SSLP320 - Propagation of a crack X-FEM emerging requested in Mode I

Summary

The purpose of this test is to validate the calculation of the stress intensity factors (K_I and K_{II}) and the way of propagation of crack with X-FEM in 2D, within the framework of linear elasticity.

This test brings into play a rectangular plate with a crack leading, and subjected to a loading of traction to the edges inferior and superior of the plate.

Three methods to manage the propagation of cracks X-FEM are available. Each one of them is the object of a modeling.

Three modelings are considered:

- modeling a: method grid,
- modeling b: method simplex,
- modeling C: method upwind,
- modeling D: geometrical method,

The relevance of the results is evaluated by comparison of the factors of intensity of the constraints with the analytical values.

One finds a variation enters K_I and K_I theoretical lower than 1.13 % for the method grid, 1.12 % for the method simplex, 1.13 % for the method upwind, 1.1 % for the geometrical method and 1.1 % for the method upwind & FM.

1 Problem of reference

1.1 Geometry

The structure 2d is a rectangular plate ($LX=10\,m$, $LY=30\,m$), comprising an emerging crack [Figure 1.1-a]. The length of the initial crack is $a=5\,m$.

One calls “lower line”, the line in $y=0$ and “higher line”, the line in $y=Ly$.

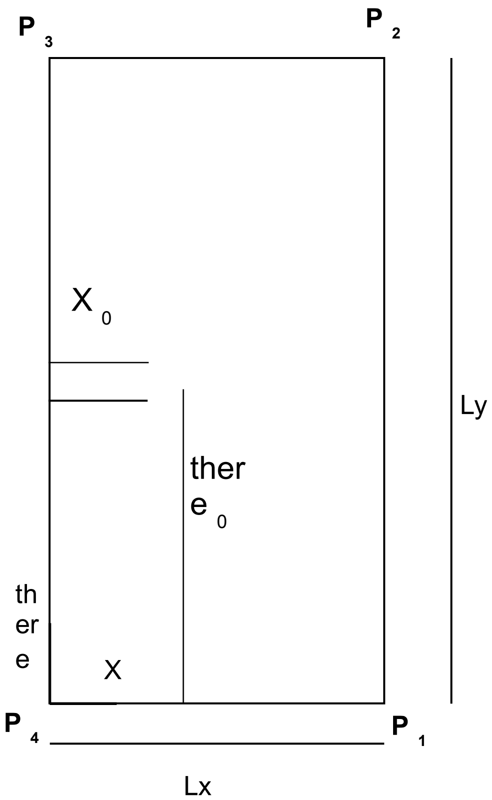

Figure 1.1-a : geometry of the fissured plate

Noted nodes $P1$ and $P4$ on Figure 1.1-a are used to impose the boundary conditions, which is clarified in the paragraph [§1.3].

1.2 Properties of material

Young modulus: $E=205\,10^9\,Pa$
Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

The loading consists in applying a force distributed to the lines lower and higher $p=106\,Pa$ and in the direction of the normal external to surface.

In order to block the rigid modes, displacements of the nodes are blocked P_1 , P_2 , P_3 and P_4 as follows:

- $DY^{P4}=DY^{P1}=0$;

- $DX^{P4}=0$.

2 Reference solution

2.1 Method of calculating

Analytical expressions of the stress intensity factors K_I and K_{II} are functions of the force distributed p, length of the crack has, the width of the plate Lx :

$$K_I = p\sqrt{\pi a}\, f\left(\frac{a}{Lx}\right)$$

$$K_{II} = 0$$

where the function f can be given several different manners. We choose that obtained by [1], and which is true for $\frac{a}{Lx} < 0{,}6$:

$$f\left(\frac{a}{Lx}\right) = 1{,}12 - 0{,}231\left(\frac{a}{Lx}\right) + 10{,}55\left(\frac{a}{Lx}\right)^2 - 21{,}72\left(\frac{a}{Lx}\right)^3 + 30{,}39\left(\frac{a}{Lx}\right)^4$$

One advances the crack thanks to the law of Paris:

$\frac{da}{dN} = C\Delta K^m$ where has is the length of crack, C and m are constants of material, ΔK between two FICs is the difference consecutive and N is the number of cycles.

