive force then $-F_0$ and a bending moment $-e_z F_0$ on the loose lead of the plate.

With the principle of superposition, the expression of the normal constraint is:

$$\sigma_x=\frac{-F_0}{tl}\left(1+\frac{12 e_z z}{t^2}\right)$$

If one neglects the effects of the Poisson's ratio the field of displacement is given by:

$$\begin{cases} u(x,y,z)=\dfrac{-F_0}{E_b tl}\left(1+\dfrac{12 e_z z}{t^2}\right)x \\ v(x,y,z)=0 \\ w(x,y,z)=\dfrac{F_0}{E_b tl}\left(\dfrac{6 e_z}{t^2}x^2\right) \end{cases}$$

Displacements are given at the loose lead of the beam to the node D, that is to say in $(4, 0.5, 0)$:

$$\begin{cases} u(x,y,z)=-0.375\text{mm} \\ v(x,y,z)=0 \\ w(x,y,z)=16.875\text{mm} \end{cases}$$

By superposition, the theoretical maximum displacement of the plate prestressed under the compressive force is of:

$$f_{tot}=f_p+w$$

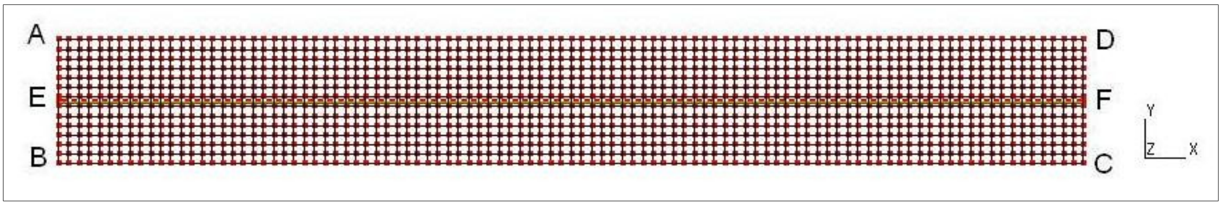
$$f_{tot}=-0.101677\text{m}$$

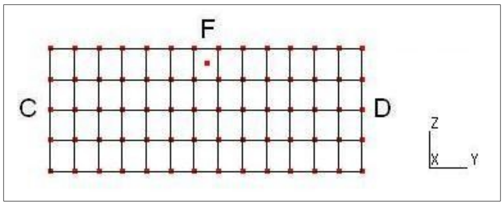
2.2 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling





- Modeling: `3D`
- Type of finite elements: Hexahedron (HEXA8), dimension of $0.05\mathrm{m}$ for the height and of $0.04\mathrm{m}$ for the length and of $0.04\mathrm{m}$ to the maximum for the width.

The plate is cut out in 13 elements over its width, 100 over its length and 4 on its thickness.
The steel reinforcement is modelled by elements `BAR`. The cable is discretized in 101 elements.

3.2 Sizes tested and results

One tests the value of displacement to the node D along the axis z .

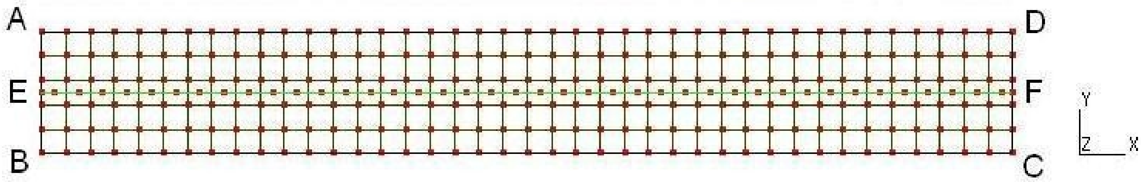
Node	Component	Value of reference (m)	Precision
D	`DZ`	0,000000 -0.101677	$2.0\mathrm{E}-2$

One tests also the tension in several elements of the cable.

