

SSLP317- Validation of macro-order RAFF_XFEM on a multi-fissured plate

Summary:

The purpose of this document is to validate the macro order RAFF_XFEM [U7.03.51] which makes it possible to obtain a field "of error" *a priori* in order to feed a process of refinement of grid.

1 Problem of reference

1.1 Geometry

A square plate is considered 2D of with dimensions unit, centered in the beginning.

This plate comprises 2 cracks and 2 inclusions (interfaces):

- an inclusion of the type notches length $L_e=0,6$ and of ray $R_e=0,05$, centered in $E(0; -0,3)$,
- a circular inclusion of ray $R_c=0,05$ centered in the beginning,
- a non-opening horizontal rectilinear crack length $L_A=0,2$ centered in $A(0,3 ; 0,25)$,
- a non-opening horizontal rectilinear crack length $L_B=0,2$ centered in $B(-0,2 ; 0,25)$.

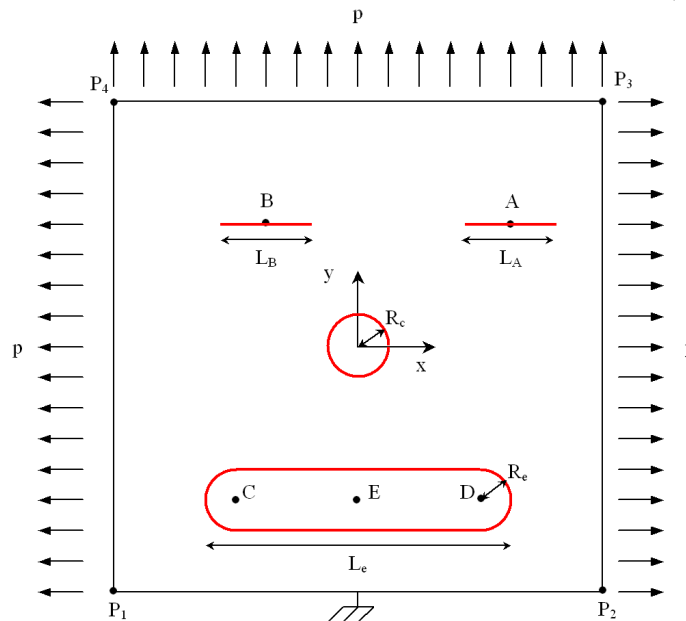


Figure 1.1-1: diagram of the multi-fissured plate

1.2 Properties of material

The material has the following properties:

- Young modulus: $E = 205\,000\text{ MPa}$
- Poisson's ratio: $\nu = 0,3$

1.3 Boundary conditions and loadings

Mechanical calculation imports little in this test because one only seeks to test the criterion of refinement *a priori*. Mechanical calculation is given only on a purely illustrative basis in modeling A.

One considers a loading of traction of $p = 1\text{ MPa}$ on the faces left, right-hand side and higher of the plate. The lower face is embedded.

1.4 Short description of various modelings

All modelings return within the framework of the representation of the cracks and the interfaces by method X-FEM associated with the level sets.

Modeling A calculates an indicator outdistances some (fields with the nodes). The test will relate to the value of this indicator in various nodes. The methodology of refinement of grid used is more simple (criterion expressed as a percentage of refined meshes), but is not inevitably optimal in term of many meshes.

Modeling B calculates an indicator outdistances some (fields with the nodes). The interest of this modeling is to show a more optimal methodology in term of many meshes. Tests will relate to the size of the meshes in the refined zone.

Modeling C calculates an indicator by zone (by meshes). Methodology presented is succeeded and most robust. It is this indicator and this methodology which one advises.

2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Modeling A

The test relates to the value of the indicator of error at exit of RAFF_XFEM. One notes $I(M)$ the value of this indicator at the point M unspecified.

At the point P_1 in bottom on the left of the structure, the indicator is worth opposite distance to the point of the notch nearest, that is to say $I(P_1) = -(CP_1 - R_e)$.