With the digital values of the test:

Pas de propagation: $0{,}25\,m$

Lx : $10\,m$

2.2 Sizes and results of reference

Reference		
$a(m)$	$K_I(Pa.m^{0,5})$	$K_{II}(Pa.m^{0,5})$
2.5	$4.205998\ 10^6$	0
2.75	$4.63286\ 10^6$	0
3	$5.09492\ 10^6$	0
3.25	$5.59908\ 10^6$	0
3.5	$6.15349\ 10^6$	0
3.75	$6.76776\ 10^6$	0
4	$7.4531\ 10^6$	0
4.25	$8.2224\ 10^6$	0
4.5	$9.0905\ 10^6$	0
4.75	$1.0074\ 10^7$	0
5	$1.1192\ 10^7$	0
5.25	$1.2465\ 10^7$	0
5.5	$1.3916\ 10^7$	0
5.75	$1.55716\ 10^7$	0
6	$1.74586\ 10^7$	0

Table 2.2-1 : values of reference for K_I and K_{II}

2.3 Uncertainties on the solution

No, analytical solution.

2.4 Bibliographical references

[1] TADA H., PARIS P., IRWIN G.: The stress analysis of aces, Handbook. Del Research Corporation, Hellertown, Pennsylvania, 1973.

3 Modeling A

3.1 Characteristics of modeling

In this modeling, the method grid is tested for the propagation of crack. The level-sets are determined by orthogonal projection on the segments composing the crack.

3.2 Characteristics of the grid

The structure is modelled by a "healthy" grid regular composed of 40×101 QUAD4, respectively along the axes x, y. The crack is represented by a succession of SEG2, independently of the grid of the structure.

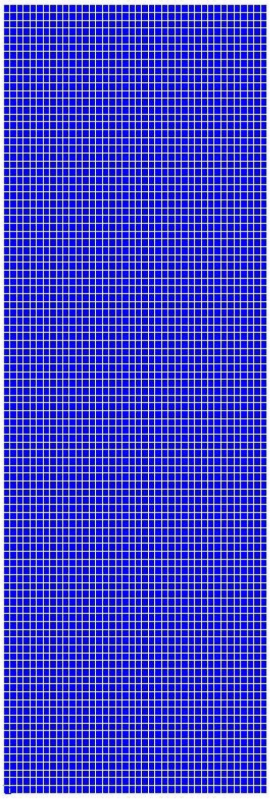

Figure 3.2-a : grid of the fissured plate

3.3 Sizes tested and results

For each step of propagation ($2{,}5\,m$), one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.
One also tests the ordinate of the bottom of crack given by PROPA_FISS.

3.3.1 Results on K_I :

Identification	*Code_Aster*	Reference	difference
CALC_G			
KI_1	4.17448 10^6	4.205998 10^6	-0,749%
KI_2	4.60197 10^6	4.63286 10^6	-0,667%
KI_3	5.0668 10^6	5.09492 10^6	-0,552%
KI_4	5.575 10^6	5.59908 10^6	-0,43%
KI_5	6.1334 10^6	6.15349 10^6	-0,326%
KI_6	6.7499 10^6	6.76776 10^6	-0,264%
KI_7	7.4338 10^6	7.4531 10^6	-0,259%
KI_8	8.19599 10^6	8.2224 10^6	-0,322%
KI_9	9.0497 10^6	9.0905 10^6	-0,449%
KI_10	1.0011 10^7	1.0074 10^7	-0,627%
KI_11	1.1099 10^7	1.1192 10^7	-0,828%
KI_12	1.2339 10^7	1.2465 10^7	-1,011%
KI_13	1.37603 10^7	1.3916 10^7	-1,121%
KI_14	1.54018 10^7	1.55716 10^7	-1,09%
KI_15	1.7313 10^7	1.74586 10^7	-0,834%

3.3.2 Results on K_{II} :

For this test, it be wished that K_{II} that is to say lower than $10^{-4} K_I$. Thus, one makes sure that K_{II} is rather close to zero, the value of reference.

