SSLS125 - Buckling of a free cylinder under external pressure

Summary:

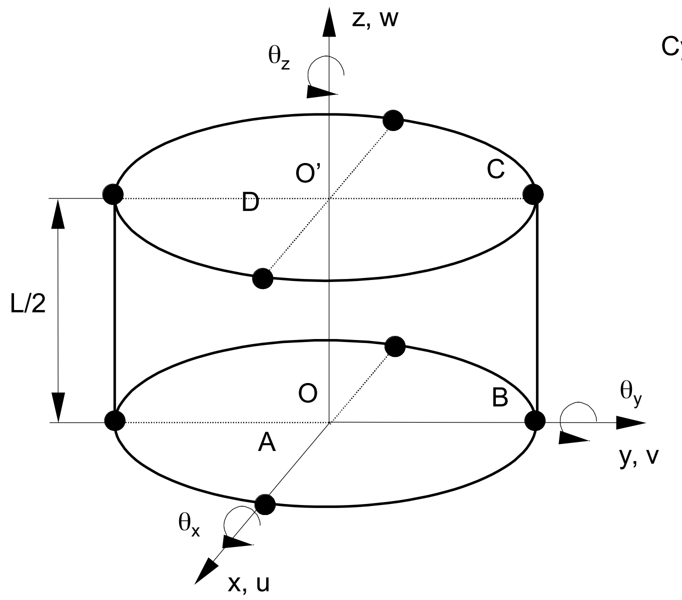
This test represents a calculation of stability of a free thin cylindrical envelope at its ends subjected to an external pressure. One calculates the critical loads leading to the elastic buckling of Euler. The geometrical matrix of rigidity used in the resolution of the problem to the eigenvalues is that which is due to the initial constraints.

It makes it possible to validate modeling finite elements SHB (linear elements SHB8 and SHB6 and quadratic elements SHB20 and SHB15)

The critical load and the clean mode obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



Cylindre libre aux extrémités

The symmetry of the problem makes it possible to model a quarter of cylinder length L , with conditions of symmetry specific to the lower edge.

$L=2\text{m}$
Average radius $R=2\text{m}$
Thickness $e=0.02\text{m}$

1.2 Properties of material

The properties of material constituting the plate are:

$E=2.10^{11}\,Pa$ Young modulus
$\nu=0.3$ Poisson's ratio

1.3 Boundary conditions and loadings

- Loading:
  - pressure uniformly distributed of $p_{cr}=1.Pa$ on the cylindrical part.
  - Conditions of symmetry:
    - on AB : DZ = 0
    - on BC : DX=0
    - on DA : DY=0

1.4 Initial conditions

Without object

2 Reference solution

2.1 Method of calculating used for the reference solution

The critical pressure is given in [bib1] or [bib2] by the following expression:

$$P_{CR}=\frac{E}{12.(1-\nu^2)}.n^2.(\frac{e}{R})^3$$

with n number of the mode (here $n=2,4,6$)

2.2 Results of reference

Critical pressures (in Pa) are:

Mode (N)	Reference
2	73260
4	293040
6	659340

2.3 Uncertainties on the solution

Analytical solution

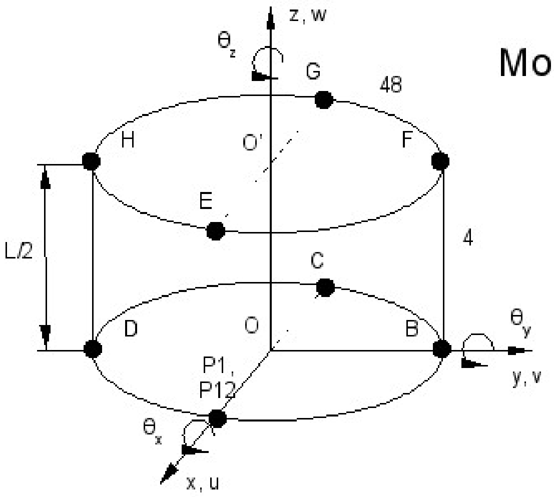
2.4 Bibliographical references

1) S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, page 500, second edition, DUNOD 1966.