At the point P_2 in bottom on the right of the structure, the indicator is worth opposite distance to the point of the notch nearest, that is to say $I(P_2) = -(DP_2 - R_e)$.

At the point P_3 in top on the right of the structure, the indicator is worth opposite distance to the right end of the crack on the right, that is to say $I(P_3) = -A' P_3$ where $A' = A + \frac{L_A}{2} \vec{x}$.

At the point P_4 in top on the left of the structure, the indicator is worth opposite distance to the left end of the crack on the left, that is to say $I(P_4) = -B' P_4$ where $B' = B - \frac{L_B}{2} \vec{x}$.

2.1.2 Modelings B and C

The test relates to the value of the diameter of smallest nets. If h_0 is the initial size of the meshes, h_c the size targets meshes after refinement, then the minimal number of call to Lobster to reach h_c is

$nb_raff = E(n) + 1$, with $n = \frac{\ln(h_0) - \ln(h_c)}{\ln(2)}$. After refinement, the size of the most refined meshes is

$$h = \frac{h_0}{2^{nb_raff}}$$

2.2 Results of reference

2.2.1 Modeling A

With the digital values used in the test, one finds:

$$I(P_1) = -(\sqrt{0,25^2 + 0,2^2} - 0,05) \approx -0,27015621187164246$$

$$I(P_2) = -(\sqrt{0,25^2 + 0,2^2} - 0,05) \approx -0,27015621187164246$$

$$I(P_3) = -\sqrt{0,25^2 + 0,1^2} \approx -0,26925824035672524$$

$$I(P_4) = -\sqrt{0,25^2 + 0,2^2} \approx -0,32015621187164245$$

2.2.2 Modelings B and C

With $h_0 = \frac{\sqrt{2}}{20}$ and $h_c = \frac{h_0}{10}$, one finds $h = 0,0044194$.

3 Modeling A

3.1 Characteristics of modeling

The interfaces and the cracks are defined by level sets.

The initial grid is refined 3 times of continuation, using the indicator in distance provided by RAFF_XFEM. Each time, one refines 20% of the meshes most sullied with "error".

3.2 Characteristics of the grid

The initial grid is healthy: it consists of 20×20 linear quadrangles.

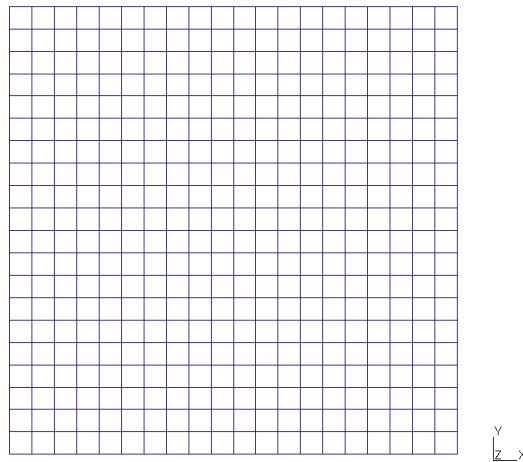


Figure 3.2-1: initial healthy grid

3.3 Sizes tested and results

One tests the value of the indicator of error at exit of RAFF_XFEM at the points P_1 , P_2 , P_3 and P_4 , for each refined grid.

| Identification | Reference | Type of reference | % tolerance |
|----------------|-----------|-------------------|-------------|
| $I(P_1)$ | -0,270156 | 'ANALYTICAL' | 0,1% |
| $I(P_2)$ | -0,270156 | 'ANALYTICAL' | 0,1% |
| $I(P_3)$ | -0,269258 | 'ANALYTICAL' | 0,1% |
| $I(P_4)$ | -0,320156 | 'ANALYTICAL' | 0,1% |

For information, the grid obtained after refinement is presented on Figure 3.3-1. It is noted that for the interfaces, they are not only the meshes close to the interfaces which are refined, but all the meshes contained in inclusion. That is due to the way in which Lobster manages the connection by conformity of the cut out quadrangles. This phenomenon does not appear with triangles. This is why in the continuation of modelings, one will prefer to start from an initial grid of triangles.

