# SSLS110 - Stability of a compressed square plate

**Summary:**

A homogeneous isotropic linear plate square elastic simply pressed on its four sides is subjected to a linear compressive force acting on two on its sides.

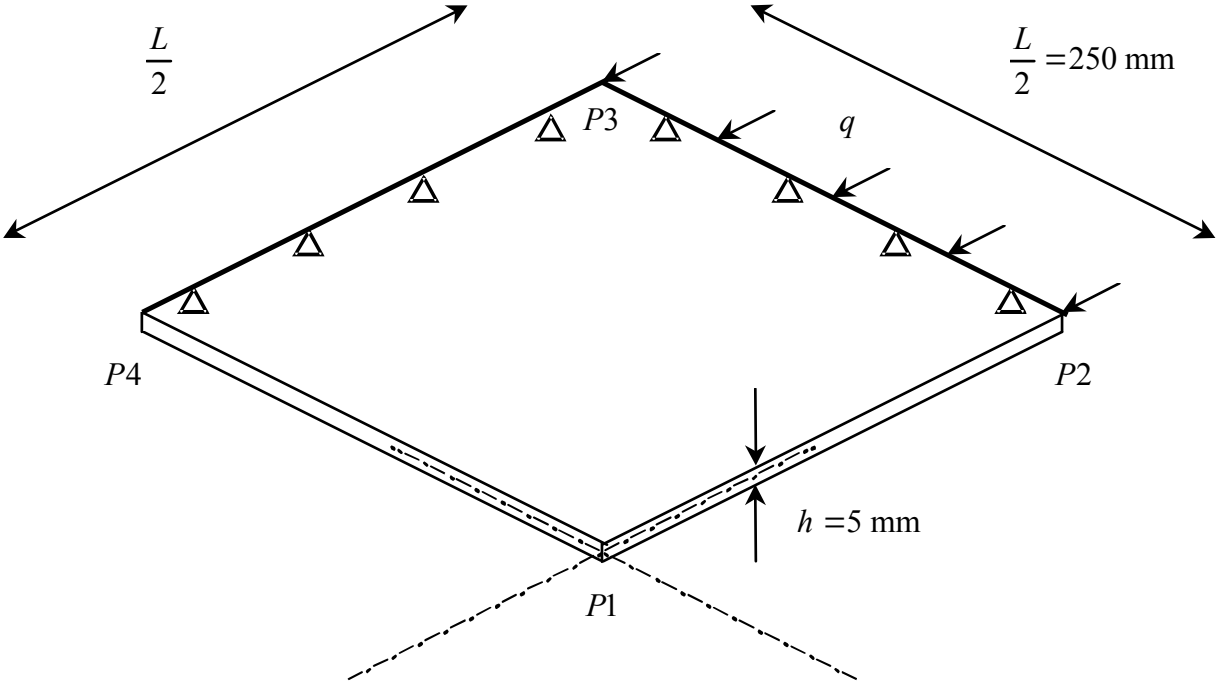
One calculates the critical loads leading to the elastic buckling of the plate. The geometrical matrix of rigidity used in the resolution of the problem to the eigenvalues is that which is due to the initial constraints.

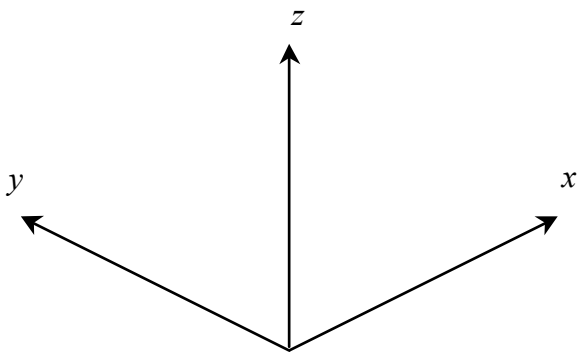
- linear elastic mechanics,
- buckling of a hull,
- interest of the test: calculation of the geometrical matrix of rigidity of the elements DKT, DKTG and COQUE_3D,
- 6 modelings.

# 1 Problem of reference

## 1.1 Geometry

Because of the geometrical and physical symmetry of the problem, only the quarter of the plate is modelled. By taking account the conditions of symmetry, one can collect only the only symmetrical modes of buckling.




## 1.2 Material properties

$E=2.1\,E\,5\,Mpa$ .

$\nu=0.3$

The transverse coefficient of shearing for the plate is worth $A_{CIS}=5/6$ .

## 1.3 Boundary conditions and loadings

Boundary conditions: $P2P3$ : $DZ=0.$

$P3P4$ : $DZ=0.$

Symmetry $P1P2$ : $DY=0.$ $DRX=0.$ $DRZ=0.$

$P4P1$ : $DX=0.$ $DRY=0.$ $DRZ=0.$

Loading:

Linear compressive force $q$ on $P2P3$

## 1.4 Remarks

It is not possible to solve the problem of deformation of compression without introducing the conditions of symmetry. Indeed, to impose boundary conditions of symmetry for a quarter of plate amounts eliminating the modes from rigid body for the complete plate.