Mesh	Value of reference (N)	Precision
$M564$	3.75×10^5	$1.0\mathrm{E}-8$
$M562$	3.75×10^5	$1.0\mathrm{E}-8$
$M547$	3.75×10^5	$1.0\mathrm{E}-8$

4 Modeling B

4.1 Characteristics of modeling



- Modeling: DKT
- Type of finite elements: Quadrangle (QUAD4), dimension of $0.1\mathrm{m}$.

The plate is then cut out in 5 elements over its width and 40 over its length.
The steel reinforcement is modelled by elements BAR . The cable is discretized in 41 elements since each node of the cable is in the middle of an element quadrangle.

4.2 Sizes tested and results

One tests the value of displacement to the node D following the axis z.

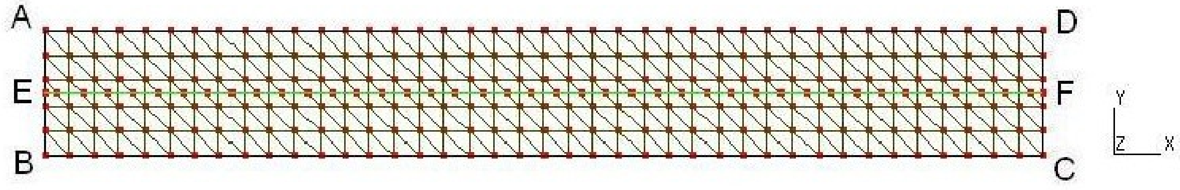
Node	Component	Value of reference (m)	Precision
D	DZ	-0.101677 0,000000	$1.0\mathrm{E}-2$

One tests also the tension in several elements of the cable.

Mesh	Value of reference (N)	Precision
$M119$	3.75×10^{5}	$1.0\mathrm{E}-8$
$M122$	3.75×10^{5}	$1.0\mathrm{E}-8$
$M130$	3.75×10^{5}	$1.0\mathrm{E}-8$

5 Modeling C

5.1 Characteristics of modeling



- Modeling: DKT
- Type of finite elements: Triangle (TRIA3), dimension of $0.1\mathrm{m}$.

Two elements triangles are created starting from an element quadrangle of the model of preceding modeling.
The plate is then cut out in 5 elements over its width and 40 over its length.
The steel reinforcement is modelled by elements BAR. The cable is discretized in 41 elements.

5.2 Sizes tested and results

One tests the value of displacement to the node D following the axis z .

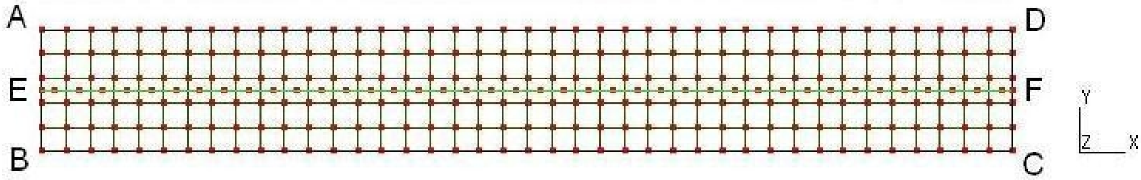
Node	Component	Value of reference (m)	Precision
D	DZ	-0.101677 0,000000	$1.0\mathrm{E}-2$

One tests also the tension in several elements of the cable.

Mesh	Value of reference (N)	Precision
$M110$	3.75×10^{5}	$1.0\mathrm{E}-8$
$M116$	3.75×10^{5}	$1.0\mathrm{E}-8$
$M127$	3.75×10^{5}	$1.0\mathrm{E}-8$

6 Modeling D

6.1 Characteristics of modeling



- Modeling: Q4GG
- Type of finite elements: Quadrangle (QUAD4), dimension of 0.1m .

The plate is then cut out in 5 elements over its width and 40 over its length.
The steel reinforcement is modelled by elements BAR. The cable is discretized in 41 elements.

6.2 Sizes tested and results

One tests the value of displacement to the node D along the axis z .