Identification	*Code_Aster*	Reference
CALC_G		
KII_1	-2.7313 10^2	0
KII_2	-8.5062 10^1	0
KII_3	-2.6061 10^2	0
KII_4	1.5995 10^2	0
KII_5	-2.7309 10^2	0
KII_6	-2.3176 10^2	0
KII_7	-3.1276 10^2	0
KII_8	3.1327 10^2	0
KII_9	-3.8393 10^2	0
KII_10	-4.1916 10^2	0
KII_11	-4.986 10^2	0
KII_12	-5.6998 10^2	0
KII_13	-6.7642 10^2	0
KII_14	-7.9542 10^2	0
KII_15	-9.5344 10^2	0

3.3.3 Results on the ordinate of the bottom of crack:

It is checked that the coordinates in ordinate of the successive funds of crack are close to the initial value. This checking gives the same indications as the test on K_{II} .

Identification	*Code_Aster*	Reference	Difference
CALC_G			
y_1	15	15	0%
y_2	15	15	$2.18\ 10^{-4}$ %
y_3	15	15	$2.8\ 10^{-4}$ %
y_4	15	15	$4.51\ 10^{-4}$ %
y_5	15	15	$5.47\ 10^{-4}$ %
y_6	15	15	$6.95\ 10^{-4}$ %
y_7	15	15	$8.1\ 10^{-4}$ %
y_8	15	15	$9.5\ 10^{-4}$ %
y_9	15.0002	15	0,001%
y_10	15.0002	15	0,001%
y_11	15.0002	15	0,001%
y_12	15.0002	15	0,002%
y_13	15.0002	15	0,002%
y_14	15.0003	15	0,002%
y_15	15.0003	15	0,002%

