SSLP319 - Propagation of two cracks X-FEM emerging solicited in mode I

Summary:

The goal of this test is to check that the operator PROPA_FISS draft correctly cases of multi-cracking.
It is about a plate $2D$ containing two cracks, each one made up of only one bottom. Several propagations are calculated by the operator PROPA_FISS. It is checked that the factors of intensity of the constraints of the propagated cracks are correct for a propagation in mode I.

1 Problem of reference

1.1 Geometry

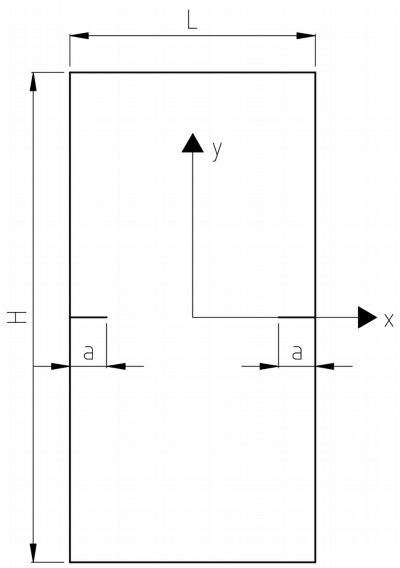

Figure 1.1-a: geometry of the fissured plate

Geometrical dimensions of the fissured plate:

width $L=1000\,mm$
height $H=2000\,mm$

Initial length of the cracks: $a_0=300\,mm$.
The cracks are positioned in the middle of the height of the plate ($H/2$).

1.2 Properties of material

Young modulus $E=206000\,MPa$
Poisson's ratio $\nu=0.33$

1.3 Boundary conditions and loadings

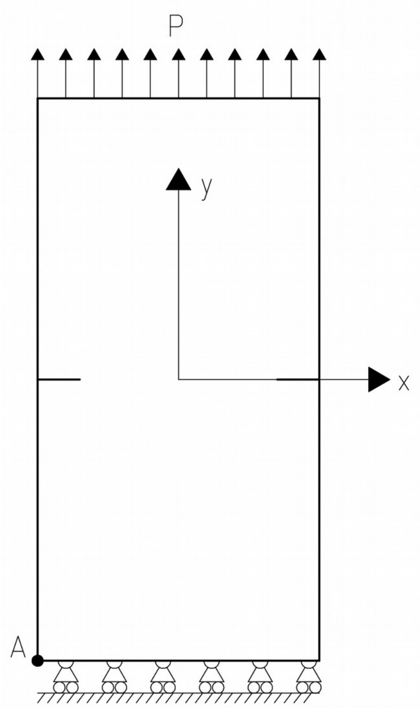

Figure 1.3-a: boundary conditions and loadings

Boundary conditions:

Not A : $\Delta X=\Delta Y=0$

Points of the lower end of the plate: $\Delta Y=0$

Loading:

Pressure applied at the higher end of the plate: $P=1\ MPa$

Three propagations are calculated by imposing a maximum advance of the cracks equalizes with $30\ mm$. As a consequence of the symmetry of the geometry, boundary conditions and loading, the advances of the two cracks are always equal in advance imposed maximum.

2 Reference solution

2.1 Method of calculating

Three propagations of the crack are calculated. The two cracks always advance same distance and their factors of intensity of the constraints are always equal between them.
One can calculate the factors of intensity of the constraints by using the following equations [bib1]:

$$K_I = Y \cdot P \cdot \sqrt{a}$$

$$Y = 1.99 + 0.76 \cdot \frac{a}{L} - 8.48 \cdot \left(\frac{a}{L}\right)^2 + 27.36 \cdot \left(\frac{a}{L}\right)^3$$

$$K_{II} = 0$$

2.2 Sizes and results of reference

For the three propagations calculated in the tests, the half-length of the crack is the following one:

Propagation	a **[** mm **]**
1	330.0
2	360.0
3	390.0

Table 2.1

The value of K_I waited is thus the following one for each propagated bottom:

Propagation	K_I **[** $Pa\sqrt{mm}$ **]**
1	3.7992E+07
2	4.1791E+07
3	4.6316E+07

Table 2.2

The value of K_{II} waited is always equal to zero.