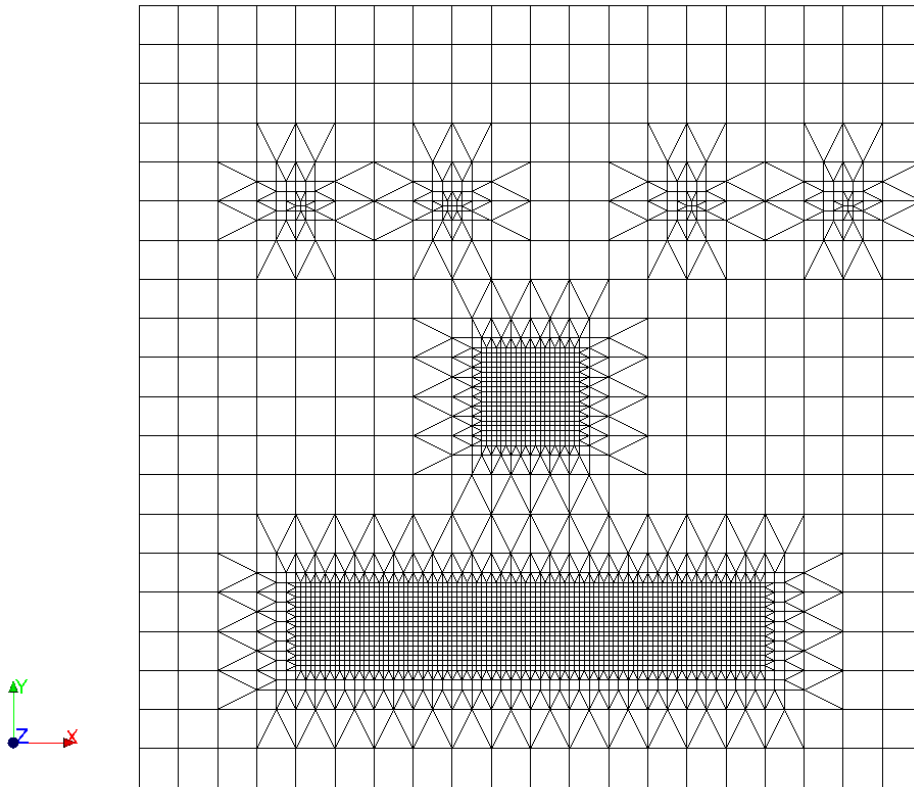


Figure 3.3-1: refined healthy grid

4 Modeling B

4.1 Characteristics of modeling

The interfaces and the cracks are defined by level sets.
The indicator in distance provided by RAFF_XFEM is used.

The interest of this modeling is to present a methodology more optimal than that used in modeling A. the idea is better to control the numbers of meshes to be refined and cuts it meshes in the end of the process of refinement.

Knowing the initial size of the meshes, and by setting a target size, one will refine as many times as necessary in order to obtaining in the zones of adequate interest of the meshes of size.

4.2 Characteristics of the grid

The initial grid is healthy: it consists of 20×20 linear quadrangles, which one cross into two in order to obtain from the triangles (see explanation to the §3.3). This grid is presented on Figure 4.2-1.

