SSLS123 - Sphere under uniform external pressure

Summary:

One treats the case of the sphere under uniform pressure external in linear elasticity, which makes it possible to evaluate the quality of the modeling of the compressive forces.

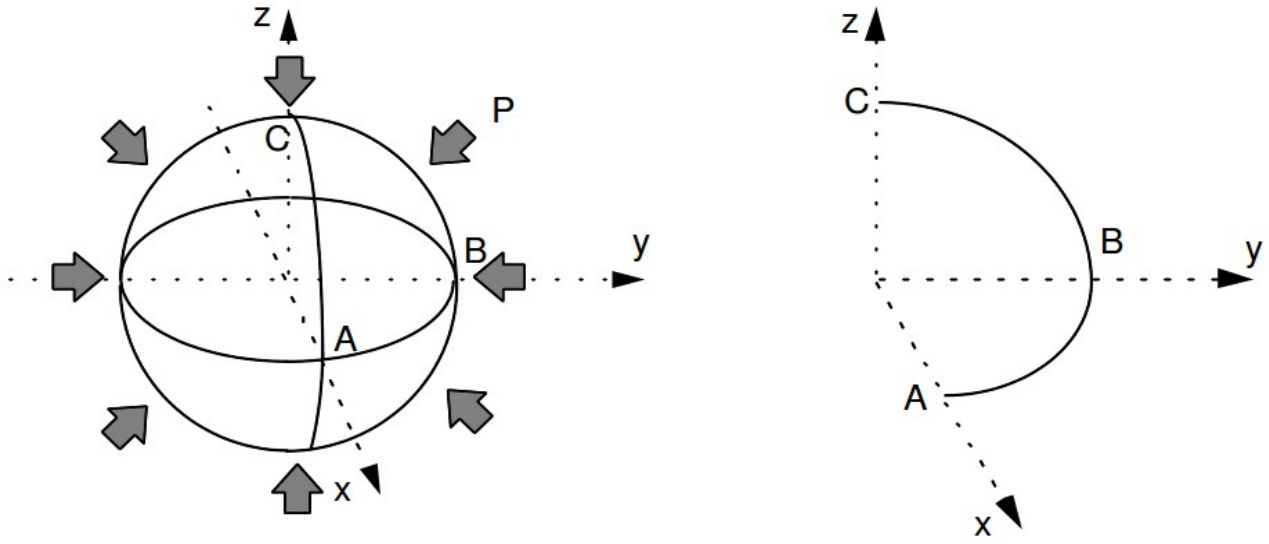
The values tested are radial displacements at the points of intersection with the axes.

One has 2 modelings:

- A: elements 3D in HEXA8
- B: elements SHB8

1 Problem of reference

1.1 Geometry



Ray	$R=10\text{m}$
Thickness	$t=0.04\text{m}$

Coordinates of the points:

	A	B	C
x	10.	0.	0.
y	0.	10.	0.
z	0.	0.	10.

1.2 Material properties

$E=6.825\,10^{7}\ Pa$
$\nu=0.3$

1.3 Boundary conditions and loadings

On a quarter of the hemisphere:

Side AC symmetry compared to the plan xz
Side BC symmetry compared to the plan yz
Side AB symmetry compared to the plan xy

Uniform external pressure $P=1.\text{Pa}$

2 Reference solution

2.1 Method of calculating used for the reference solution

Radial displacement in any node of the sphere under external pressure is given by:

$$U_r = B \cdot r + \frac{C}{r^2}$$

With

$$B = \frac{1-2\nu}{E} \cdot \frac{r_e^3}{r_i^3 - r_e^3} \cdot P \quad \text{and} \quad C = \frac{1+\nu}{2E} \cdot \frac{r_i^3 r_e^3}{r_i^3 - r_e^3} \cdot P$$