2.3 Bibliographical references

[1] D.Broek, “Elementary engineering fractures mechanics”, Martinus Nijhoff Publishers, The La Hague, The Netherlands, 1982

1 Modeling A

1.1 Characteristics of modeling

The method fast marching `UPWIND` is used by `PROPA_FISS` to solve the equations of propagation of the crack.
No auxiliary grid is not used. That is possible because the grid of the structure is very regular.

1.2 Characteristics of the grid

The structure is modelled by a grid made up of 1440 elements `QUAD4` (see Figure 1.2-a).

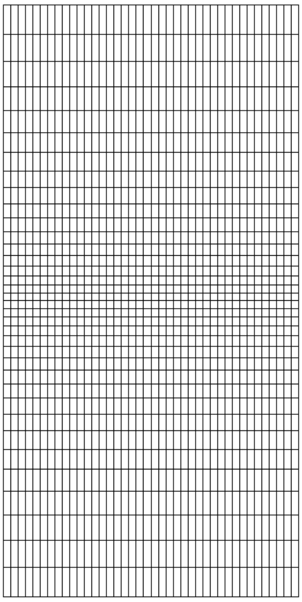

Figure 1.2-a: grid of the structure

The grid is very coarse to reduce the computing time. It is refined more in the zone of propagation of the crack. In this zone, the dimension of the elements is of $25\times25\,mm$. The largest element used has a dimension equalizes with $25\times100\,mm$.

1.3 Sizes tested and results

One tests the values of K_I and K_{II} for the two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equalizes with 5% for the values of K_I . On the other hand, to check if the value of K_{II} is worthless, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is worthless if its value is lower than 1% value of K_I . Indeed, in this case one can neglect the value of K_{II} .

Propagation	**Crack**	K_I **reference [** $Pa\sqrt{mm}$ **]**	**Tolerance**
1	left	3.7992E+07	<5%
	right-hand side	3.7992E+07	<5%
2	left	4.1791E+07	<5%
	right-hand side	4.1791E+07	<5%

3	left	4.6316E+07	<5%
	right-hand side	4.6316E+07	<5%

Propagation	Crack	K_{II} reference [$Pa\sqrt{mm}$]	Tolerance [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$
2	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$
3	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$

1.4 Remarks

All the values tested are in the tolerances used. That means that the method UPWIND calculate correctly at the same time the position of the two cracks and the level sets.
The error obtained on the values of K_I is almost worthless and the values of K_{II} are always about 0.01% values of K_I. The got results are thus very satisfactory.

2 Modeling B

2.1 Characteristics of modeling

Method SIMPLEX is used by PROPA_FISS to solve the equations of propagation of the crack. **No auxiliary grid** is not used.

2.2 Characteristics of the grid

One uses the same grid as that of modeling A .

2.3 Sizes tested and results

One tests the values of K_I and K_{II} for the two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equalizes with 5% for the values of K_I . On the other hand, to check if the value of K_{II} is worthless, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is worthless if its value is lower than 1% value of K_I . Indeed, in this case one can neglect the value of K_{II} .

Propagation	**Crack**	K_I **reference** [$Pa\sqrt{mm}$]	**Tolerance**
1	left	3.7992E+07	<5%
	right-hand side	3.7992E+07	<5%
2	left	4.1791E+07	<5%
	right-hand side	4.1791E+07	<5%
3	left	4.6316E+07	<5%
	right-hand side	4.6316E+07	<5%

Propagation	**Crack**	K_{II} **reference** [$Pa\sqrt{mm}$]	**Tolerance** [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$
2	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$
3	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$

2.4 Remarks

All the values tested are in the tolerances used. That means that the method SIMPLEX calculate correctly at the same time the position of the two cracks and the level sets.

The error obtained on the values of K_I is almost worthless and the values of K_{II} are always about 0.01% values of K_I. The got results are thus very satisfactory.

3 Modeling C

3.1 Characteristics of modeling

Method GRID is used by PROPA_FISS. The operator CALC_G is used for the calculation of the factors of intensities of the constraints.

3.2 Characteristics of the grid

One uses the same grid as that of modeling A .

3.3 Sizes tested and results

In this modeling, the keyword is not used COMP_LINE of PROPA_FISS, but one gives as starter PROPA_FISS a table containing several moments. One thus tests by means of computer that the calculation of the cycle is correct.

One tests the values of K_I and K_{II} for the two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equalizes with 1% for the values of K_I . On the other hand, to check if the value of K_{II} is worthless, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is worthless if its value is lower than $0,1\%$ value of K_I. Indeed, in this case one can neglect the value of K_{II}.