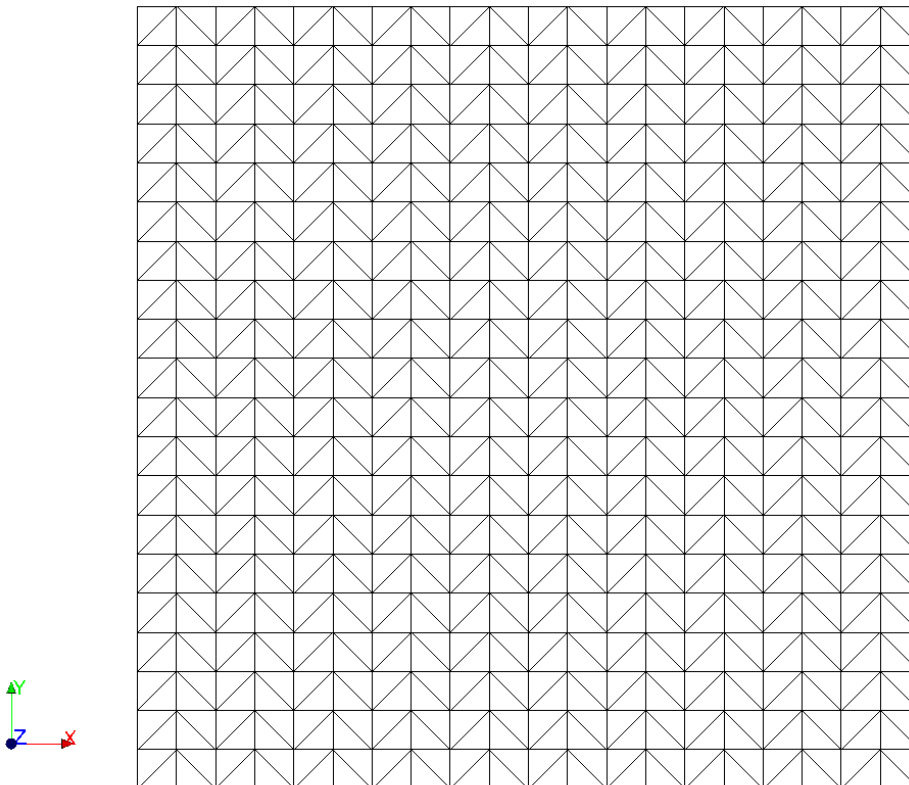


Figure 4.2-1: initial healthy grid

One applies the procedure of refinement as described in [U2.05.02] for the indicator in distance. This procedure makes it possible to better control the size and the number of mesh with refinement. After refinement, one obtains the grid presented on Figure 4.2-2. It is interesting to note that the meshes strictly included in the zone of interest (for example around the bottom n°1 of the crack centered in A) do not have all the same size (see Figure 4.2-3).