# 2 Reference solution

## 2.1 Reference solution

- Critical loads

The analytical solution obtained with a thin theory of section in isotropic homogeneous linear elasticity [bib1] without taking into account of the transverse energy of shearing determines $i$ ème critical load:

$$q_{crI}=\frac{D\pi^2}{L^2}\left(i+\frac{1}{i}\right)^2$$

with:

$D=\frac{E\,h^3}{12\left(1-\nu^2\right)}$ : the coefficient of rigidity of inflection of the hull

$h$ : the thickness

$L$ : the length on the side of the square plate.

- Deformation of membrane

The analytical expression of the deformation of membrane along the axis $X$ is the following one:

$$e_{xx}=\frac{q\times L}{(h\times L\times E)}$$

The result of reference was calculated with $q=1.N/mm$

## 2.2 Results of reference

Certain modes corresponding to the critical loads of the analytical solution are not symmetrical and cannot be collected with the conditions of symmetry for a quarter of plate. The Values of the critical loads obtained thus correspond to the first 3 symmetrical modes of buckling:

- Mode 1 of the quarter of the plate = Mode 1 of all the plate
- Mode 2 of the quarter of the plate = Mode 3 of all the plate
- Mode 3 of the quarter of the plate = Mode 5 of all the plate

## 2.3 Uncertainty on the solution

Exact solution for a theory of plate without transverse shearing.

## 2.4 Bibliographical references

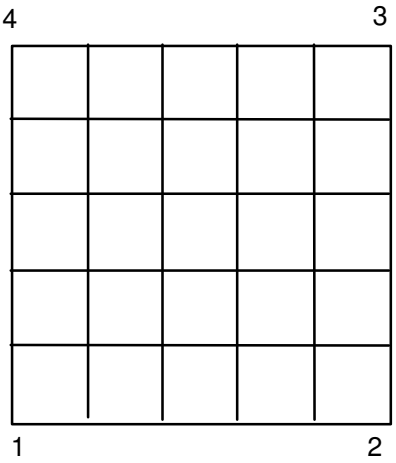
1) J.G. EISLEY "Mechanics of Elastic Structures: Classical years Finite Methods Element". Prentice Hall, Englewood Cliffs N.J. 07632 (19XX).

2) "Stability of Square Punt Biaxial Under Loading". The SAMCEF software User' S Manuals V7.1. (1998).

# 3 Modeling A

## 3.1 Characteristics of modeling

MEC3QU9H (COQUE_3D)




modeling COQUE_3D

## 3.2 Characteristics of the grid

Many nodes: 121

Many meshs and types: 25 QUAD9

## 3.3 Values tested

- Critical loads

| Identification | Type of reference | Reference | Tolerance % |
|---|---|---|---|
| mode 1 | 'ANALYTICAL' | – 3.79600E+02 | 1.0 |
| mode 2 | 'ANALYTICAL' | – 1.05444E+03 | 0.6 |
| mode 3 | 'ANALYTICAL' | – 2.56609E+03 | 0.4 |

- Deformation of membrane

| Identification | | | Type of reference | Reference | Tolérance % |
|---|---|---|---|---|---|
| Value | Mesh | Node | | | |
| *EXX* | *MA000045* | *NC000003* | 'ANALYTICAL' | – 9.5238095E-07 | 1.e-4 |

| Identification | | | Type of reference | Reference | Tolérance % |
|---|---|---|---|---|---|
| Value | Mesh | Not | | | |
| *EXX* | *MA000045* | 1 | 'ANALYTICAL' | – 9.5238095E-07 | 1.e-4 |

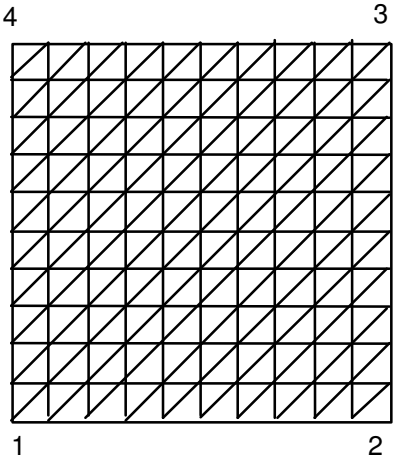
## 3.4 Remarks

Energy due to transverse shearing is not neglected.