2) BO O. ALMROTH, D.O. BRUSH: Buckling of bars, punts and shells, page 173, Mc Graw-Hill, New York, 1975.

3 Modeling A

3.1 Characteristics of modeling



3.2 Characteristics of the grid

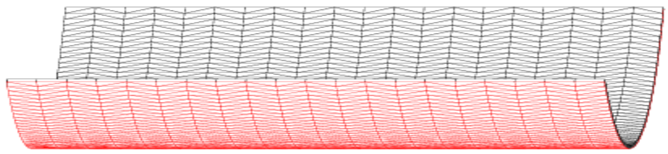
Many nodes: 882
Many meshs and types: 400 HEXA8

3.3 Sizes tested and results

Identification	Mode (N)	Reference	*Aster*	% difference
Critical pressure (Pa)	2	73260	72492	1.05
	4	293040	293481	0.15
	6	659340	673600	2.2

4 Modeling B

4.1 Characteristics of modeling



4.2 Characteristics of the grid

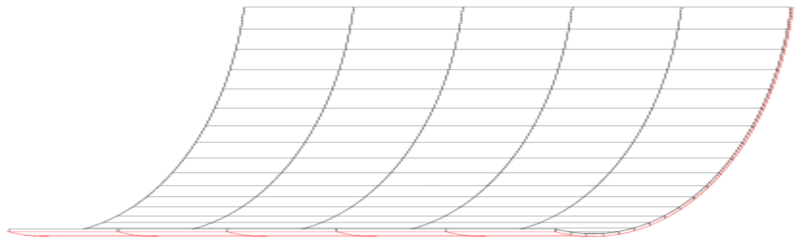
Many nodes: 2982 (20 elements in the height, 70 on the circumference)
Many meshs and types: 2800 `PENTA6`

4.3 Sizes tested and results

Identification	Mode (N)	Reference	*Aster*	% difference
Critical pressure (Pa)	2	73260	75544	3.1
	4	293040	302291	3.1
	6	659340	680524	3.2

5 Modeling C

5.1 Characteristics of modeling



5.2 Characteristics of the grid

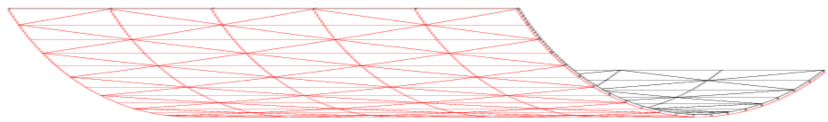
Many nodes: 828 (5 elements in the height, 20 on the circumference)
Many meshs and types: 100 HEXA20

5.3 Sizes tested and results

Identification	Mode (N)	Reference	*Aster*	% difference
Critical pressure (Pa)	2	73260	71996	-1.7
	4	293040	287912	-1.7
	6	659340	647176	-1.8

6 Modeling D

6.1 Characteristics of modeling



6.2 Characteristics of the grid

Many nodes: 1028 (5 elements in the height, 20 on the circumference)
Many meshs and types: 200 `PENTA15`

6.3 Sizes tested and results

Identification	Mode (N)	Reference	*Aster*	% difference
Critical pressure (Pa)	2	73260	73235	-0.03
	4	293040	293111	0.02
	6	659340	659800	0.07

7 Summary of the results

The got results are satisfactory. Uncertainties on the critical pressure do not exceed 3% . It should be noted that to obtain this precision, the element SHB6 need for a grid finer has than the other elements (3000 nodes instead of 1000).

The modal deformation obtained corresponds well to the expected circumferential mode: $n=2$ for two modelings.

This test made it possible to test modeling SHB in linear buckling of Euler of a mean structure subjected to an external pressure.