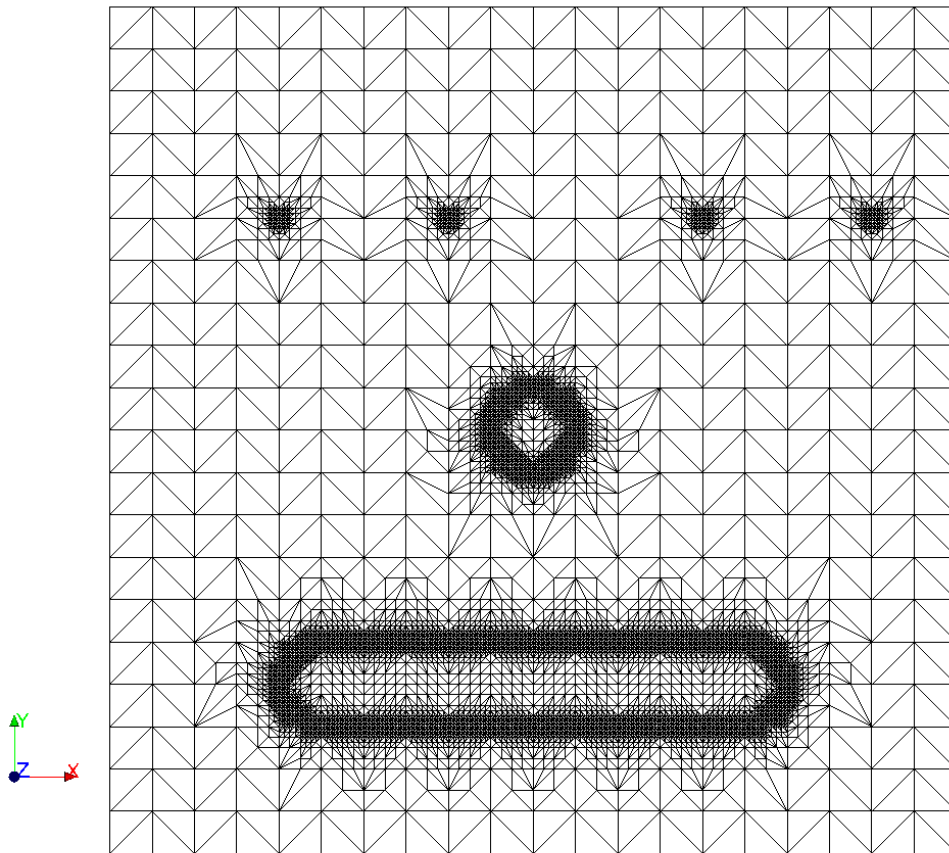


Figure 4.2-2: refined healthy grid

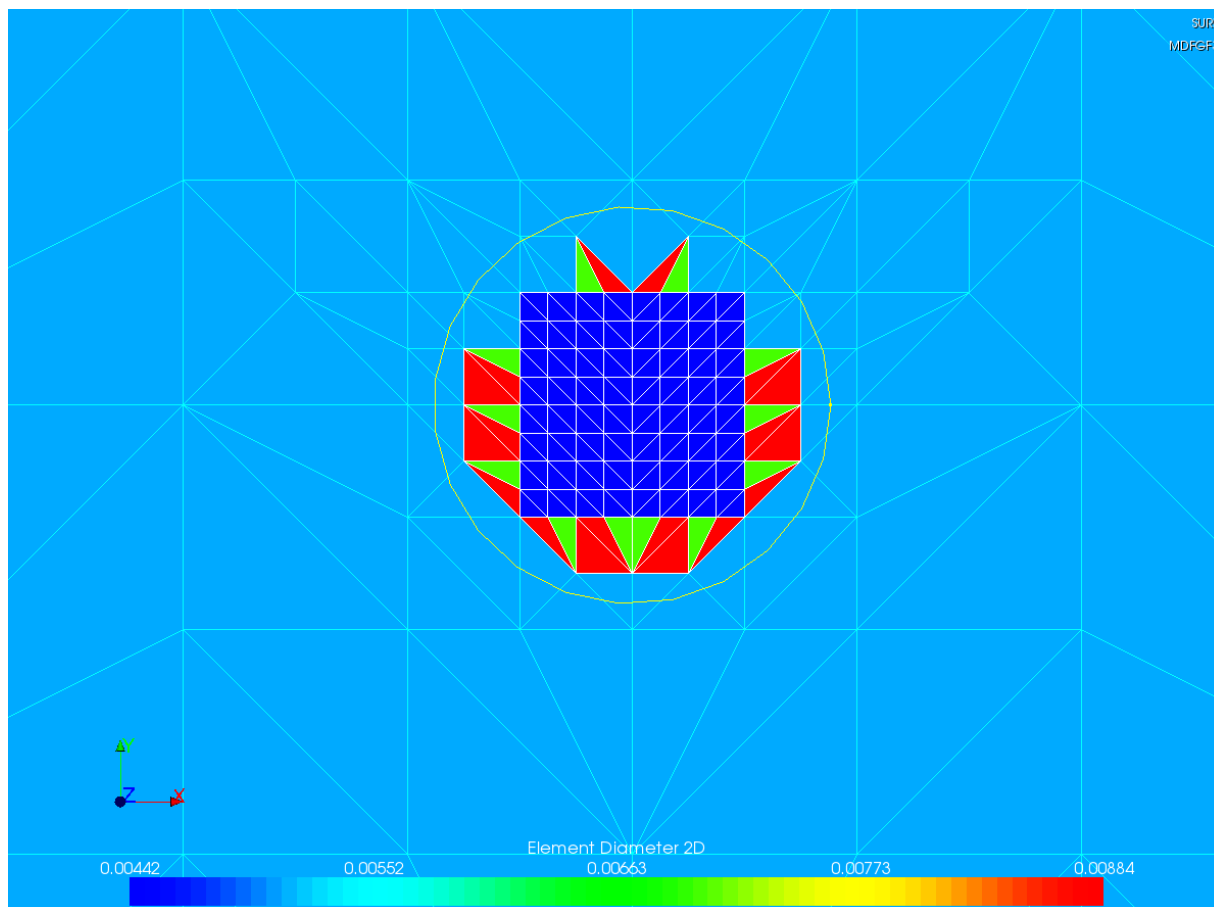


Figure 4.2-3: refined healthy grid - zoom around the bottom of crack n°1 of the crack centered in A and map of sizes in the zone of interest