Propagation	Crack	K_I reference [$Pa\sqrt{mm}$]	Tolerance
1	left	3,80E+007	<1%
	right-hand side	3,80E+007	<1%
2	left	4,18E+007	<1%
	right-hand side	4,18E+007	<1%
3	left	4,63E+007	<1%
	right-hand side	4,63E+007	<1%

Propagation	Crack	K_{II} reference [$Pa\sqrt{mm}$]	Tolerance [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Réf}/1000$
	right-hand side	0	< $K_{I\,Réf}/1000$
2	left	0	< $K_{I\,Réf}/1000$
	right-hand side	0	< $K_{I\,Réf}/1000$
3	left	0	< $K_{I\,Réf}/1000$
	right-hand side	0	< $K_{I\,Réf}/1000$

3.4 Remarks

All the values tested are in the tolerances used. That means that the method GRID calculate correctly at the same time the position of the two cracks and the level sets.
The error obtained on the values of K_I is lower than 1% and values of K_{II} are always lower than $K_I/1000$. The got results are thus very satisfactory.

4 Modeling D

4.1 Characteristics of modeling

Method GRID is used by PROPA_FISS. The operator POST_K1_K2_K3 is used for the calculation of the factors of intensities of the constraints.

4.2 Characteristics of the grid

One uses the same grid as that of modeling A .

4.3 Sizes tested and results

One tests the values of K_I and K_{II} for the two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equalizes with 13% for the values of K_I . On the other hand, to check if the value of K_{II} is worthless, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is worthless if its value is lower than 1% value of K_I . Indeed, in this case one can neglect the value of K_{II} .

Propagation	Crack	K_I reference [$Pa\sqrt{mm}$]	Tolerance
1	left	3.7993E+07	<13%
	right-hand side	3.7993E+07	<13%
2	left	4.179E+07	<13%
	right-hand side	4.179E+07	<13%
3	left	4.632E+07	<13%
	right-hand side	4.632E+07	<13%

Propagation	Crack	K_{II} reference [$Pa\sqrt{mm}$]	Tolerance [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$
2	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$
3	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$

4.4 Remarks

All the values tested are in the tolerances used. That means that the method GRID calculate correctly at the same time the position of the two cracks and the level sets.

The error obtained on the values of K_I is almost worthless and the values of K_{II} are always about 0.01% values of K_I. The got results are thus very satisfactory.

5 Modeling E

5.1 Characteristics of modeling

Method GEOMETRICAL is used by PROPA_FISS to update the position of the crack. **No auxiliary grid** is not used.

5.2 Characteristics of the grid

One uses the same grid as that of modeling A .

5.3 Sizes tested and results

One tests the values of K_I and K_{II} for the two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equalizes with 5% for the values of K_I . On the other hand, to check if the value of K_{II} is worthless, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is worthless if its value is lower than 1% value of K_I . Indeed, in this case one can neglect the value of K_{II} .

Propagation	Crack	K_I reference [$Pa\sqrt{mm}$]	Tolerance
1	left	3.7992E+07	<5%
	right-hand side	3.7992E+07	<5%
2	left	4.1791E+07	<5%
	right-hand side	4.1791E+07	<5%
3	left	4.6316E+07	<5%
	right-hand side	4.6316E+07	<5%

Propagation	Crack	K_{II} reference [$Pa\sqrt{mm}$]	Tolerance [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$
2	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$
3	left	0	< $K_{I\,Aster}/100$
	right-hand side	0	< $K_{I\,Aster}/100$

5.4 Remarks

All the values tested are in the tolerances used. That means that the method GEOMETRICAL calculate correctly at the same time the position of the two cracks and the level sets.

The error obtained on the values of K_I is almost worthless and the values of K_{II} are always about 0.01% values of K_I. The got results are thus very satisfactory.

6 Summary of the results

All methods of the operator PROPA_FISS used (UPWIND, SIMPLEX, GRID and GEOMETRICAL) allowed to calculate well the position of two existing cracks in the same model and propagating in mode I. The factors of intensity of the constraints were calculated correctly and the methods used calculate correctly the level sets with each propagation.
The got results make it possible to validate the implementation of multi-cracking (case of several cracks with only one bottom of crack each one) in the operator PROPA_FISS.