3.4 Complementary results

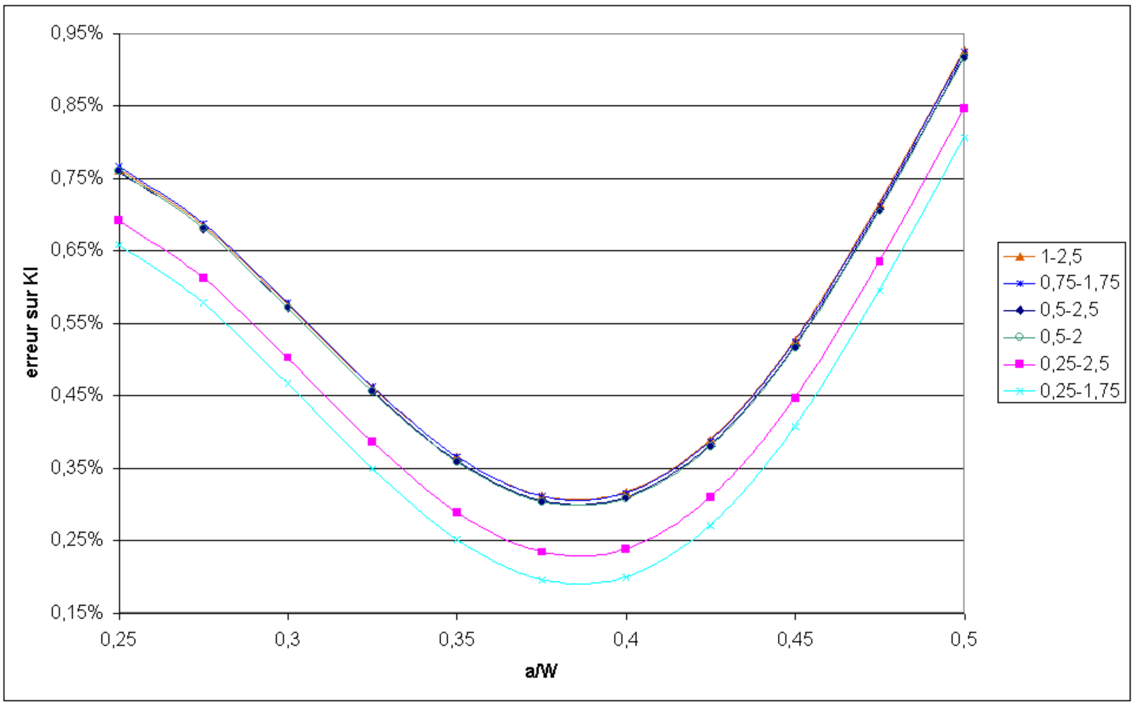

Figure 3.4-a : Influence of the choice of crowns IH and RS on the error on KI

We can see here that the configuration most adapted for the choice of RI and RS (crowns lower and higher of the field theta) is: $RI=2*L_0$ and $RS=7*L_0$ where L_0 is the smallest edge of the grid.

4 Modeling B

4.1 Characteristics of modeling

In this modeling, the method simplex is tested for the propagation of crack.
The level-sets are determined by resolution of the equations of reactualization.

4.2 Characteristics of the grid

One uses here the same grid as in modeling A.

4.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

4.3.1 Results on K_I :

Identification	*Code_Aster*	Reference	difference
CALC_G			
KI_1	4.1749 10^6	4.205998 10^6	0,73%
KI_2	4.6025 10^6	4.63286 10^6	0,65%
KI_3	5.0675 10^6	5.09492 10^6	0,54%
KI_4	5.5758 10^6	5.59908 10^6	0,41%
KI_5	6.1344 10^6	6.15349 10^6	0,31%
KI_6	6.7511 10^6	6.76776 10^6	0,24%
KI_7	7.4352 10^6	7.4531 10^6	0,24%
KI_8	8.1976 10^6	8.2224 10^6	0,30%
KI_9	9.0516 10^6	9.0905 10^6	0,42%
KI_10	1.0013 10^6	1.0074 10^7	0,60%
KI_11	1.1101 10^6	1.1192 10^7	0,80%
KI_12	1.2341 10^6	1.2465 10^7	0,98%
KI_13	1.37608 10^6	1.3916 10^7	1,09%
KI_14	1.5405 10^6	1.55716 10^7	1,06%
KI_15	1.7317 10^6	1.74586 10^7	0,80%

4.3.2 Results on K_{II} :

Identification	*Code_Aster*	Reference
CALC_G		
KII_1	-294.543283239	0
KII_2	140.53299141	0
KII_3	-92.1854404834	0
KII_4	31.5966858116	0
KII_5	-22.0812184567	0
KII_6	1.80888843609	0
KII_7	-14.6528361549	0
KII_8	-12.9336699382	0
KII_9	-21.8747247036	0
KII_10	-27.5009059699	0
KII_11	-36.8193114189	0
KII_12	-47.1435134216	0
KII_13	-60.5512354886	0
KII_14	-77.2532857738	0
KII_15	-98.7961435219	0

5 Modeling C

5.1 Characteristics of modeling

In this modeling, the method upwind fast marching UPWIND is tested for the propagation of crack.
The level-sets are determined by resolution of the equations of reactualization per diagram to the finished differences.

5.2 Characteristics of modeling

One uses here the same grid as in modeling A (§2.1).