where $r_i = R - \frac{t}{2}$ and $r_e = R + \frac{t}{2}$

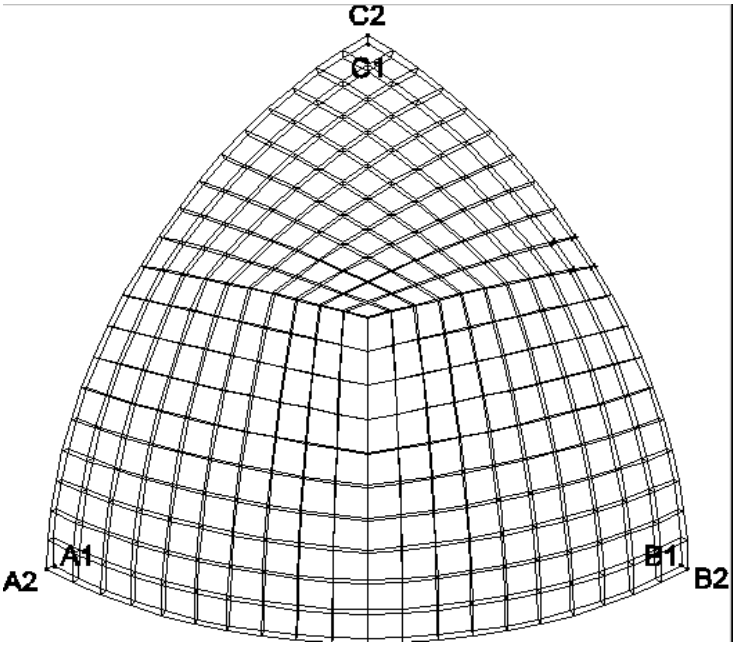
2.2 Results of reference

Displacement of the point A according to x, displacement of the point B according to y, displacement of the point C according to z.

3 Modeling A

3.1 Characteristics of modeling

Element of hull SHB



Modeling of a quarter of the sphere in SHB

Names of the nodes:

Not *A1*	*N40*	Not *A2*	*N42*
Not *B1*	*N01*	Not *B2*	*N02*
Not *C1*	*N662*	Not *C2*	*N658*

3.2 Characteristics of the grid

Many nodes: 662
Many meshs and types: 300 SHB for the sphere and 300 QUAD4 for external surface.

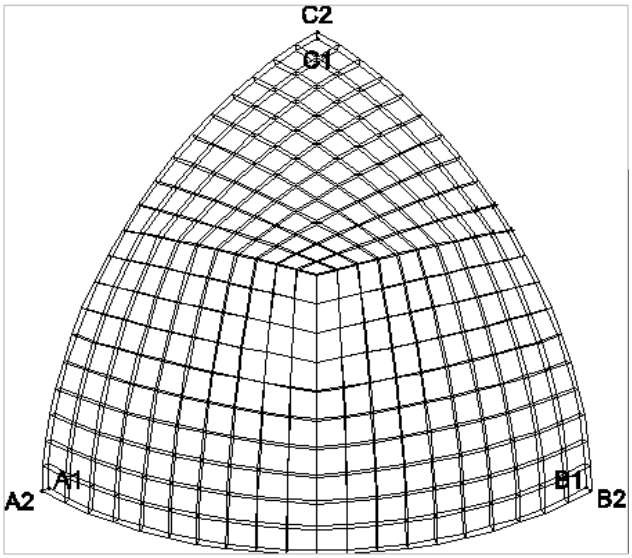
3.3 Values tested and results of modeling A

Identification	Reference	*Aster*	% difference
Not *A2* displacement *u*	$-1.28279.10^{-5}$	$-1.27928.10^{-5}$	0.27
Not *B2* displacement *v*	$-1.28279.10^{-5}$	$-1.27929.10^{-5}$	0.27
Not *C2* displacement *w*	$-1.28279.10^{-5}$	$-1.3034.10^{-5}$	1.7

4 Modeling B

4.1 Characteristics of modeling

Voluminal element 3D HEXA8



Modeling of a quarter of the sphere in HEXA8.

Names of the nodes:

Not *A1*	*N40*	Not *A2*	*N42*
Not *B1*	*N01*	Not *B2*	*N02*
Not *C1*	*N662*	Not *C2*	*N658*

4.2 Characteristics of the grid

Many nodes: 662
Many meshs and types: 300 HEXA8 for the sphere and 300 QUAD4 for external surface.

4.3 Values tested and results of modeling B

Identification	Reference	*Aster*	% difference
Not *A2* displacement *u*	$-1.28279.10^{-5}$	$-1.28298.10^{-5}$	0,015
Not *B2* displacement *v*	$-1.28279.10^{-5}$	$-1.28298.10^{-5}$	0,015
Not *C2* displacement *w*	$-1.28279.10^{-5}$	$-1.28662.10^{-5}$	0.30

5 Summary of the results

The results are in conformity with the reference solution.

One could expect to find exactly same displacement at the three points A, B and C. The difference at the point C comes from the not-symmetry of the grid. Grid is slightly more distorted around this point, which explains the fall of precision, which remains nevertheless very good, as well for element HEXA8 as for the SHB8.