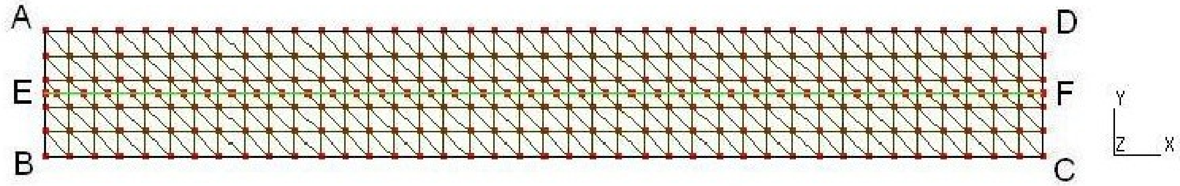
Node	Component	Value of reference (m)	Precision
D	DZ	0,000000 -0.101677	$1.0\text{E}-2$

One tests also the tension in several elements of the cable.

Mesh	Value of reference (N)	Precision
$M119$	3.75×10^5	$1.0\text{E}-8$
$M122$	3.75×10^5	$1.0\text{E}-8$
$M130$	3.75×10^5	$1.0\text{E}-8$

7 Modeling E

7.1 Characteristics of modeling



- Modeling: Q4GG
- Type of finite elements: Triangle (TRIA3), dimension of 0.1m .

Two elements triangles are created starting from an element quadrangle of the model of preceding modeling.
The plate is then cut out in 5 elements over its width and 40 over its length.
The steel reinforcement is modelled by elements BAR. The cable is discretized in 41 elements.

7.2 Sizes tested and results

One tests the value of displacement to the node D along the axis z .

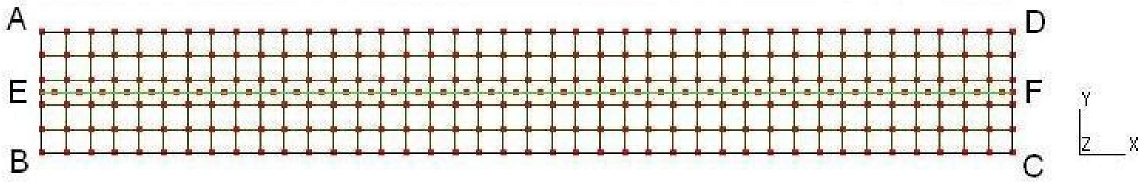
Node	Component	Value of reference (m)	Precision
D	DZ	0,000000 -0.101677	$1.0\text{E}-2$

One tests also the tension in several elements of the cable.

Mesh	Value of reference (N)	Precision
$M110$	3.75×10^5	$1.0\text{E}-8$
$M116$	3.75×10^5	$1.0\text{E}-8$
$M127$	3.75×10^5	$1.0\text{E}-8$

8 Modeling F

8.1 Characteristics of modeling



- Modeling: Q4GG
- Type of finite elements: Quadrangle (QUAD4), dimension of $0.1\mathrm{m}$.

The plate is then cut out in 5 elements over its width and 40 over its length. It is modelled by the nonlinear law GLRC_DAMAGE.
The steel wire rope is modelled by elements BAR. The cable is discretized in 41 elements.

8.2 Sizes tested and results

One tests the value of displacement to the node D along the axis z.

Node	Component	Value of reference (m)	Precision
D	DZ	0,000000 -0.101677	$1.0\mathrm{E}-2$

One tests also the tension in several elements of the cable.

Mesh	Value of reference (N)	Precision
$M119$	3.75×10^5	$1.0\mathrm{E}-8$
$M122$	3.75×10^5	$1.0\mathrm{E}-8$
$M130$	3.75×10^5	$1.0\mathrm{E}-8$

9 Synthesis

The results got with modeling A correspond well to the analytical solution suggested starting from the theory of the beams. That thus validates the choice of the model of beam like reference.

In modelings B with E (DKT and Q4GG), values of the constraints in the cable in exit of CALC_PRECONT are well those expected. In addition, the values of arrow obtained are almost identical to the analytical solution. It is noticed that the percentage of error is less important with modeling DKT for this case of study, but especially that the error on the arrow is quite less than with modeling 3D .

Modeling F makes it possible to validate that the calculation of prestressing is correct when the concrete is modelled by the law GLRC_DAMAGE .

This validates the use of the macro-order CALC_PRECONT for the hulls.