4.3 Sizes tested and results

One tests the value of the minimal size of the meshes.

| Identification | Reference | Type of reference | % tolerance |
|----------------|-----------|-------------------|-------------|
| $\min(h)$ | 0,004419 | 'ANALYTICAL' | 0,1% |

This test will be improved when one can better test the cards.

5 Modeling C

5.1 Characteristics of modeling

The interfaces and the cracks are defined by level sets.
The indicator by zone provided by RAFF_XFEM is used.

The interest of this modeling is to present succeeded methodology the pus, which uses the indicator by zone.

5.2 Characteristics of the grid

The initial grid is the same one as that of modeling B. After refinement, one obtains the refined grid presented on Figure 5.2-1. It is interesting to note the good regularity of the sizes of the meshes in the zone of interest (see Figure 5.2-2).

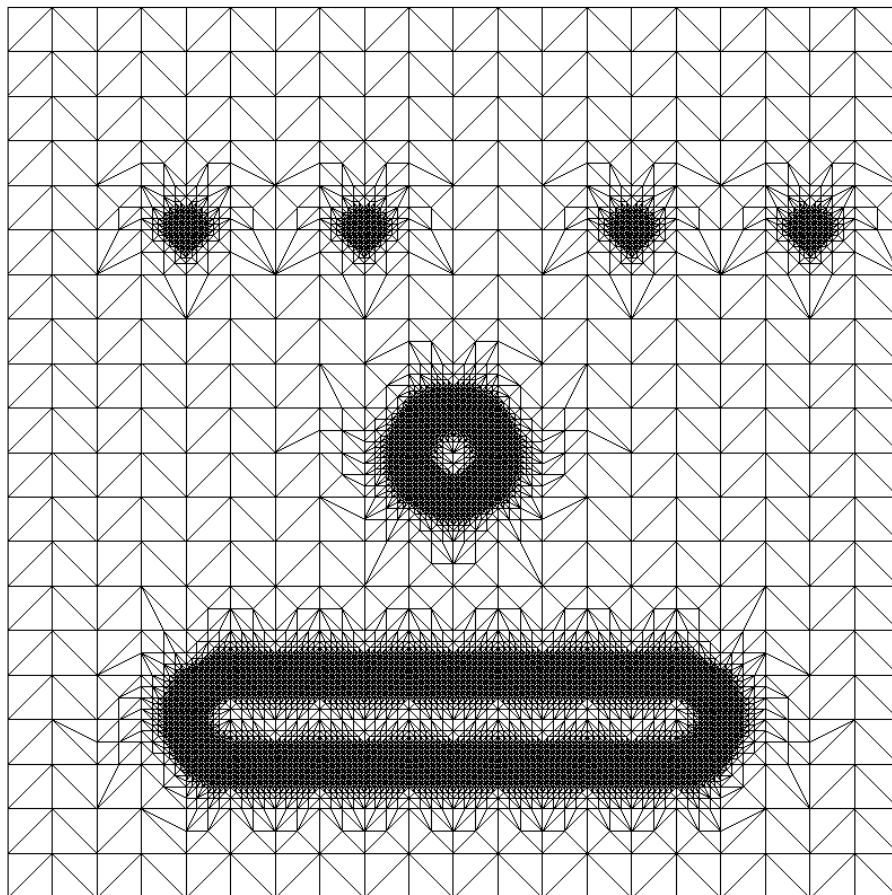


Figure 5.2-1: refined healthy grid

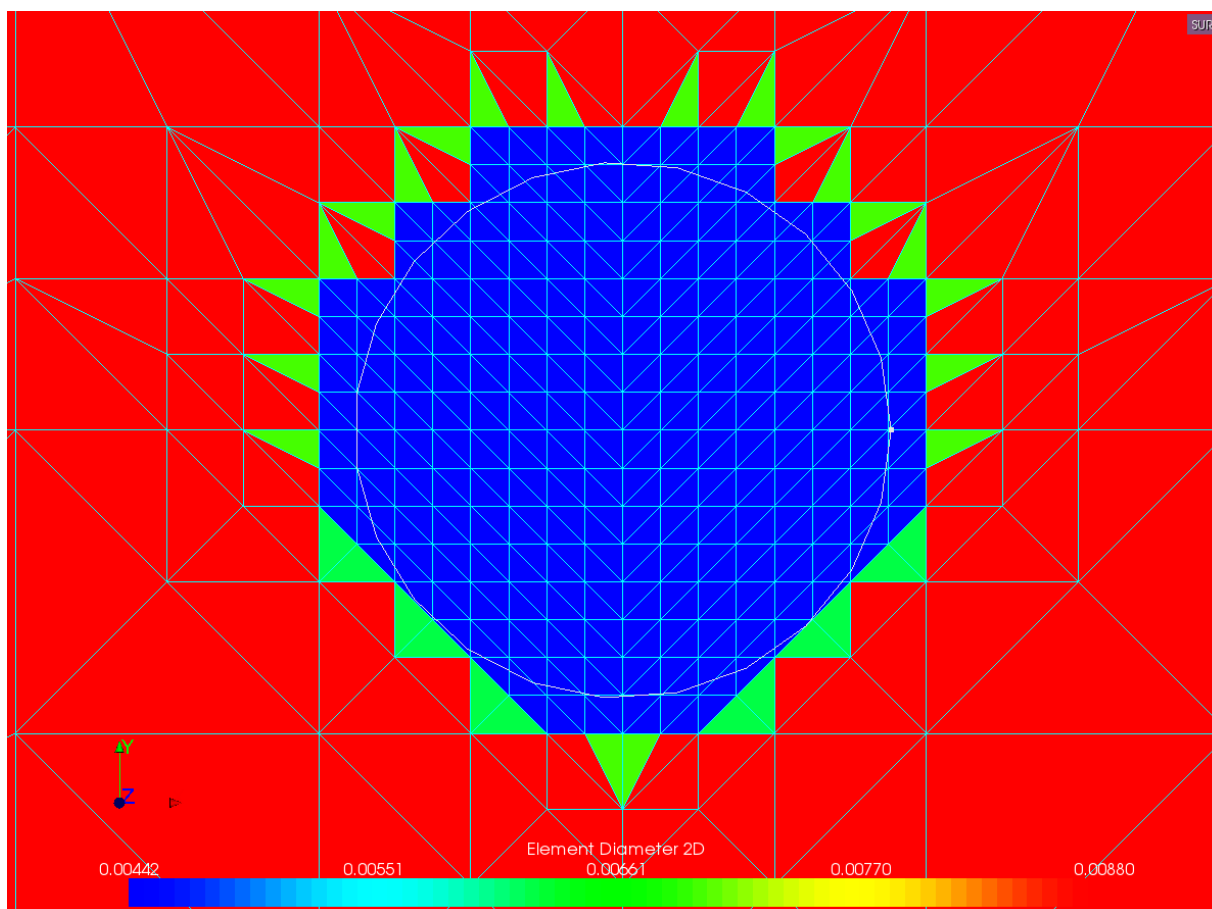


Figure 5.2-2: refined healthy grid - zoom around the bottom of crack n°1 of the crack centered in A and map of sizes in the zone of interest

5.3 Sizes tested and results

One tests the value of the minimal size of the meshes.

| Identification | Reference | Type of reference | % tolerance |
|----------------|-----------|-------------------|-------------|
| $\min(h)$ | 0,004419 | 'ANALYTICAL' | 0,1% |

This test will be improved when one can better test the cards.

6 Summary of the results

This test made it possible to validate the macro-order RAFF_XFEM.
This order is also validated for interfaces.