# 4 Modeling B

## 4.1 Characteristics of modeling

MEC3TR7H (COQUE_3D)




modeling COQUE_3D

## 4.2 Characteristics of the grid

Many nodes: 641

Many meshs and types: 200 TRIA7

## 4.3 Values tested

- Critical loads

| Identification | Type of reference | Reference | Tolerance % |
|---|---|---|---|
| mode 1 | 'ANALYTICAL' | – 3.79600E+02 | 0.6 |
| mode 2 | 'ANALYTICAL' | – 1.05444E+03 | 0.3 |
| mode 3 | 'ANALYTICAL' | – 2.56609E+03 | 0.7 |

- Deformation of membrane

| Identification | | | Type of reference | Reference | Tolérance % |
|---|---|---|---|---|---|
| Value | Mesh | Node | | | |
| *EXX* | *MA000240* | *NC000003* | 'ANALYTICAL' | – 9.5238095E-07 | 1.e-4 |

| Identification | | | Type of reference | Reference | Tolérance % |
|---|---|---|---|---|---|
| Value | Mesh | Not | | | |
| *EXX* | *MA000240* | 1 | 'ANALYTICAL' | – 9.5238095E-07 | 1.e-4 |

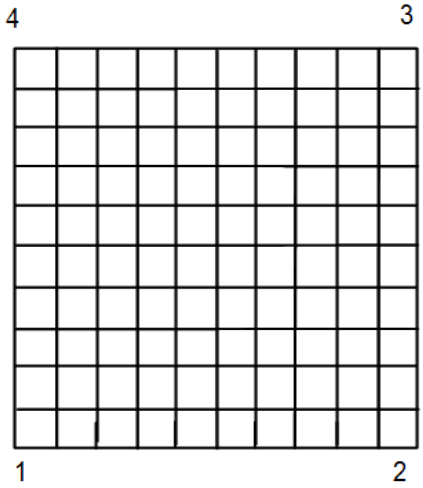
## 4.4 Remarks

Energy due to transverse shearing is not neglected.

# 5 Modeling C

## 5.1 Characteristics of modeling

MEDKQU4 (modeling DKT)




## 5.2 Characteristics of the grid

Many nodes: 121

Many meshs and types: 100 QUAD4

## 5.3 Values tested

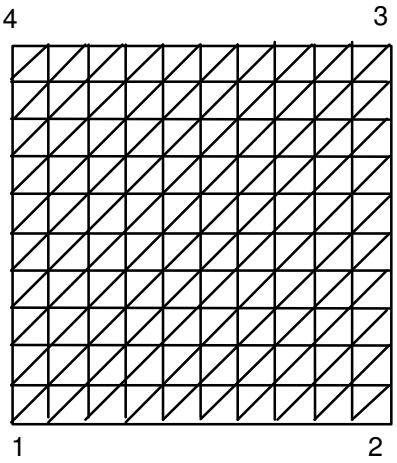
Critical loads

| Identification | Type of reference | Reference | Tolerance % |
|---|---|---|---|
| mode 1 | 'ANALYTICAL' | – 3.79600E+02 | 3.0 |
| mode 2 | 'ANALYTICAL' | – 1.05444E+03 | 2.0 |
| mode 3 | 'ANALYTICAL' | – 2.56609E+03 | 5.5 |

# 6 Modeling D

## 6.1 Characteristics of modeling

MEDKTR3 (modeling DKT)




## 6.2 Characteristics of the grid

Many nodes: 121

Many meshs and types: 200 TRIA3

## 6.3 Values tested

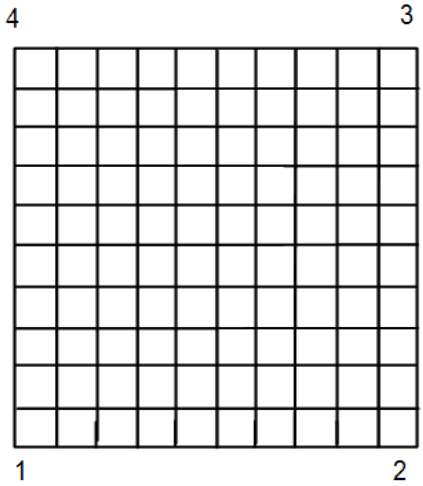
Critical loads

| Identification | Type of reference | Reference | Tolerance % |
|---|---|---|---|
| mode 1 | 'ANALYTICAL' | – 3.79600E+02 | 0.01 |
| mode 2 | 'ANALYTICAL' | – 1.05444E+03 | 2.0 |
| mode 3 | 'ANALYTICAL' | – 2.56609E+03 | 5.0 |

# 7 Modeling E

## 7.1 Characteristics of modeling

MEDKQU4 (modeling DKTG)




## 7.2 Characteristics of the grid

Many nodes: 121

Many meshs and types: 100 QUAD4

## 7.3 Values tested

Critical loads

| Identification | Type of reference | Reference | Tolerance % |
|---|---|---|---|
| mode 1 | 'ANALYTICAL' | – 3.79600E+02 | 3.0 |
| mode 2 | 'ANALYTICAL' | – 1.05444E+03 | 2.0 |
| mode 3 | 'ANALYTICAL' | – 2.56609E+03 | 5.5 |

# 8 Summary of the results

Got results:

- Modeling COQUE_3D : are very satisfactory for the two types of elements, QUAD9 and TRIA7, even if it is necessary to employ one more a large number of elements triangles.
- Modelings DKT and DKTG : except for mode 1 (meshs TRIA3), are less good than modeling COQUE_3D but are nevertheless satisfactory for the two types of elements, QUAD4 and TRIA3.