5.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

5.3.1 Results on KI:

Identification	*Code_Aster*	Reference	difference
CALC_G			
KI_1	$4.174911\ 10^6$	$4.205998\ 10^6$	0,739%
KI_2	$4.602545\ 10^6$	$4.632857\ 10^6$	0,654%
KI_3	$5.067525\ 10^6$	$5.094923\ 10^6$	0,538%
KI_4	$5.575881\ 10^6$	$5.599079\ 10^6$	0,414%
KI_5	$6.134452\ 10^6$	$6.153487\ 10^6$	0,309%
KI_6	$6.751139\ 10^6$	$6.767759\ 10^6$	0,24%
KI_7	$7.435204\ 10^6$	$7.453097\ 10^6$	0,24%
KI_8	$8.197634\ 10^6$	$8.222429\ 10^6$	0,302%
KI_9	$9.051616\ 10^6$	$9.090524\ 10^6$	0,428%
KI_10	$1.0013134\ 10^7$	$1.0074102\ 10^7$	0,605%
KI_11	$1.1101774\ 10^7$	$1.1191940\ 10^7$	0,805%
KI_12	$1.2341792\ 10^7$	$1.2464967\ 10^7$	0,99%
KI_13	$1.3763566\ 10^7$	$1.3916354\ 10^7$	1,098%
KI_14	$1.5405615\ 10^7$	$1.5571606\ 10^7$	1,065%
KI_15	$1.7317423\ 10^7$	$1.7458645\ 10^7$	0,809%

5.3.2 Results on KII:

Identification	*Code_Aster*	Reference
CALC_G		
KII_1	-294.54	0
KII_2	-310.00	0
KII_3	-330.90	0
KII_4	-357.05	0
KII_5	-389.44	0
KII_6	-428.98	0
KII_7	-476.90	0
KII_8	-534.75	0
KII_9	-604.47	0
KII_10	-688.57	0
KII_11	-790.30	0
KII_12	-913.84	0
KII_13	-1064.72	0
KII_14	-1250.30	0
KII_15	-1480.53	0

6 Modeling D

6.1 Characteristics of modeling

In this modeling, the geometrical method is tested for the propagation of crack.

6.2 Characteristics of modeling

One uses here the same grid as in modeling A.

6.3 Sizes tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by `CALC_G`.

6.3.1 Results on KI:

Identification	*Code_Aster*	Reference	difference
CALC_G			
KI_1	4.205998 10^6	4.205998 10^6	0,739%
KI_2	4.602538 10^6	4.632857 10^6	0,654%
KI_3	5.067526 10^6	5.094923 10^6	0,538%
KI_4	5.575879 10^6	5.599079 10^6	0,414%
KI_5	6.134453 10^6	6.153487 10^6	0,309%
KI_6	6.751140 10^6	6.767759 10^6	0,246%
KI_7	7.435205 10^6	7.453097 10^6	0.240%
KI_8	8.197636 10^6	8.222429 10^6	0,302%
KI_9	9.051618 10^6	9.090524 10^6	0,428%
KI_10	1.0013136 10^7	1.0074102 10^7	0.61%
KI_11	1.1101777 10^7	1.1191940 10^7	0.805%
KI_12	1.2341796 10^7	1.2464967 10^7	0,99%
KI_13	1.3763571 10^7	1.3916354 10^7	1.098%
KI_14	1.5405621 10^7	1.5571606 10^7	1,065%
KI_15	1.7317438 10^7	1.7458645 10^7	0,808%

6.3.2 Results on KII:

Identification	*Code_Aster*	Reference
CALC_G		
KII_1	-294.543283255	0
KII_2	140.53345257	0
KII_3	-92.1815420989	0
KII_4	31.588424802	0
KII_5	-22.0719386916	0
KII_6	1.80251453051	0
KII_7	-14.6478888153	0
KII_8	-12.9364628022	0
KII_9	-21.8732802716	0
KII_10	-27.5019633622	0
KII_11	-36.819305336	0
KII_12	-47.1442361154	0
KII_13	-60.5518993996	0
KII_14	-77.2544634291	0
KII_15	-98.7975849383	0

7 Summaries of the results

One can compare the computing time for the same number of steps of propagation (15) of the three methods.

Grid	Method	Time (s)
40×101	Grid	21.7
	Simplex	18.8
	Upwind	23.7
	geometrical	21.2

The results make it possible to validate on a simple case the calculation of the stress intensity factors in mode I for elements X-FEM for the four methods grid, geometrical simplex, upwind and.